IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper N

Difficulty Rating: 3.9850/0.6948

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 20 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

The straight line L_1 passes through the points $A(1,4)$ and $B(3,9)$.

a) Find an equation for L_1, giving the answer in the form $ax+by+c=0$, where a, b and c are integers. **(3)**

The straight line L_2 is perpendicular to L_1 and passes through the points B and C.

Given the point C has coordinates $(13,k)$, find …

b) … the value of k. **(4)**

c) … the area of the triangle ABC. **(4)**

Question 2

The sum of £840 is to be shared equally amongst n qualifying individuals.

It was later found that 6 of those n individuals did not actually qualify so the share of the rest increased by £45.

Find the value of n. **(7)**

Question 3

It is asserted that

" The difference of the squares of two non consecutive positive integers can never be a prime number ".

a) Prove the validity of the above assertion. **(3)**

The difference between two consecutive square numbers is 163.

b) Given further that 163 is a prime number find the above mentioned consecutive square numbers. **(4)**

Question 4

$$x^2 + y^2 - 10x + 4y + 9 = 0$$

The circle with the above equation has radius r and has its centre at the point C.

a) Determine the value of r and the coordinates of C. **(3)**

b) Find the coordinates of the points where the circle intersects the x axis. **(2)**

The point $P(3,2)$ lies on the circle.

c) Show that an equation of the tangent to the circle at P is

$$x - 2y + 1 = 0.$$ **(4)**

Question 5

A curve has equation

$$y = x - 8\sqrt{x}, \ x \in \mathbb{R}, \ x \geq 0.$$

The curve meets the x axis at the origin and at the point P.

a) Determine the coordinates of P. **(3)**

The point Q, where $x = 4$, lies on the curve. **(6)**

b) Find an equation of the normal to curve at Q.

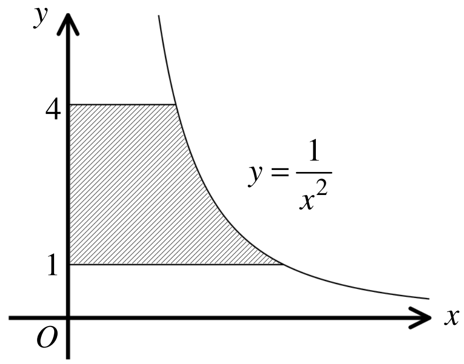
c) Show that the normal to the curve at Q does not meet the curve again. **(4)**

Question 6

$$T = 240 - 5 + 237 - 5 + 234 - 5 + 231 - \ ... + 6 - 5 + 3 - 5.$$

Show clearly that $T = 9320$. **(5)**

Question 7



The figure above shows part of the graph of the curve with equation

$$y = \frac{1}{x^2}, \; x \in \mathbb{R}, \; x > 0.$$

Find the area bounded by the curve, the y axis, and the straight lines with equations $y = 1$ and $y = 4$. **(7)**

Question 8

The algebraic expression $\frac{1}{\sqrt[3]{1+x}}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Expand $\frac{1}{\sqrt[3]{1+x}}$ up and including the term in x^3. **(3)**

b) Use the expansion of part **(a)** to find the expansion of $\left(1+\frac{3}{4}x\right)^{-\frac{1}{3}}$ up and including the term in x^3. **(2)**

c) Hence find the expansion of $\sqrt[3]{\frac{256}{4+3x}}$ up and including the term in x^3. **(3)**

Question 9

Solve the following equation

$$\sum_{r=0}^{\infty} (\sin x)^{2r} = 2\tan x .$$

You may assume that the left hand side of the equation converges. **(8)**

Question 10

Solve the differential equation

$$\frac{dy}{dx} = y^2\sqrt{x} , \ x \neq 0 , \ y \neq 0 ,$$

with $y = -2$ at $x = 1$.

Give the answer in the form $y = \dfrac{A}{1 + Bx^{\frac{3}{2}}}$, where A and B are integers. **(7)**

Question 11

The function f is given by

$$f : x \mapsto 3 + \frac{2}{x-2}, \ x \in \mathbb{R}, \ x > 2 .$$

a) Sketch a detailed graph of f. **(4)**

b) Find an expression for $f^{-1}(x)$ as a single fraction, in its simplest form. **(5)**

c) Find the domain and range of $f^{-1}(x)$. **(3)**

d) Find the value of x that satisfy the equation $f(x) = f^{-1}(x)$ **(4)**

Question 12

Relative to a fixed origin, the points P and Q have position vectors $9\mathbf{j}-2\mathbf{k}$ and $7\mathbf{i}-8\mathbf{j}+11\mathbf{k}$, respectively.

a) Find the distance between the points P and Q. **(2)**

b) Find the position vector of the point M, where M is the midpoint of PQ. **(2)**

The points P and Q are vertices of a cube, so that PQ is one of the longest diagonals of the cube.

c) Show that the length of one of the sides of the cube is 13 units. **(4)**

d) Show that the origin O lies inside the cube. **(3)**

Question 13

The obtuse angles θ and φ satisfy the equation

$$\sin 6\theta° + \cos 4\varphi° = -2.$$

Find the possible values of θ and φ. **(8)**

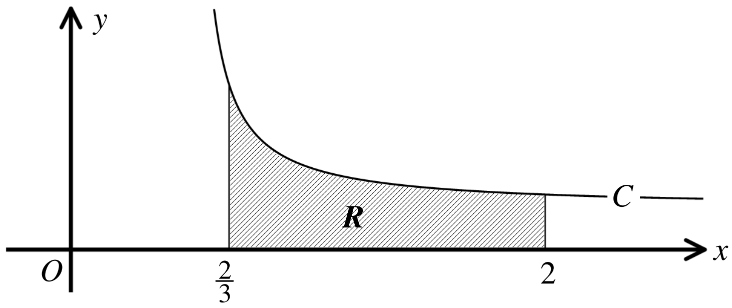
Question 14

Find, in exact form where appropriate, the solutions for each of the following equations.

a) $e^{1-x} = 3e$. **(4)**

b) $e^{w} - 3 = \dfrac{8}{e^{w}-1}$. **(6)**

Question 15



The figure above shows the curve with parametric equations

$$x = \frac{1}{1+t}, \quad y = \frac{1}{1-t}, \quad -1 < t < 1.$$

The region R, shown shaded in the figure, is bounded by the curve, the x axis and the straight lines with equations $x = \frac{2}{3}$ and $x = 2$.

a) Show that the area of R can be found by the parametric integral

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{(1-t)(1+t)^2} \, dt,$$

and hence find the exact area of R. **(10)**

b) Determine a Cartesian equation of the curve, in the form $y = f(x)$, and by evaluating a suitable integral in Cartesian verify the answer given to part **(a)**. **(10)**

Question 16

$$y = 3\tan^3 2x, \ x \in \mathbb{R}, \ x \neq \frac{1}{4}n\pi.$$

Determine, by showing detailed workings, the value of $\frac{dy}{dx}$, at $x = \arctan\frac{1}{2}$. **(10)**

Question 17

A curve C is given by the implicit equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$

a) Show clearly that

$$\frac{dy}{dx} = -\frac{x+2y}{2x+2y}. \qquad (4)$$

b) Find the coordinates of the turning points of C. **(5)**

c) Show further that

$$1 + 4\frac{dy}{dx} + 2\left(\frac{dy}{dx}\right)^2 + 2(x+y)\frac{d^2y}{dx^2} = 0. \qquad (4)$$

d) Hence determine the nature of these turning points. **(3)**

Question 18

Use a suitable trigonometric manipulation to find an exact simplified answer for the following integral.

$$\int_0^{\frac{\pi}{3}} \frac{1}{\left(\cos x + \sqrt{3}\sin x\right)^2}\, dx. \qquad (12)$$

Question 19

Prove by first principles, and by using the small angle approximations for $\sin x$ and $\cos x$, that

$$\frac{d}{dx}(\tan x) = \sec^2 x. \qquad (7)$$

Question 20

$$ax^3 + ax^2 + ax + b = 0,$$

where a and b are non zero real constants.

Given that $x = b$ is a root of the above equation, determine the range of the possible values of a.

(8)

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