IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper O

Difficulty Rating: 4.1800/0.7692

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 18 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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Question 1

The points A, B and C have coordinates $(-6,5)$, $(0,7)$ and $(8,3)$, respectively.

The straight line L_1 is parallel to BC and passes through the point A.

a) Show that an equation for L_1 is

$$x+2y=4. \qquad (4)$$

The straight line L_2 passes through the point C is perpendicular to BC.

b) Find an equation for L_2. **(2)**

The lines L_1 and L_2 meet at the point D.

c) Show that the distance BD is 10 units. **(4)**

Question 2

A function is defined by

$$f(x)=\sqrt{e^x-1}, \ x \geq 0.$$

a) Find, showing detailed workings, the value of …

i. … $f(\ln 5)$. **(2)**

ii. … $f'(\ln 5)$. **(4)**

The inverse function of $f(x)$ is $g(x)$.

b) Determine an expression for $g(x)$. **(3)**

c) State, with justification, the value of $g'(2)$. **(2)**

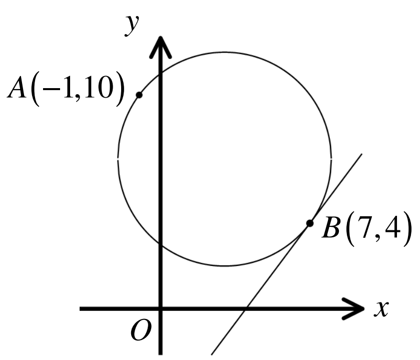
Question 3

$$f(x) = 2x^2 + 9x - 5$$

a) Given that when $f(x)$ is divided by $(2x-k)$ the remainder is 13, find the possible values of k. **(4)**

b) Given further that when $f(x)$ is divided by $(x-2k)$ the remainder is 121, find the value of k. **(4)**

Question 4



The figure above shows a circle that passes through the points $A(-1,10)$ and $B(7,4)$.

a) Given that AB is a diameter of the circle show that an equation for this circle is given by

$$x^2 + y^2 - 6x - 14y + 33 = 0.$$ **(4)**

The tangent to the circle at B meets the y axis at the point D.

b) Show that the coordinates of D are $\left(0, -\frac{16}{3}\right)$. **(5)**

Question 5

Find the solutions of the equation

$$\left(x^2-x-3\right)^2-12\left(x^2-x-3\right)+27=0. \qquad \textbf{(6)}$$

Question 6

The algebraic expression $\sqrt[3]{1-3x}$ is to be expanded as an infinite convergent series, in ascending powers of x.

a) Find the first 4 terms in the series expansion of $\sqrt[3]{1-3x}$. **(3)**

b) State the range of values of x for which the expansion is valid. **(1)**

c) By substituting a suitable value for x in the expansion, show **clearly** that

$$\sqrt[3]{997}\approx 9.989989983. \qquad \textbf{(4)}$$

Question 7

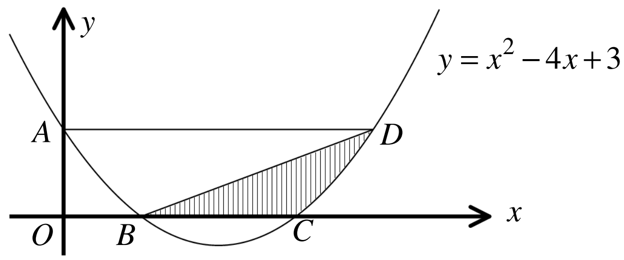
Show that the value of x in the following expression

$$\ln x=\frac{3\ln 2}{2\ln 2-1}$$

satisfies the logarithmic equation

$$2+\log_2 x=2\ln\left(\frac{x}{\sqrt{e}}\right), \quad x>0. \qquad \textbf{(8)}$$

Question 8



The figure above shows a quadratic curve with equation

$$y = x^2 - 4x + 3.$$

The points A, B and C are the points where the curve meets the coordinate axes.

The point D lies on the curve so that AD is parallel to the x axis.

Calculate the exact area of the shaded region, bounded by the curve, the x axis and the straight line segment BD.

(9)

Question 9

A geometric series has first term a and common ratio r.

The ratio of the sum of the first 5 terms of the series, to the sum of the reciprocals of the first 5 terms of the series, is 49.

Given further that the sum of the first and third term of the series is 35, determine the value of a and the two possible values of r.

(9)

Question 10

A curve has equation $y = f(x)$ given by

$$f(x) = \frac{3x-1}{x+2}, \ x \in \mathbb{R}, \ x \neq 2.$$

a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of …

- … all the points where the curve meets the coordinate axes.
- … the equations of the two asymptotes of the curve. **(4)**

A different curve has equation $y = g(x)$ given by

$$g(x) = \frac{1}{x} + k, \ x \in \mathbb{R}, \ x \neq 0, \text{ where } k \text{ is a constant.}$$

The graph of $f(x)$ meets the graph of $g(x)$ at the points A and B.

b) Given that A lies on the x axis determine …

i. … the value of k.

ii. … the coordinates of B. **(7)**

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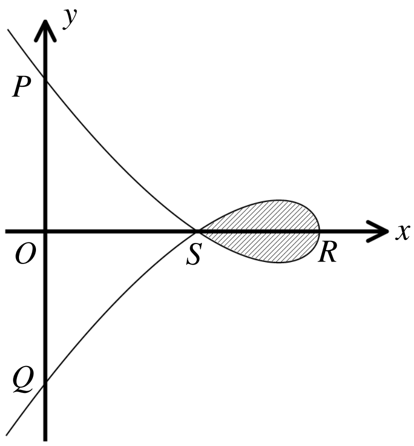
Question 11



The figure above shows the **re-entrant** curve C with parametric equations

$$x = 27 - 3t^2, \quad y = 5t\left(4 - t^2\right), \quad t \in \mathbb{R}.$$

The curve meets the y axis at P and Q, and the x axis at R and S.

a) Determine …

i. … the value of t at the points P, Q, R and S. **(4)**

ii. … the Cartesian coordinates of the points P, Q, R and S. **(4)**

b) Given that C is symmetrical about the x axis, show that the area enclosed by the loop of C, shown shaded in the figure above, is 256 square units. **(5)**

c) Find a Cartesian equation of C, in the form $y^2 = f(x)$. **(4)**

Question 12

$$y = \arcsin x, \quad -1 \le y \le 1.$$

a) Show clearly that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}. \qquad (5)$$

The point $P\left(\frac{1}{6}, k\right)$, where k is a constant, lies on the curve with equation

$$\arcsin 3x + 2\arcsin y = \frac{\pi}{2}, \; |x| \le \frac{1}{3}, \; |y| \le 1.$$

b) Find the value of the gradient at P. **(5)**

Question 13

$$I = \int_1^2 \frac{1}{x^2 - x\sqrt{x^2-1}} \, dx.$$

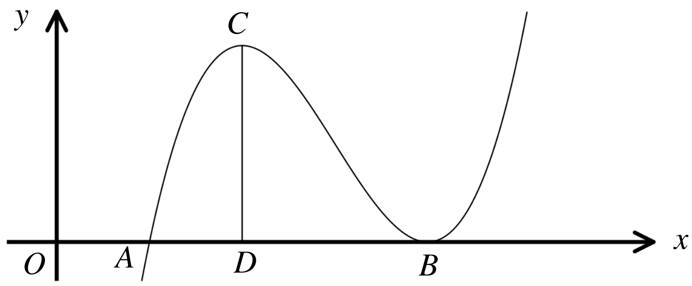
a) Show that the substitution $x = \sec\theta$ transforms I to

$$I = \int_0^{\frac{1}{3}\pi} \frac{\tan\theta}{\sec\theta - \tan\theta} \, d\theta. \qquad (4)$$

b) Hence use trigonometric identities to show that

$$I = 1 + \sqrt{3} - \frac{1}{3}\pi. \qquad (7)$$

Question 14



The figure above shows a cubic curve whose coefficient of x^3 is 1.

The curve crosses the x axis at $A(a,0)$ and touches the x axis at $B(b,0)$, where a and b are positive constants.

The point C is a local maximum of the curve.

The point D lies on the x axis so that CD is parallel to the y axis.

Show, with a detailed method, that

$$|AB| = 3|AD|.$$ **(10)**

Question 15

$$y = \frac{1+\cos x}{1+\sin x}, \; 0 \le x < 2\pi, \; x \ne \frac{3}{2}\pi.$$

Determine, with full justification, the coordinates of the minimum point of y. **(14)**

Question 16

Water is leaking out of a hole at the base of a cylindrical barrel with constant cross sectional area and a height of 1 m.

It is given that t minutes after the leaking started, the volume of the water left in the barrel is $V \text{ m}^3$, and its height is $h \text{ m}$.

It is assumed that the water is leaking out, in m^3 per minute, at a rate proportional to the square root of the volume of the water left in the barrel.

a) Show clearly that

$$\frac{dh}{dt} = -B\sqrt{h},$$

where B is a positive constant. **(4)**

The barrel was initially full and 5 minutes later half its contents have leaked out.

b) Solve the differential equation to show that

$$\sqrt{h} = 1 - \frac{1}{10}\left(2-\sqrt{2}\right)t.$$ **(7)**

c) Show further that

$$t = 5\left(2+\sqrt{2}\right)\left(1-\sqrt{h}\right).$$ **(4)**

d) If T is the time taken for the barrel to empty, find h when $t = \frac{1}{2}T$. **(4)**

Question 17

Differentiate from first principles

$$\frac{x}{x+1}, \quad x \neq -1.$$ **(9)**

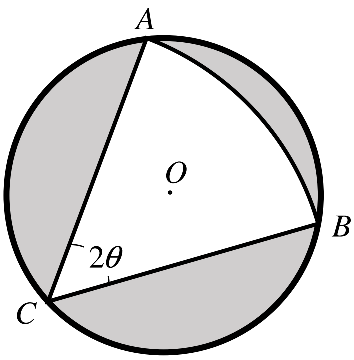
Question 18

$$2x\tan x = 1, \ x \neq \frac{1}{2}n\pi, \ n \in \mathbb{N}$$

a) Show that the above equation has a solution in the interval $(0.6, 0.7)$. **(2)**

b) Use the Newton Raphson method to find the solution of this equation, correct to 5 decimal places. **(7)**

The figure below shows a circle, centre at O. The points A, B and C lie on the circumference of this circle. A circular sector ABC, subtending an angle of 2θ at C, is inscribed in this circle.



c) Determine the greatest proportion of the area of the circle, which can be covered by this sector.

You may give the answer as a percentage, correct to two decimal places **(12)**