IYGB GCE

Mathematics SYN

Advanced Level

Synoptic Paper T

Difficulty Rating: 4.2200/0.7865

Time: 3 hours

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This synoptic practice paper follows closely the Advanced Level Pure Mathematics Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. (**2**).
There are 21 questions in this question paper.
The total mark for this paper is 200.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The points A and B have coordinates $(1,1)$ and $(5,7)$, respectively.

a) Find an equation for the straight line l_1 which passes through A and B. **(3)**

The straight line l_2 with equation

$$2x+3y=18$$

meets l_1 at the point C.

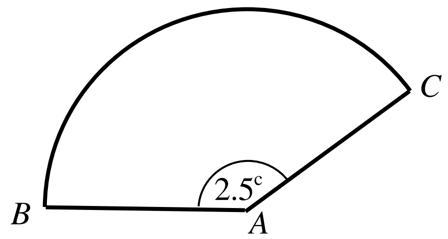
b) Determine the coordinates of C. **(3)**

The point D, where $x=-3$, lies on l_2.

c) Show clearly that

$$|AD|=|BD|.$$ **(4)**

Question 2



The figure above shows a circular sector ABC subtending an angle of 2.5 radians at the point A.

Given that the area of the sector is 45 cm^2, find its perimeter. **(4)**

Question 3

A geometric series has first term 20480 and its sum to infinity is 81920.

a) Show that the common ratio of the series is $\frac{3}{4}$. **(2)**

b) Calculate the difference between the fifth and the sixth term of the series. **(2)**

c) Determine the smallest number of terms that should be added so that their total **exceeds** 80000. **(5)**

Question 4

A circle C has equation

$$4x^2 + 4y^2 - 8x + 24y - 5 = 0$$

a) Find the coordinates of the centre of the circle. **(3)**

b) Determine the size of the radius of the circle, giving the answer in the form $k\sqrt{5}$, where k is a rational constant. **(1)**

The point P has coordinates $(8,11)$.

The straight line L passes through P and touches the circle at the point Q.

c) Calculate the distance PQ. **(4)**

Question 5

Solve the following trigonometric equation

$$\tan x + \cot x = 8\cos 2x, \quad 0 \le x < \pi,$$

where x is measured in radians. **(8)**

Question 6

The polynomials $f(x)$ and $g(x)$ are defined in terms of the constants a and b

$$f(x) = a(x^3 + 1) - bx(x+1)$$

$$g(x) = bx^3 - 5x^2 - 2a(x-1).$$

a) Given that $(x-2)$ is a factor of **both** $f(x)$ and $g(x)$, determine the value of a and the value of b. **(4)**

b) Factorize both $f(x)$ and $g(x)$, and hence show that $f(x)$ and $g(x)$, have another linear common factor. **(4)**

Question 7

$$f(x) = \frac{12}{\sqrt{1-2x}}, \quad x \in \mathbb{R}, \quad x \le \frac{1}{2}.$$

Use a quadratic approximation for $f(x)$ to solve the equation

$$f(x) = 16 - 67x - 2x^2.$$ **(7)**

Question 8

A curve has parametric equations

$$x = 1 - \cos\theta, \quad y = \sin\theta \sin 2\theta \quad , \quad 0 \le \theta \le \pi .$$

Determine in exact form the coordinates of the stationary points of the curve.

No credit will be given for methods involving a Cartesian form of this curve. **(10)**

Question 9

A sequence u_1, u_2, u_3, u_4, u_5 ... satisfies

$$u_{n+1} = Au_n + B,$$

where A and B are non zero constants.

The second and third term of this sequence are 464 and 428, respectively.

Given further that the sequence converges to 320, find the value of the fourth term of this sequence. **(8)**

Question 10

The curve C has implicit equation

$$y = xe^y, \ x \neq 0, \ y \neq 1, \ y \neq 2.$$

Show clearly that

$$(1-y)\frac{d^2y}{dx^2} = (2-y)\left(\frac{dy}{dx}\right)^2.$$ **(10)**

Question 11

The point P lies on the curve with equation

$$xy = e^x, \quad xy > 0.$$

The tangent to the curve at P passes through the origin O.

Determine the coordinates of P. **(9)**

I.Y.G.B. www.madasmaths.com

Question 12

Find the solution interval for the following inequality.

$$x(x-4) < |5x-16|-4\,. \qquad \textbf{(11)}$$

Question 13

The function f is defined in the largest real domain by

$$f(x) \equiv (\ln x)^2, \quad x \in (0,\infty).$$

a) Sketch the graph of $f(x)$. $x \in (0,\infty)$ **(3)**

The function g is defined as

$$g(x) \equiv \ln x, \quad x \in (0,\infty).$$

b) Determine in exact simplified form the area of the finite region bounded by the graph of f and the graph of g.

You may assume that $\int \ln x \, dx = x\ln x - x + \text{constant}$. **(8)**

Question 14

Solve the trigonometric equation

$$\arcsin x + \arccos\frac{3}{5} = 2\arctan\frac{3}{4}\,. \qquad \textbf{(8)}$$

Question 15

Solve the following equation.

$$\frac{2+\sqrt{2}\,x}{x^2+\sqrt{2}\,x+1}+\frac{2-\sqrt{2}\,x}{x^2-\sqrt{2}\,x+1}=2, \quad x \in \mathbb{R}. \qquad (8)$$

Question 16

Solve the following logarithmic equation

$$5 \times 5^{\log x}+5^{2-\log x}=30, \quad x>0. \qquad (8)$$

Question 17

Water is leaking out of a hole at the base of a cylindrical barrel with constant cross sectional area and a height of H m.

It is given that t minutes after the leaking started, the volume of the water left in the barrel is V m^3, and the height of the water is h m.

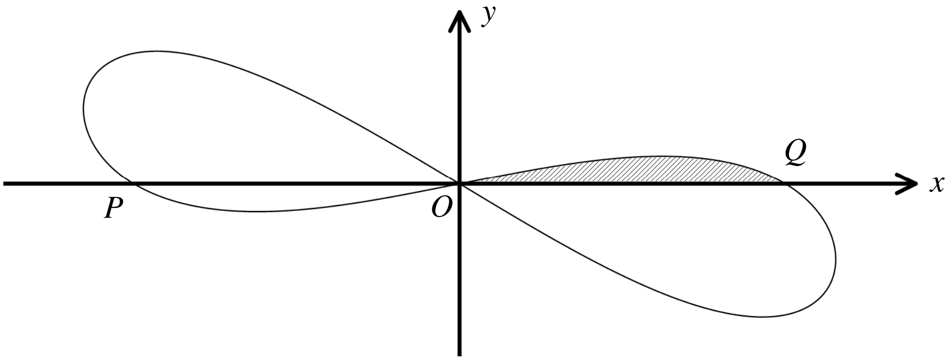
It is assumed that the water is leaking out, in m^3 per minute, at a rate proportional to the square root of the height of the water still left in the barrel.

The barrel was initially full and T minutes later all the water has leaked out.

Show by a complete calculus method that

$$h=H\left(1-\frac{t}{T}\right)^2, \quad 0 \le t \le T. \qquad (12)$$

Question 18



The figure above shows a curve with parametric equations

$$x = \cos\theta, \ \ y = \sin 2\theta - \cos\theta, \ \ 0 \le \theta < 2\pi .$$

The curve, which has rotational symmetry about the origin O, crosses the x axis at the points P, Q and O.

The finite region bounded by the curve, for which $x \ge 0$, $y \ge 0$, and the x axis is shown shaded in the figure.

Show, with detailed workings, that …

a) … the area of shaded region is $\frac{5}{24}$. **(10)**

b) … the area enclosed by the two loops of the curve is $\frac{8}{3}$. **(6)**

c) … a Cartesian equation of the curve is

$$4x^2\left(1 - x^2\right) = \left(x + y\right)^2 .$$ **(4)**

Question 19

A cubic curve with equation

$$y = x^3 - 3x^2 - 9x + 3, \quad x \in \mathbb{R},$$

is odd about some point P.

Find the coordinates of P and use transformation arguments to justify the assertion that the curve is odd about P. **(8)**

Question 20

It is given that

$$\int_1^k \frac{\left(\sqrt{3}k + \sqrt{3x}\right)^2}{kx^3}\, dx = a - \sqrt{k},$$

where a and k are integers.

Use algebra to determine the value of a. **(12)**

Question 21

The point P, whose y coordinate is 2, lies on the curve with equation

$$y = \frac{k + 8x\sqrt{x}}{12x}, \quad x \in \mathbb{R}, \quad x > 0,$$

where k is a non zero constant.

The tangent to the curve at P is parallel to the straight line with equation

$$6x + y = 17.$$

Determine the value of k. **(11)**
