# ALGEBRAIC FRACTIONS

# SIMPLIFYING RATIONAL EXPRESSIONS

**Question 1**

Simplify the following algebraic fractions.

**a)** $\dfrac{x^2-4x}{x^2-6x+8}$

**b)** $\dfrac{y^2+2y-15}{y^2-7y+12}$

**c)** $\dfrac{t^2+12t+36}{t^2+t-30}$

**d)** $\dfrac{w^2+4w-12}{w^2+9w+18}$

$\boxed{\dfrac{x}{x-2}}, \boxed{\dfrac{y+5}{y-4}}, \boxed{\dfrac{t+6}{t-5}}, \boxed{\dfrac{w-2}{w-3}}$

(a) $\dfrac{x^2-4x}{x^2-6x+8} = \dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}(x-2)} = \dfrac{x}{x-2}$ //

(b) $\dfrac{y^2+2y-15}{y^2-7y+12} = \dfrac{(y+5)\cancel{(y-3)}}{\cancel{(y-3)}(y-4)} = \dfrac{y+5}{y-4}$ //

(c) $\dfrac{t^2+12t+36}{t^2+t-30} = \dfrac{\cancel{(t+6)}(t+6)}{\cancel{(t+6)}(t-5)} = \dfrac{t+6}{t-5}$ //

(d) $\dfrac{w^2+4w-12}{w^2+9w+18} = \dfrac{(w-2)\cancel{(w+6)}}{(w+3)\cancel{(w+6)}} = \dfrac{w-2}{w+3}$ //

**Question 2**

Simplify the following algebraic fractions.

**a)** $\dfrac{x^2+10x+24}{x^2+5x-6}$

**b)** $\dfrac{y^2-7y+12}{y^2-8y+15}$

**c)** $\dfrac{t^2-6t-16}{t^2-11t+24}$

**d)** $\dfrac{w^2-3w-40}{w^2-12w+32}$

$\boxed{\dfrac{x+4}{x-1}}, \boxed{\dfrac{y-4}{y-5}}, \boxed{\dfrac{t+2}{t-3}}, \boxed{\dfrac{w+5}{w-4}}$

(a) $\dfrac{x^2+10x+24}{x^2+5x-6} = \dfrac{(x+4)\cancel{(x+6)}}{(x-1)\cancel{(x+6)}} = \dfrac{x+4}{x-1}$

(b) $\dfrac{y^2-7y+12}{y^2-8y+15} = \dfrac{\cancel{(y-3)}(y-4)}{\cancel{(y-3)}(y-5)} = \dfrac{y-4}{y-5}$

(c) $\dfrac{t^2-6t-16}{t^2-11t+24} = \dfrac{\cancel{(t-8)}(t+2)}{\cancel{(t-8)}(t-3)} = \dfrac{t+2}{t-3}$

(d) $\dfrac{w^2-3w-40}{w^2-12w+32} = \dfrac{(w+5)\cancel{(w-8)}}{(w-4)\cancel{(w-8)}} = \dfrac{w+5}{w-4}$

**Question 3**

Simplify the following algebraic fractions:

**a)** $\dfrac{x^2-4x}{x^2-5x+4}$

**b)** $\dfrac{y^2+4y-12}{y^2+8y-20}$

**c)** $\dfrac{t^2-6t-16}{t^2+7t+10}$

**d)** $\dfrac{w^2-36}{w^2+10w+24}$

$\boxed{\dfrac{x}{x-1}}$ , $\boxed{\dfrac{y+6}{y+10}}$ , $\boxed{\dfrac{t-8}{t+5}}$ , $\boxed{\dfrac{w-6}{w+4}}$

(a) $\dfrac{x^2-4x}{x^2-5x+4} = \dfrac{x\cancel{(x-4)}}{(x-1)\cancel{(x-4)}} = \dfrac{x}{x-1}$ //

(b) $\dfrac{y^2+4y-12}{y^2+8y-20} = \dfrac{\cancel{(y-2)}(y+6)}{\cancel{(y-2)}(y+10)} = \dfrac{y+6}{y+10}$ //

(c) $\dfrac{t^2-6t-16}{t^2+7t+10} = \dfrac{\cancel{(t+2)}(t-8)}{\cancel{(t+2)}(t+5)} = \dfrac{t-8}{t+5}$ //

(d) $\dfrac{w^2-36}{w^2+10w+24} = \dfrac{(w-6)\cancel{(w+6)}}{\cancel{(w+6)}(w+4)} = \dfrac{w-6}{w+4}$ //

**Question 4**

Simplify the following algebraic fractions:

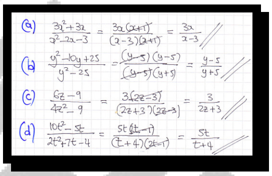
**a)** $\dfrac{3x^2+3x}{x^2-2x-3}$

**b)** $\dfrac{y^2-10y+25}{y^2-25}$

**c)** $\dfrac{6z-9}{4z^2-9}$

**d)** $\dfrac{10t^2-5t}{2t^2+7t-4}$

$\boxed{\dfrac{3x}{x-3}}, \boxed{\dfrac{y-5}{y+5}}, \boxed{\dfrac{3}{2z+3}}, \boxed{\dfrac{5t}{t+4}}$



(a) $\dfrac{3x^2+3x}{x^2-2x-3} = \dfrac{3x(x+1)}{(x-3)(x+1)} = \dfrac{3x}{x-3}$

(b) $\dfrac{y^2-10y+25}{y^2-25} = \dfrac{(y-5)(y-5)}{(y-5)(y+5)} = \dfrac{y-5}{y+5}$

(c) $\dfrac{6z-9}{4z^2-9} = \dfrac{3(2z-3)}{(2z+3)(2z-3)} = \dfrac{3}{2z+3}$

(d) $\dfrac{10t^2-5t}{2t^2+7t-4} = \dfrac{5t(2t-1)}{(t+4)(2t-1)} = \dfrac{5t}{t+4}$

## Question 5

Simplify the following algebraic fractions:

**a)** $$\frac{2x^2-50}{8x^2-39x-5}$$

**b)** $$\frac{12y^2-36y+15}{12y^2-3}$$

**c)** $$\frac{2z^2-3z+1}{z^2-1}$$

**d)** $$\frac{5w^2+14w-3}{50w^2-2}$$

$$\boxed{\frac{2(x+5)}{8x-1}}, \quad \boxed{\frac{2y-5}{2y+1}}, \quad \boxed{\frac{2z-1}{z+1}}, \quad \boxed{\frac{w+3}{2(5w+1)}}$$

(a) $$\frac{2x^2-50}{8x^2-39x-5} = \frac{2(x^2-25)}{(8x+1)(x-5)} = \frac{2\cancel{(x-5)}(x+5)}{(8x+1)\cancel{(x-5)}} = \frac{2(x+5)}{8x+1}$$

(b) $$\frac{12y^2-36y+15}{12y^2-3} = \frac{\cancel{3}(4y^2-12y+5)}{\cancel{3}(4y^2-1)} = \frac{\cancel{(2y-1)}(2y-5)}{\cancel{(2y-1)}(2y+1)} = \frac{2y-5}{2y+1}$$

(c) $$\frac{2z^2-3z+1}{z^2-1} = \frac{\cancel{(z-1)}(2z-1)}{\cancel{(z-1)}(z+1)} = \frac{2z-1}{z+1}$$

(d) $$\frac{5w^2+14w-3}{50w^2-2} = \frac{(5w-1)(w+3)}{2(25w^2-1)} = \frac{\cancel{(5w-1)}(w+3)}{2\cancel{(5w-1)}(5w+1)} = \frac{w+3}{2(5w+1)}$$

**Question 6**

Simplify the following algebraic fractions:

**a)** $\dfrac{3x^2-7x+4}{x^2-1}$

**b)** $\dfrac{y^3-8}{3y^2-8y+4}$

**c)** $\dfrac{20-5w}{6w^2-24w}$

**d)** $\dfrac{2t^3+t^2-13t+6}{t^2+t-6}$

$\boxed{\dfrac{3x-4}{x+1}}$, $\boxed{\dfrac{y^2+2y+4}{3y-2}}$, $\boxed{-\dfrac{5}{6w}}$, $\boxed{2t+1}$

(a) $\dfrac{3x^2-7x+4}{x^2-1} = \dfrac{(3x-4)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \dfrac{3x-4}{x+1}$ //

(b) $\dfrac{y^3-8}{3y^2-8y+4} = \dfrac{\cancel{(y-2)}(y^2+2y+4)}{(3y-2)\cancel{(y-2)}} = \dfrac{y^2+2y+4}{3y-2}$

(c) $\dfrac{20-5w}{6w^2-24w} = \dfrac{5(4-w)}{6w(w-4)} = \dfrac{-5(-4+w)}{6w(w-4} = \dfrac{-5\cancel{(w-4)}}{6w\cancel{(w-4)}} = -\dfrac{5}{6w}$ //

(d) $\dfrac{2t^3+t^2-13t+6}{t^2+t-6} = \dfrac{(2t-1)\cancel{(t-2)}\cancel{(t+3)}}{\cancel{(t-2)}\cancel{(t+3)}} = 2t-1$ //

**Question 7**

Simplify the following algebraic fractions:

**a)** $\dfrac{4x^2+4x-3}{4x^3-x}$

**b)** $\dfrac{y^3-8}{3y^2-8y+4}$

**c)** $\dfrac{20-5w}{6w^2-24w}$

**d)** $\dfrac{(2t+1)(3t^2-6t)}{6t^3-19t^2+9t+10}$

$\boxed{\dfrac{2x+3}{x(2x+1)}}$, $\boxed{\dfrac{y^2+2y+4}{3y-2}}$, $\boxed{-\dfrac{5}{6w}}$, $\boxed{\dfrac{3t}{3t-5}}$

# ADDING ALGEBRAIC FRACTIONS

**Question 1**

Simplify the following expressions into a single fraction.

**a)** $\dfrac{1}{x}+\dfrac{2}{y}$

**b)** $\dfrac{4}{3x}-\dfrac{1}{x}$

**c)** $\dfrac{1}{t^2}+\dfrac{2}{3t}+\dfrac{5}{6}$

**d)** $\dfrac{1}{2}(w+2)+\dfrac{1}{3}(w-4)$

$\boxed{\dfrac{2x+y}{xy}}$, $\boxed{\dfrac{1}{3x}}$, $\boxed{\dfrac{5t^2+4t+6}{6t^2}}$, $\boxed{\dfrac{5w-2}{6}}$

(a) $\dfrac{1}{x}+\dfrac{2}{y}=\dfrac{1\times y+2x}{xy}=\dfrac{y+2x}{xy}$

(b) $\dfrac{4}{3x}-\dfrac{1}{x}=\dfrac{4-1\times 3}{3x}=\dfrac{1}{3x}$

(c) $\dfrac{1}{t^2}+\dfrac{2}{3t}+\dfrac{5}{6}=\dfrac{1\times 6+2\times 2t+5\times t^2}{6t^2}=\dfrac{6+4t+5t^2}{6t^2}$

(d) $\dfrac{1}{2}(w+2)+\dfrac{1}{3}(w-4)=\dfrac{w+2}{2}+\dfrac{w-4}{3}=\dfrac{3(w+2)}{6}+\dfrac{2(w-4)}{6}$

$=\dfrac{3w+6+2w-8}{6}=\dfrac{5w-2}{6}$

**Question 2**

Simplify the following expressions into a single fraction.

**a)** $\dfrac{1}{x+2}+\dfrac{3}{x+1}$

**b)** $\dfrac{4}{y+1}-\dfrac{3}{y-2}$

**c)** $\dfrac{2}{2t+1}-\dfrac{1}{t+3}$

**d)** $\dfrac{3}{2(w+1)}+\dfrac{1}{4(w-1)}$

$$\boxed{\frac{4x+7}{(x+1)(x+2)}},\ \boxed{\frac{y-11}{(y+1)(y-2)}},\ \boxed{\frac{5}{(2t+1)(t+3)}},\ \boxed{\frac{7w-5}{4(w+1)(w-1)}}$$

(a) $\dfrac{1}{x+2}+\dfrac{3}{x+1}=\dfrac{1(x+1)+3(x+2)}{(x+2)(x+1)}=\dfrac{x+1+3x+6}{(x+2)(x+1)}=\dfrac{4x+7}{(x+2)(x+1)}$

(b) $\dfrac{4}{y+1}-\dfrac{3}{y-2}=\dfrac{4(y-2)-3(y+1)}{(y+1)(y-2)}=\dfrac{4y-8-3y-3}{(y+1)(y-2)}=\dfrac{y-11}{(y+1)(y-2)}$

(c) $\dfrac{2}{2t+1}-\dfrac{1}{t+3}=\dfrac{2(t+3)-1(2t+1)}{(2t+1)(t+3)}=\dfrac{\cancel{2t}+6-\cancel{2t}-1}{(2t+1)(t+3)}=\dfrac{5}{(2t+1)(t+3)}$

(d) $\dfrac{3}{2(w+1)}+\dfrac{1}{4(w-1)}=\dfrac{3\times 2(w-1)+1(w+1)}{4(w+1)(w-1)}=\dfrac{6w-6+w+1}{4(w+1)(w-1)}=\dfrac{7w-5}{4(w+1)(w-1)}$

**Question 3**

Simplify the following into a single fraction.

**a)** $\dfrac{2}{x+1}+\dfrac{3}{x-2}$

**b)** $\dfrac{5}{y-1}-\dfrac{4}{y+2}$

**c)** $\dfrac{4}{2t-1}-\dfrac{2}{t+3}$

**d)** $\dfrac{w}{w^2-1}+\dfrac{2}{w+1}$

$$\boxed{\frac{5x-1}{(x+1)(x-2)}},\ \boxed{\frac{y+14}{(y+1)(y-2)}},\ \boxed{\frac{14}{(2t-1)(t+3)}},\ \boxed{\frac{3w-2}{(w+1)(w-1)}}$$

(a) $\dfrac{2}{x+1}+\dfrac{3}{x-2}=\dfrac{2(x-2)+3(x+1)}{(x+1)(x-2)}=\dfrac{2x-4+3x+3}{(x+1)(x-2)}=\dfrac{5x-1}{(x+1)(x-2)}$

(b) $\dfrac{5}{y-1}-\dfrac{4}{y+2}=\dfrac{5(y+2)-4(y-1)}{(y-1)(y+2)}=\dfrac{5y+10-4y+4}{(y-1)(y+2)}=\dfrac{y+14}{(y-1)(y+2)}$

(c) $\dfrac{4}{2t-1}-\dfrac{2}{t+3}=\dfrac{4(t+3)-2(2t-1)}{(2t-1)(t+3)}=\dfrac{\cancel{4t}+12-\cancel{4t}+2}{(2t-1)(t+3)}=\dfrac{1}{(2t-1)(t+3)}$

(d) $\dfrac{w}{w^2-1}+\dfrac{2}{w+1}=\dfrac{w}{(w-1)(w+1)}+\dfrac{2}{w+1}=\dfrac{w+2(w-1)}{(w-1)(w+1)}=\dfrac{w+2w-2}{(w-1)(w+1)}=\dfrac{3w-2}{(w-1)(w+1)}$

**Question 4**

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form.

**a)** $\dfrac{2}{x-1}-\dfrac{6}{(x-1)(2x+1)}$

**b)** $\dfrac{2y+5}{y+3}-\dfrac{1}{(y+3)(y+2)}$

**c)** $\dfrac{t+3}{(t+1)(t+2)}-\dfrac{t+1}{(t+2)(t+3)}$

**d)** $\dfrac{2}{w-2}+\dfrac{3w}{w^2-4}-\dfrac{5}{w+2}$

$\boxed{\dfrac{4}{2x+1}}$, $\boxed{\dfrac{2y+3}{y+2}}$, $\boxed{\dfrac{4}{(t+1)(t+3)}}$, $\boxed{\dfrac{14}{(w-2)(w+2)}=\dfrac{14}{w^2-4}}$

(a) $\dfrac{2}{x-1}-\dfrac{6}{(x-1)(2x+1)}=\dfrac{2(2x+1)-6}{(x-1)(2x+1)}=\dfrac{4x+2-6}{(x-1)(2x+1)}=\dfrac{4x-4}{(x-1)(2x+1)}$

$=\dfrac{4\cancel{(x-1)}}{\cancel{(x-1)}(2x+1)}=\dfrac{4}{2x+1}$

(b) $\dfrac{2y+5}{y+3}-\dfrac{1}{(y+3)(y+2)}=\dfrac{(2y+5)(y+2)-1}{(y+3)(y+2)}=\dfrac{2y^2+4y+5y+10-1}{(y+3)(y+2)}$

$=\dfrac{2y^2+9y+9}{(y+3)(y+2)}=\dfrac{\cancel{(y+3)}(2y+3)}{\cancel{(y+3)}(y+2)}=\dfrac{2y+3}{y+2}$

(c) $\dfrac{t+3}{(t+1)(t+2)}-\dfrac{t+1}{(t+2)(t+3)}=\dfrac{(t+3)(t+3)-(t+1)(t+1)}{(t+1)(t+2)(t+3)}=$

$=\dfrac{\cancel{t^2}+6t+9-\cancel{t^2}-2t-1}{(t+1)(t+2)(t+3)}=\dfrac{4t+8}{(t+1)(t+2)(t+3)}$

$=\dfrac{4\cancel{(t+2)}}{(t+1)\cancel{(t+2)}(t+3)}=\dfrac{4}{(t+1)(t+3)}$

(d) $\dfrac{2}{w-2}+\dfrac{3w}{w^2-4}-\dfrac{5}{w+2}=\dfrac{2}{w-2}+\dfrac{3w}{(w-2)(w+2)}-\dfrac{5}{w+2}$

$=\dfrac{2(w+2)+3w-5(w-2)}{(w-2)(w+2)}=\dfrac{\cancel{2w}+4+\cancel{3w}-\cancel{5w}+10}{(w-2)(w+2)}$

$=\dfrac{14}{(w-2)(w+2)}$ or $\dfrac{14}{w^2-4}$

## Question 5

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form.

**a)** $\dfrac{4x}{x^2-9}-\dfrac{2}{x+3}$

**b)** $\dfrac{y-10}{(y-3)(y+4)}-\dfrac{y-8}{(y-3)(2y-1)}$

**c)** $1+\dfrac{4t}{2t-5}-\dfrac{15}{2t^2-7t+5}$

**d)** $\dfrac{2w^2}{(w+1)^3}+\dfrac{3w}{(w+1)^2}-\dfrac{4}{w+1}$

$\boxed{\dfrac{2}{x-3}}$, $\boxed{\dfrac{y-14}{(y+4)(2y-1)}}$, $\boxed{\dfrac{3t+2}{t-1}}$, $\boxed{\dfrac{w^2-5w-4}{(w+1)^3}}$

(a) $\dfrac{4x}{x^2-9}-\dfrac{2}{x+3}=\dfrac{4x}{(x+3)(x-3)}-\dfrac{2}{x+3}=\dfrac{4x-2(x-3)}{(x+3)(x-3)}$

$=\dfrac{4x-2x+6}{(x+3)(x-3)}=\dfrac{2x+6}{(x+3)(x-3)}=\dfrac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}=\dfrac{2}{x-3}$

(b) $\dfrac{y-10}{(y-3)(y+4)}-\dfrac{y-8}{(y-3)(2y-1)}=\dfrac{(y-10)(2y-1)-(y-8)(y+4)}{(y-3)(y+4)(2y-1)}$

$=\dfrac{2y^2-21y+10-y^2+4y+32}{(y-3)(y+4)(2y-1)}$

$=\dfrac{y^2-17y+42}{(y-3)(y+4)(2y-1)}=\dfrac{\cancel{(y-3)}(y-14)}{\cancel{(y-3)}(y+4)(2y-1)}$

$=\dfrac{y-14}{(y+4)(2y-1)}$

(c) $1+\dfrac{4t}{2t-5}-\dfrac{15}{2t^2-7t+5}=1+\dfrac{4t}{2t-5}-\dfrac{15}{(2t-5)(t-1)}$

$=\dfrac{1(2t-5)(t-1)+4t(t-1)-15}{(2t-5)(t-1)}$

$=\dfrac{2t^2-7t+5+4t^2-4t-15}{(2t-5)(t-1)}$

$=\dfrac{6t^2-11t-10}{(2t-5)(t-1)}=\dfrac{\cancel{(2t-5)}(3t+2)}{\cancel{(2t-5)}(t-1)}$

$=\dfrac{3t+2}{t-1}$

(d) $\dfrac{2w^2}{(w+1)^3}+\dfrac{3w}{(w+1)^2}-\dfrac{4}{w+1}=\dfrac{2w^2+3w(w+1)-4(w+1)^2}{(w+1)^3}$

$=\dfrac{2w^2+3w^2+3w-4(w^2+2w+1)}{(w+1)^3}$

$=\dfrac{2w^2+3w^2+3w-4w^2-8w-4}{(w+1)^3}$

$=\dfrac{w^2-5w-4}{(w+1)^3}$

**Question 6**

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form.

**a)** $\dfrac{1}{x+4}-\dfrac{2(x-1)}{3x^2+14x+8}$

**b)** $\dfrac{y}{y^2-9}-\dfrac{1}{y^2-4y+3}$

**c)** $1+\dfrac{3t+2}{3t^2-t-2}$

**d)** $\dfrac{2w+2}{w^2-2w-3}-\dfrac{w+1}{w-3}$

$\boxed{\dfrac{1}{3x+2}}$, $\boxed{\dfrac{y+1}{(y-1)(y+3)}}$, $\boxed{\dfrac{t}{t-1}}$, $\boxed{-1}$

(a) $\dfrac{1}{x+4}-\dfrac{2(x-1)}{3x^2+14x+8}=\dfrac{1}{x+4}-\dfrac{2(x-1)}{(3x+2)(x+4)}$

$=\dfrac{1(3x+2)-2(x-1)}{(3x+2)(x+4)}=\dfrac{3x+2-2x+2}{(3x+2)(x+4)}$

$=\dfrac{\cancel{x+4}}{(3x+2)\cancel{(x+4)}}=\dfrac{1}{3x+2}$

(b) $\dfrac{y}{y^2-9}-\dfrac{1}{y^2-4y+3}=\dfrac{y}{(y+3)(y-3)}-\dfrac{1}{(y-3)(y-1)}$

$=\dfrac{y(y-1)-(y+3)}{(y+3)(y-3)(y-1)}$

$=\dfrac{y^2-y-y-3}{(y+3)(y-3)(y-1)}=\dfrac{y^2-2y-3}{(y+3)(y-3)(y-1)}$

$=\dfrac{(y+1)\cancel{(y-3)}}{(y+3)\cancel{(y-3)}(y-1)}=\dfrac{y+1}{(y-1)(y+3)}$

(c) $1+\dfrac{3t+2}{3t^2-t-2}=1+\dfrac{\cancel{3t+2}}{\cancel{(3t+2)}(t-1)}=1+\dfrac{1}{t-1}$

$=\dfrac{(t-1)+1}{t-1}=\dfrac{t-1+1}{t-1}=\dfrac{t}{t-1}$

(d) $\dfrac{2w+2}{w^2-2w-3}-\dfrac{w-1}{w-3}=\dfrac{2\cancel{(w+1)}}{\cancel{(w+1)}(w-3)}-\dfrac{w-1}{w-3}$

$=\dfrac{2}{w-3}-\dfrac{w-1}{w-3}=\dfrac{2-(w-1)}{w-3}$

$=\dfrac{2-w+1}{w-3}=\dfrac{3-w}{w-3}=-\dfrac{\cancel{w-3}}{\cancel{w-3}}$

$=-1$

**Question 7**

Prove that

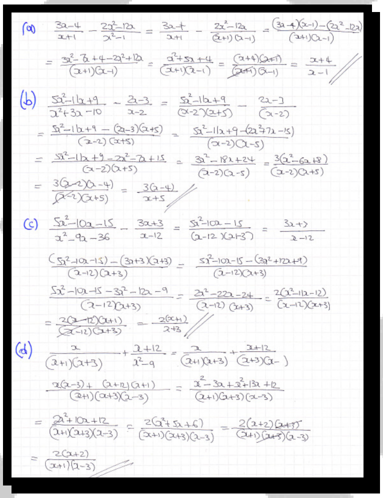
**a)** $\dfrac{3x-4}{x+1}-\dfrac{2x^2-12x}{x^2-1}\equiv\dfrac{x+4}{x-1}$

**b)** $\dfrac{5x^2-11x+9}{x^2+3x-10}-\dfrac{2x-3}{x-2}\equiv\dfrac{3(x-4)}{x+5}$

**c)** $\dfrac{5x^2-10x-15}{x^2-9x-36}-\dfrac{3x+3}{x-12}\equiv\dfrac{2(x+1)}{x+3}$

**d)** $\dfrac{x}{(x+1)(x+3)}+\dfrac{x+12}{x^2-9}\equiv\dfrac{2(x+2)}{(x+1)(x-3)}$

proof



**Question 8**

Prove that

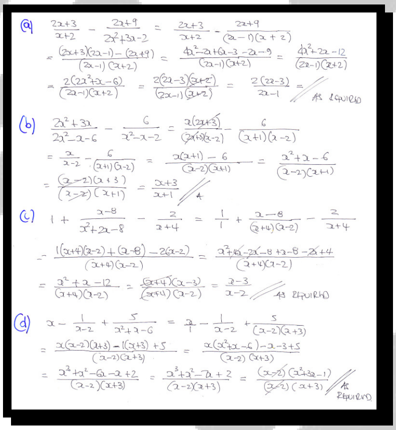
**a)** $\dfrac{2x+3}{x+2}-\dfrac{2x+9}{2x^2+3x-2}\equiv\dfrac{2(2x-3)}{2x-1}$

**b)** $\dfrac{2x^2+3x}{2x^2-x-6}-\dfrac{6}{x^2-x-2}\equiv\dfrac{x+3}{x+1}$

**c)** $1+\dfrac{x-8}{x^2+2x-8}-\dfrac{2}{x+4}\equiv\dfrac{x-3}{x-2}$

**d)** $x-\dfrac{1}{x-2}+\dfrac{5}{x^2+x-6}\equiv\dfrac{x^2+3x-1}{x+3}$

$\boxed{\text{proof}}$



(a) $\dfrac{2x+3}{x+2}-\dfrac{2x+9}{2x^2+3x-2}=\dfrac{2x+3}{x+2}-\dfrac{2x+9}{(2x-1)(x+2)}$

$=\dfrac{(2x+3)(2x-1)-(2x+9)}{(2x-1)(x+2)}=\dfrac{4x^2-2x+6x-3-2x-9}{(2x-1)(x+2)}=\dfrac{4x^2+2x-12}{(2x-1)(x+2)}$

$=\dfrac{2(2x^2+x-6)}{(2x-1)(x+2)}=\dfrac{2(2x-3)\cancel{(x+2)}}{(2x-1)\cancel{(x+2)}}=\dfrac{2(2x-3)}{2x-1}$ // AS REQUIRED

(b) $\dfrac{2x^2+3x}{2x^2-x-6}-\dfrac{6}{x^2-x-2}=\dfrac{x\cancel{(2x+3)}}{\cancel{(2x+3)}(x-2)}-\dfrac{6}{(x+1)(x-2)}$

$=\dfrac{x}{x-2}-\dfrac{6}{(x+1)(x-2)}=\dfrac{x(x+1)-6}{(x-2)(x+1)}=\dfrac{x^2+x-6}{(x-2)(x+1)}$

$=\dfrac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}(x+1)}=\dfrac{x+3}{x+1}$ // A

(c) $1+\dfrac{x-8}{x^2+2x-8}-\dfrac{2}{x+4}=\dfrac{1}{1}+\dfrac{x-8}{(x+4)(x-2)}-\dfrac{2}{x+4}$

$=\dfrac{1(x+4)(x-2)+(x-8)-2(x-2)}{(x+4)(x-2)}=\dfrac{x^2+4x-2x-8+x-8-2x+4}{(x+4)(x-2)}$

$=\dfrac{x^2+x-12}{(x+4)(x-2)}=\dfrac{\cancel{(x+4)}(x-3)}{\cancel{(x+4)}(x-2)}=\dfrac{x-3}{x-2}$ // AS REQUIRED

(d) $x-\dfrac{1}{x-2}+\dfrac{5}{x^2+x-6}=\dfrac{x}{1}-\dfrac{1}{x-2}+\dfrac{5}{(x-2)(x+3)}$

$=\dfrac{x(x-2)(x+3)-1(x+3)+5}{(x-2)(x+3)}=\dfrac{x(x^2+x-6)-x-3+5}{(x-2)(x+3)}$

$=\dfrac{x^3+x^2-6x-x+2}{(x-2)(x+3)}=\dfrac{x^3+x^2-7x+2}{(x-2)(x+3)}=\dfrac{\cancel{(x-2)}(x^2+3x-1)}{\cancel{(x-2)}(x+3)}$ // AS REQUIRED

**Question 9**

Prove that

**a)** $\dfrac{2}{x-1}+\dfrac{x-11}{x^2+3x-4}\equiv\dfrac{3}{x+4}$

**b)** $\dfrac{3x^2-x-2}{x^2-1}-\dfrac{1}{x^2+x}\equiv\dfrac{3x-1}{x}$

**c)** $\dfrac{4x-1}{2(x-1)}-\dfrac{3}{2(x-1)(2x-1)}-2\equiv\dfrac{3}{2x-1}$

**d)** $1-\dfrac{1}{x-2}+\dfrac{3}{x^2-x-2}\equiv\dfrac{x}{x+1}$

proof

(a) $\dfrac{2}{x-1}+\dfrac{x-11}{x^2+3x-4}=\dfrac{2}{x-1}+\dfrac{x-11}{(x-1)(x+4)}=\dfrac{2(x+4)+(x-11)}{(x-1)(x+4)}$

$=\dfrac{2x+8+x-11}{(x-1)(x+4)}=\dfrac{3x-3}{(x-1)(x+4)}=\dfrac{3\cancel{(x-1)}}{\cancel{(x-1)}(x+4)}=\dfrac{3}{x+4}$

(b) $\dfrac{3x^2-x-2}{x^2-1}-\dfrac{1}{x^2+x}=\dfrac{(3x+2)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}-\dfrac{1}{x(x+1)}=\dfrac{3x+2}{x+1}-\dfrac{1}{x(x+1)}$

$=\dfrac{(3x+2)x-1}{x(x+1)}=\dfrac{3x^2+2x-1}{x(x+1)}=\dfrac{(3x-1)\cancel{(x+1)}}{x\cancel{(x+1)}}$

$=\dfrac{3x-1}{x}$

(c) $\dfrac{4x-1}{2(x-1)}-\dfrac{3}{2(x-1)(2x-1)}-2=\dfrac{(4x-1)(2x-1)-3-2\times2(x-1)(2x-1)}{2(x-1)(2x-1)}$

$=\dfrac{\cancel{8x^2}-6x+1-3-\cancel{8x^2}+12x-4}{2(x-1)(2x-1)}$

$=\dfrac{6x-6}{2(x-1)(2x-1)}=\dfrac{6\cancel{(x-1)}}{2\cancel{(x-1)}(2x-1)}=\dfrac{3}{2x-1}$

(d) $1-\dfrac{1}{x-2}+\dfrac{3}{x^2-x-2}=1-\dfrac{1}{x-2}+\dfrac{3}{(x-2)(x+1)}$

$=\dfrac{1(x-2)(x+1)-1(x+1)+3}{(x-2)(x+1)}$

$=\dfrac{x^2-x-\cancel{2}-x-\cancel{1}+\cancel{3}}{(x-2)(x+1)}=\dfrac{x^2-2x}{(x-2)(x+1)}$

$=\dfrac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}=\dfrac{x}{x+1}$

**Question 10**

Prove that

**a)** $\dfrac{x}{x-2}-\dfrac{2}{x^2-3x+2}\equiv\dfrac{x+1}{x-1}$

**b)** $\dfrac{x}{x+2}-\dfrac{2}{x^2+3x+2}-\dfrac{1}{x+1}\equiv\dfrac{x-2}{x+1}$

**c)** $\dfrac{4x-1}{2x^2-x-6}-\dfrac{2(x+1)}{x^2+2x-8}\equiv\dfrac{5}{(2x+3)(x+4)}$

**d)** $x-\dfrac{12}{x^2+2x-3}+\dfrac{3}{x-1}\equiv\dfrac{x^2+3x+3}{x+3}$

proof

(a) $\dfrac{x}{x-2}-\dfrac{2}{x^2-3x+2}=\dfrac{x}{x-2}-\dfrac{2}{(x-2)(x-1)}$

$=\dfrac{x(x-1)-2}{(x-2)(x-1)}=\dfrac{x^2-x-2}{(x-2)(x-1)}=\dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x-1)}$

$=\dfrac{x+1}{x-1}$ // AS REQUIRED

(b) $\dfrac{x}{x+2}-\dfrac{2}{x^2+3x+2}-\dfrac{1}{x+1}=\dfrac{x}{x+2}-\dfrac{2}{(x+1)(x+2)}-\dfrac{1}{x+1}$

$=\dfrac{x(x+1)-2-1(x+2)}{(x+1)(x+2)}=\dfrac{x^2+\cancel{x}-2-\cancel{x}-2}{(x+1)(x+2)}=\dfrac{x^2-4}{(x+1)(x+2)}$

$=\dfrac{(x-2)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}}=\dfrac{x-2}{x+1}$ // AS REQUIRED

(c) $\dfrac{4x-1}{2x^2-x-6}-\dfrac{2(x+1)}{x^2+2x-8}=\dfrac{4x-1}{(x-2)(2x+3)}-\dfrac{2(x+1)}{(x+4)(x-2)}$

$=\dfrac{(4x-1)(x+4)-2(x+1)(2x+3)}{(x-2)(2x+3)(x+4)}=\dfrac{4x^2+15x-4-2(2x^2+5x+3)}{(x-2)(2x+3)(x+4)}$

$=\dfrac{\cancel{4x^2}+15x-4-\cancel{4x^2}-10x-6}{(x-2)(2x+3)(x+4)}=\dfrac{5x-10}{(x-2)(2x+3)(x+4)}$

$=\dfrac{5\cancel{(x-2)}}{\cancel{(x-2)}(2x+3)(x+4)}=\dfrac{5}{(2x+3)(x+4)}$ // AS REQUIRED

(d) $x-\dfrac{12}{x^2+2x-3}+\dfrac{3}{x-1}=x-\dfrac{12}{(x+3)(x-1)}+\dfrac{3}{x-1}=$

$=\dfrac{x(x+3)(x-1)-12+3(x+3)}{(x+3)(x-1)}=\dfrac{x^3+2x^2-\cancel{3x}-12+\cancel{3x}+9}{(x+3)(x-1)}=$

$=\dfrac{x^3+2x^2-3}{(x+3)(x-1)}=\dfrac{\cancel{(x-1)}(x^2+3x+3)}{\cancel{(x-1)}(x+3)}=\dfrac{x^2+3x+3}{x+3}$ // AS REQUIRED

NOTE $(x-1)(x^2+3x+3)=x^3+3x^2+\cancel{3x}-x^2-\cancel{3x}-3=x^3+2x^2-3$

# ALGEBRAIC DIVISIONS

## Question 1

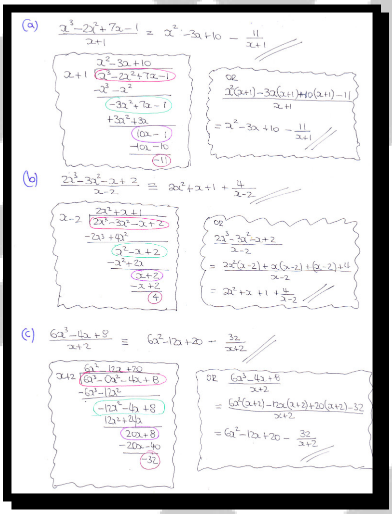
Find the values of the constants in the each of the following identities.

**a)** $$\frac{x^3-2x^2+7x-1}{x+1} \equiv ax^2+bx+c+\frac{d}{x+1}$$

**b)** $$\frac{2x^3-3x^2-x+2}{x-2} \equiv Ax^2+Bx+C+\frac{D}{x-2}$$

**c)** $$\frac{6x^3-4x+8}{x+2} \equiv Px^2+Qx+R+\frac{S}{x+2}$$

$\boxed{(a,b,c,d)=(1,-3,10,-11)}$, $\boxed{(A,B,C,D)=(2,1,1,4)}$, $\boxed{(P,Q,R,S)=(6,12,20,-32)}$



**Question 2**

Find the values of the constants in the each of the following identities.

**a)** $$\frac{x^3+2x^2-3x+1}{x-2} \equiv ax^2+bx+c+\frac{d}{x-2}$$

**b)** $$\frac{2x^3-x^2+3x-2}{x+1} \equiv Ax^2+Bx+C+\frac{D}{x+1}$$

**c)** $$\frac{2x^3-3x^2+x-4}{x-1} \equiv Px^2+Qx+R+\frac{S}{x-1}$$

$\boxed{(a,b,c,d)=(1,4,5,11)}$, $\boxed{(A,B,C,D)=(2,-3,6,-8)}$, $\boxed{(P,Q,R,S)=(2,-1,0,-4)}$

**Question 3**

Find the values of the constants in the each of the following identities.

**a)** $\dfrac{3x^3 - x^2 - 20x + 31}{x+3} \equiv ax^2 + bx + c + \dfrac{d}{x+3}$

**b)** $\dfrac{4x^3 - 7x^2 - 30x - 21}{x-4} \equiv Ax^2 + Bx + C + \dfrac{D}{x-4}$

**c)** $\dfrac{2x^3 - 7x + 2}{x+1} \equiv Px^2 + Qx + R + \dfrac{S}{x+1}$

$\boxed{(a,b,c,d) = (3,-10,10,1)}$, $\boxed{(A,B,C,D) = (4,9,6,3)}$, $\boxed{(P,Q,R,S) = (2,-2,-5,7)}$

**Question 4**

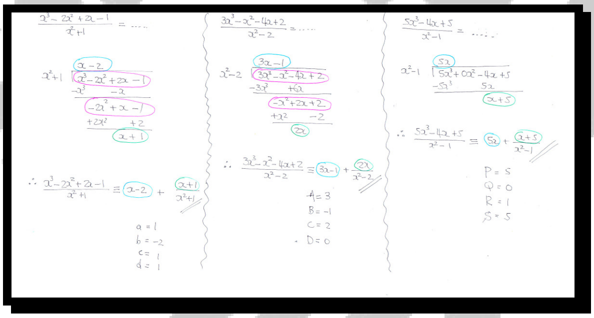
Find the values of the constants in the each of the following identities.

**a)** $\dfrac{x^3-2x^2+2x-1}{x^2+1} \equiv ax+b+\dfrac{cx+d}{x^2+1}$

**b)** $\dfrac{3x^3-x^2-4x+2}{x^2-2} \equiv Ax+B+\dfrac{Cx+D}{x^2-2}$

**c)** $\dfrac{5x^3-4x+5}{x^2-1} \equiv Px+Q+\dfrac{Rx+S}{x^2-1}$

$\boxed{(a,b,c,d)=(1,-2,1,1)}$, $\boxed{(A,B,C,D)=(3,-1,2,0)}$, $\boxed{(P,Q,R,S)=(5,0,1,5)}$



**Question 5**

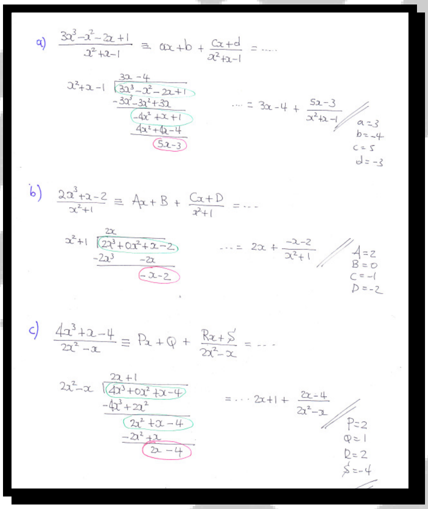
Find the values of the constants in the each of the following identities.

**a)** $$\frac{3x^3-x^2-2x+1}{x^2+x-1} \equiv ax+b+\frac{cx+d}{x^2+x-1}$$

**b)** $$\frac{2x^3+x-2}{x^2+1} \equiv Ax+B+\frac{Cx+D}{x^2+1}$$

**c)** $$\frac{4x^3+x-4}{2x^2-x} \equiv Px+Q+\frac{Rx+S}{2x^2-x}$$

$\boxed{(a,b,c,d)=(3,-4,5,-3)}$, $\boxed{(A,B,C,D)=(2,0,-1,-2)}$, $\boxed{(P,Q,R,S)=(2,1,2,-4)}$



a) $\frac{3x^3-x^2-2x+1}{x^2+x-1} \equiv ax+b+\frac{cx+d}{x^2+x-1} = \dots$

$\dots = 3x-4+\frac{5x-3}{x^2+x-1}$

$a=3$, $b=-4$, $c=5$, $d=-3$

b) $\frac{2x^3+x-2}{x^2+1} \equiv Ax+B+\frac{Cx+D}{x^2+1} = \dots$

$\dots = 2x+\frac{-x-2}{x^2+1}$

$A=2$, $B=0$, $C=-1$, $D=-2$

c) $\frac{4x^3+x-4}{2x^2-x} \equiv Px+Q+\frac{Rx+S}{2x^2-x} = \dots$

$= \dots 2x+1+\frac{2x-4}{2x^2-x}$

$P=2$, $Q=1$, $R=2$, $S=-4$

**Question 6**

Find the values of the constants in the each of the following identities.

**a)** $$\frac{2x^3+x^2-4x+1}{x^2+x-2} \equiv ax+b+\frac{c}{x+d}$$

**b)** $$\frac{2x^3+3x^2-10x-3}{x^2+x-6} \equiv Ax+B+\frac{C}{x+D}$$

**c)** $$\frac{-x^3-5x^2-4x+4}{x^2+3x+2} \equiv Px+Q+\frac{R}{x+S}$$

$$\boxed{(a,b,c,d)=(2,-1,1,2)}, \ \boxed{(A,B,C,D)=(2,1,1,-2)}, \ \boxed{(P,Q,R,S)=(-1,-2,4,1)}$$

(a) $$\frac{2x^3+x^2-4x+1}{x^2+x-2} = 2x-1+\frac{x-1}{x^2+x-2} = 2x-1+\frac{\cancel{x-1}}{\cancel{(x-1)}(x+2)} = 2x-1+\frac{1}{x+2}$$

$$\begin{array}{r} 2x-1 \\ x^2+x-2 \; \overline{)\, 2x^3+x^2-4x+1} \\ -2x^3-2x^2+4x \\ \hline -x^2 \quad +1 \\ x^2+x-2 \\ \hline x-1 \end{array}$$

(b) $$\frac{2x^3+3x^2-10x-3}{x^2+x-6} = 2x+1+\frac{x+3}{x^2-x-6} = 2x+1+\frac{\cancel{x+3}}{\cancel{(x+3)}(x-2)} = 2x+1+\frac{1}{x-2}$$

$$\begin{array}{r} 2x+1 \\ x^2+x-6 \; \overline{)\, 2x^3+3x^2-10x-3} \\ -2x^3-2x^2+12x \\ \hline x^2+2x-3 \\ -x^2-x+6 \\ \hline x+3 \end{array}$$

(c) $$\frac{-x^3-5x^2-4x+4}{x^2+3x+2} = -x-2+\frac{4x+8}{x+3x+2} = -x-2+\frac{4\cancel{(x+2)}}{\cancel{(x+2)}(x+1)} = -x-2+\frac{4}{x+1}$$

$$\begin{array}{r} -x-2 \\ x^2+3x+2 \; \overline{)\, -x^3-5x^2-4x+4} \\ x^3+3x^2+2x \\ \hline -2x^2-2x+4 \\ 2x^2+6x+4 \\ \hline 4x+8 \end{array}$$

# MULTIPLICATION AND DIVISION OF ALGEBRAIC FRACTIONS

**Question 1**

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form:

**a)** $\dfrac{2x^3+x^2}{x^2-4}\times\dfrac{x-2}{2x^2-5x-3}$

**b)** $\dfrac{18y^2-8}{12y^2}\times\dfrac{3y^2+9y}{3y^2+11y+6}$

**c)** $\dfrac{t^3+1}{2t^2+7t+5}\times\dfrac{4t^2-25}{4t^2-t+1}$

$\boxed{\dfrac{x^2}{(x+2)(x+3)}}$, $\boxed{\dfrac{3y-2}{2y}}$ $\boxed{2t-5}$

**Question 2**

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form:

**a)** $\dfrac{2x}{2x^2+3x-5}\div\dfrac{x^3}{x^2-x}$

**b)** $\dfrac{y^2-8y+15}{(y-5)^2}\div\dfrac{y^2-9}{2y^2+6y}$

$\boxed{\dfrac{2}{x(2x+5)}}$, $\boxed{\dfrac{2y}{y-5}}$

**Question 3**

Simplify the following algebraic expressions giving your final answer as a single fraction in its simplest form:

**a)** $$\frac{\frac{x^2+x-6}{x^2-9}}{\frac{x-2}{4}}$$

**b)** $$\frac{\frac{3}{x+4}+\frac{2}{x-1}}{\frac{x+1}{x-1}}$$

**c)** $$\frac{(2x+1)(2x-5)+4(2x-1)}{\frac{2x+3}{2x+1}}$$

$\boxed{\frac{4}{x-3}}$, $\boxed{\frac{5}{x+4}}$, $\boxed{4x^2-4x-3}$

# PARTIAL FRACTIONS

## Question 1

Express each of the following into partial fractions.

**a)** $\dfrac{6x}{(x-1)(x+2)}$

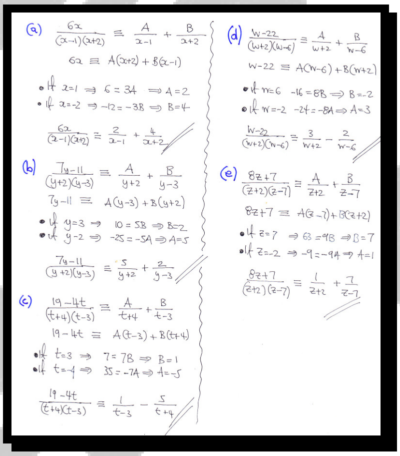
**b)** $\dfrac{7y-11}{(y+2)(y-3)}$

**c)** $\dfrac{19-4t}{(t+4)(t-3)}$

**d)** $\dfrac{w-22}{(w+2)(w-6)}$

**e)** $\dfrac{8z+7}{(z+2)(z-7)}$

$\boxed{\dfrac{2}{x-1}+\dfrac{4}{x+2}}$, $\boxed{\dfrac{2}{y-3}+\dfrac{5}{y+2}}$, $\boxed{\dfrac{1}{t-3}-\dfrac{5}{t+4}}$, $\boxed{\dfrac{3}{w+2}-\dfrac{2}{w-6}}$, $\boxed{\dfrac{1}{z+2}+\dfrac{7}{z-7}}$



(a) $\dfrac{6x}{(x-1)(x+2)} \equiv \dfrac{A}{x-1}+\dfrac{B}{x+2}$

$6x \equiv A(x+2)+B(x-1)$

- if $x=1 \Rightarrow 6=3A \Rightarrow A=2$
- if $x=-2 \Rightarrow -12=-3B \Rightarrow B=4$

$\dfrac{6x}{(x-1)(x+2)} \equiv \dfrac{2}{x-1}+\dfrac{4}{x+2}$

(b) $\dfrac{7y-11}{(y+2)(y-3)} \equiv \dfrac{A}{y+2}+\dfrac{B}{y-3}$

$7y-11 \equiv A(y-3)+B(y+2)$

- if $y=3 \Rightarrow 10=5B \Rightarrow B=2$
- if $y-2 \Rightarrow -25=-5A \Rightarrow A=5$

$\dfrac{7y-11}{(y+2)(y-3)} \equiv \dfrac{5}{y+2}+\dfrac{2}{y-3}$

(c) $\dfrac{19-4t}{(t+4)(t-3)} \equiv \dfrac{A}{t+4}+\dfrac{B}{t-3}$

$19-4t \equiv A(t-3)+B(t+4)$

- if $t=3 \Rightarrow 7=7B \Rightarrow B=1$
- if $t=-4 \Rightarrow 35=-7A \Rightarrow A=-5$

$\dfrac{19-4t}{(t+4)(t-3)} \equiv \dfrac{1}{t-3}-\dfrac{5}{t+4}$

(d) $\dfrac{w-22}{(w+2)(w-6)} \equiv \dfrac{A}{w+2}+\dfrac{B}{w-6}$

$w-22 \equiv A(w-6)+B(w+2)$

- if $w=6$ $-16=8B \Rightarrow B=-2$
- if $w=-2$ $-24=-8A \Rightarrow A=3$

$\dfrac{w-22}{(w+2)(w-6)} \equiv \dfrac{3}{w+2}-\dfrac{2}{w-6}$

(e) $\dfrac{8z+7}{(z+2)(z-7)} \equiv \dfrac{A}{z+2}+\dfrac{B}{z-7}$

$8z+7 \equiv A(z-7)+B(z+2)$

- if $z=7 \Rightarrow 63=9B \Rightarrow B=7$
- if $z=-2 \Rightarrow -9=-9A \Rightarrow A=1$

$\dfrac{8z+7}{(z+2)(z-7)} \equiv \dfrac{1}{z+2}+\dfrac{7}{z-7}$

**Question 2**

Express each of the following into partial fractions.

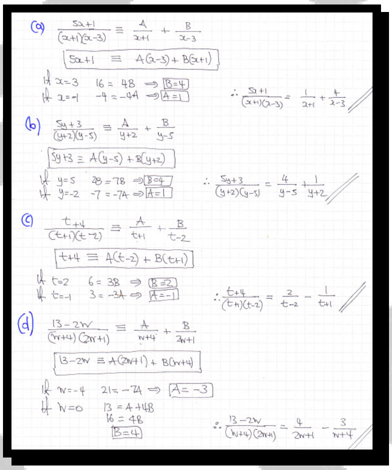
**a)** $\dfrac{5x+1}{(x+1)(x-3)}$

**b)** $\dfrac{5y+3}{(y+2)(y-5)}$

**c)** $\dfrac{t+4}{(t+1)(t-2)}$

**d)** $\dfrac{13-2w}{(w+4)(2w+1)}$

$\boxed{\dfrac{1}{x+1}+\dfrac{4}{x-3}}$ , $\boxed{\dfrac{4}{y-5}+\dfrac{1}{y+2}}$ , $\boxed{\dfrac{2}{t-2}-\dfrac{1}{t+1}}$ , $\boxed{\dfrac{4}{2w+1}-\dfrac{3}{w+4}}$


(a) $\dfrac{5x+1}{(x+1)(x-3)} \equiv \dfrac{A}{x+1}+\dfrac{B}{x-3}$

$\boxed{5x+1 \equiv A(x-3)+B(x+1)}$

If $x=3$ $\quad 16=4B \Rightarrow \boxed{B=4}$

If $x=-1$ $\quad -4=-4A \Rightarrow \boxed{A=1}$

$\therefore \dfrac{5x+1}{(x+1)(x-3)} = \dfrac{1}{x+1}+\dfrac{4}{x-3}$

(b) $\dfrac{5y+3}{(y+2)(y-5)} \equiv \dfrac{A}{y+2}+\dfrac{B}{y-5}$

$\boxed{5y+3 \equiv A(y-5)+B(y+2)}$

If $y=5$ $\quad 28=7B \Rightarrow \boxed{B=4}$

If $y=-2$ $\quad -7=-7A \Rightarrow \boxed{A=1}$

$\therefore \dfrac{5y+3}{(y+2)(y-5)} = \dfrac{4}{y-5}+\dfrac{1}{y+2}$

(c) $\dfrac{t+4}{(t+1)(t-2)} \equiv \dfrac{A}{t+1}+\dfrac{B}{t-2}$

$\boxed{t+4 \equiv A(t-2)+B(t+1)}$

If $t=2$ $\quad 6=3B \Rightarrow \boxed{B=2}$

If $t=-1$ $\quad 3=-3A \Rightarrow \boxed{A=-1}$

$\therefore \dfrac{t+4}{(t+1)(t-2)} = \dfrac{2}{t-2}-\dfrac{1}{t+1}$

(d) $\dfrac{13-2w}{(w+4)(2w+1)} \equiv \dfrac{A}{w+4}+\dfrac{B}{2w+1}$

$\boxed{13-2w \equiv A(2w+1)+B(w+4)}$

If $w=-4$ $\quad 21=-7A \Rightarrow \boxed{A=-3}$

If $w=0$ $\quad 13=A+4B$

$16=4B$

$\boxed{B=4}$

$\therefore \dfrac{13-2w}{(w+4)(2w+1)} = \dfrac{4}{2w+1}-\dfrac{3}{w+4}$

## Question 3

Express each of the following into partial fractions.

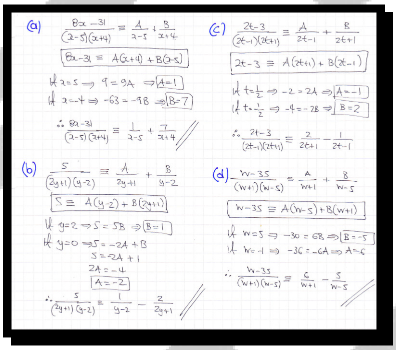
**a)** $\dfrac{8x-31}{(x-5)(x+4)}$

**b)** $\dfrac{5}{(2y+1)(y-2)}$

**c)** $\dfrac{2t-3}{(2t-1)(2t+1)}$

**d)** $\dfrac{w-35}{(w+1)(w-5)}$

$\boxed{\dfrac{7}{x+4}+\dfrac{1}{x-5}}$, $\boxed{\dfrac{2}{2y+1}-\dfrac{1}{y-2}}$, $\boxed{\dfrac{2}{2t+1}-\dfrac{1}{2t-1}}$, $\boxed{\dfrac{6}{w+1}-\dfrac{5}{w-5}}$



(a) $\dfrac{8x-31}{(x-5)(x+4)} \equiv \dfrac{A}{x-5}+\dfrac{B}{x+4}$

$\boxed{8x-31 \equiv A(x+4)+B(x-5)}$

If $x=5 \Rightarrow 9=9A \Rightarrow \boxed{A=1}$

If $x=-4 \Rightarrow -63=-9B \Rightarrow \boxed{B=7}$

$\therefore \dfrac{8x-31}{(x-5)(x+4)} \equiv \dfrac{1}{x-5}+\dfrac{7}{x+4}$ //

(b) $\dfrac{5}{(2y+1)(y-2)} \equiv \dfrac{A}{2y+1}+\dfrac{B}{y-2}$

$\boxed{5 \equiv A(y-2)+B(2y+1)}$

If $y=2 \Rightarrow 5=5B \Rightarrow \boxed{B=1}$

If $y=0 \Rightarrow 5=-2A+B$

$5=-2A+1$

$2A=-4$

$\boxed{A=-2}$

$\therefore \dfrac{5}{(2y+1)(y-2)} = \dfrac{1}{y-2}-\dfrac{2}{2y+1}$ //

(c) $\dfrac{2t-3}{(2t-1)(2t+1)} \equiv \dfrac{A}{2t-1}+\dfrac{B}{2t+1}$

$\boxed{2t-3 \equiv A(2t+1)+B(2t-1)}$

If $t=\frac{1}{2} \Rightarrow -2=2A \Rightarrow \boxed{A=-1}$

If $t=-\frac{1}{2} \Rightarrow -4=-2B \Rightarrow \boxed{B=2}$

$\therefore \dfrac{2t-3}{(2t-1)(2t+1)} \equiv \dfrac{2}{2t+1}-\dfrac{1}{2t-1}$

(d) $\dfrac{w-35}{(w+1)(w-5)} \equiv \dfrac{A}{w+1}+\dfrac{B}{w-5}$

$\boxed{w-35 \equiv A(w-5)+B(w+1)}$

If $w=5 \Rightarrow -30=6B \Rightarrow \boxed{B=-5}$

If $w=-1 \Rightarrow -36=-6A \Rightarrow A=6$

$\therefore \dfrac{w-35}{(w+1)(w-5)} \equiv \dfrac{6}{w+1}-\dfrac{5}{w-5}$ //

**Question 4**

Express each of the following into partial fractions.

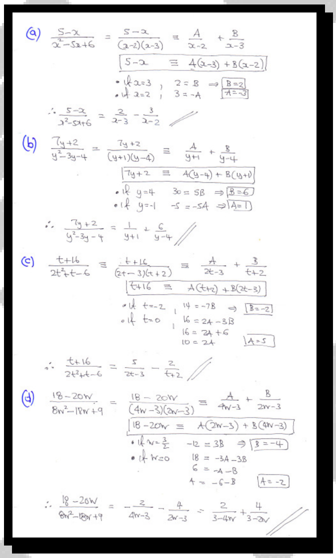
**a)** $\dfrac{5-x}{x^2-5x+6}$

**b)** $\dfrac{7y+2}{y^2-3y-4}$

**c)** $\dfrac{t+16}{2t^2+t-6}$

**d)** $\dfrac{18-20w}{8w^2-18w+9}$

$$\boxed{\frac{2}{x-3}-\frac{3}{x-2}}\ ,\ \boxed{\frac{6}{y-4}+\frac{1}{y+1}}\ ,\ \boxed{\frac{5}{2t-3}-\frac{2}{t+2}}\ ,\ \boxed{\frac{2}{3-4w}+\frac{4}{3-2w}}$$



(a) $\frac{5-x}{x^2-5x+6} = \frac{5-x}{(x-2)(x-3)} \equiv \frac{A}{x-2} + \frac{B}{x-3}$

$\boxed{5-x \equiv A(x-3)+B(x-2)}$

- if $x=3$; $2=B \Rightarrow \boxed{B=2}$
- if $x=2$; $3=-A$ $\boxed{A=-3}$

$\therefore \frac{5-x}{x^2-5x+6} = \frac{2}{x-3} - \frac{3}{x-2}$

(b) $\frac{7y+2}{y^2-3y-4} = \frac{7y+2}{(y+1)(y-4)} \equiv \frac{A}{y+1} + \frac{B}{y-4}$

$\boxed{7y+2 \equiv A(y-4)+B(y+1)}$

- if $y=4$ $30=5B \Rightarrow \boxed{B=6}$
- if $y=-1$ $-5=-5A \Rightarrow \boxed{A=1}$

$\therefore \frac{7y+2}{y^2-3y-4} = \frac{1}{y+1} + \frac{6}{y-4}$

(c) $\frac{t+16}{2t^2+t-6} \equiv \frac{t+16}{(2t-3)(t+2)} \equiv \frac{A}{2t-3} + \frac{B}{t+2}$

$\boxed{t+16 \equiv A(t+2)+B(2t-3)}$

- if $t=-2$; $14=-7B \Rightarrow \boxed{B=-2}$
- if $t=0$; $16=2A-3B$

$16=2A+6$

$10=2A$ $\boxed{A=5}$

$\therefore \frac{t+16}{2t^2+t-6} = \frac{5}{2t-3} - \frac{2}{t+2}$

(d) $\frac{18-20w}{8w^2-18w+9} = \frac{18-20w}{(4w-3)(2w-3)} \equiv \frac{A}{4w-3} + \frac{B}{2w-3}$

$\boxed{18-20w \equiv A(2w-3)+B(4w-3)}$

- if $w=\frac{3}{2}$ $-12=3B \Rightarrow \boxed{B=-4}$
- if $w=0$ $18=-3A-3B$

$6=-A-B$

$4=-6-B$ $\boxed{A=-2}$

$\therefore \frac{18-20w}{8w^2-18w+9} = -\frac{2}{4w-3} - \frac{4}{2w-3} = \frac{2}{3-4w} + \frac{4}{3-2w}$

**Question 5**

Express each of the following into partial fractions.

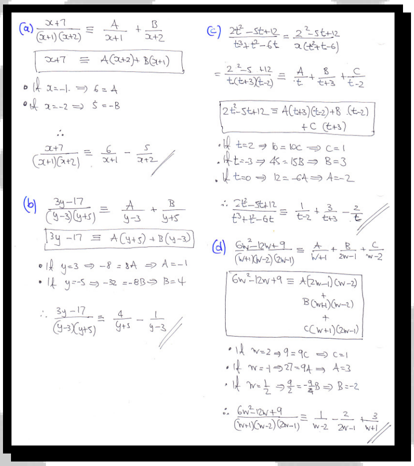
**a)** $\dfrac{x+7}{(x+1)(x+2)}$

**b)** $\dfrac{3y-17}{(y-3)(y+5)}$

**c)** $\dfrac{2t^2-5t+12}{t^3+t^2-6t}$

**d)** $\dfrac{6w^2-12w+9}{(w+1)(2w-1)(w-2)}$

$\boxed{\dfrac{6}{x+1}-\dfrac{5}{x+2}}$, $\boxed{\dfrac{4}{y+5}-\dfrac{1}{y-3}}$, $\boxed{\dfrac{1}{t-2}-\dfrac{2}{t}+\dfrac{3}{t+3}}$, $\boxed{\dfrac{1}{w-2}-\dfrac{2}{2w-1}+\dfrac{3}{w+1}}$



(a) $\dfrac{x+7}{(x+1)(x+2)} \equiv \dfrac{A}{x+1}+\dfrac{B}{x+2}$

$\boxed{x+7 \equiv A(x+2)+B(x+1)}$

- If $x=-1 \Rightarrow 6=A$
- If $x=-2 \Rightarrow 5=-B$

$\therefore \dfrac{x+7}{(x+1)(x+2)} \equiv \dfrac{6}{x+1}-\dfrac{5}{x+2}$

(b) $\dfrac{3y-17}{(y-3)(y+5)} \equiv \dfrac{A}{y-3}+\dfrac{B}{y+5}$

$\boxed{3y-17 \equiv A(y+5)+B(y-3)}$

- If $y=3 \Rightarrow -8=8A \Rightarrow A=-1$
- If $y=-5 \Rightarrow -32=-8B \Rightarrow B=4$

$\therefore \dfrac{3y-17}{(y-3)(y+5)} \equiv \dfrac{4}{y+5}-\dfrac{1}{y-3}$

(c) $\dfrac{2t^2-5t+12}{t^3+t^2-6t} = \dfrac{2t^2-5t+12}{t(t^2+t-6)}$

$= \dfrac{2t^2-5t+12}{t(t+3)(t-2)} \equiv \dfrac{A}{t}+\dfrac{B}{t+3}+\dfrac{C}{t-2}$

$\boxed{2t^2-5t+12 \equiv A(t+3)(t-2)+Bt(t-2)+Ct(t+3)}$

- If $t=2 \Rightarrow 10=10C \Rightarrow C=1$
- If $t=-3 \Rightarrow 45=15B \Rightarrow B=3$
- If $t=0 \Rightarrow 12=-6A \Rightarrow A=-2$

$\therefore \dfrac{2t^2-5t+12}{t^3+t^2-6t} \equiv \dfrac{1}{t-2}+\dfrac{3}{t+3}-\dfrac{2}{t}$

(d) $\dfrac{6w^2-12w+9}{(w+1)(w-2)(2w-1)} \equiv \dfrac{A}{w+1}+\dfrac{B}{2w-1}+\dfrac{C}{w-2}$

$\boxed{6w^2-12w+9 \equiv A(2w-1)(w-2)+B(w+1)(w-2)+C(w+1)(2w-1)}$

- If $w=2 \Rightarrow 9=9C \Rightarrow C=1$
- If $w=-1 \Rightarrow 27=9A \Rightarrow A=3$
- If $w=\frac{1}{2} \Rightarrow \frac{9}{2}=-\frac{9}{4}B \Rightarrow B=-2$

$\therefore \dfrac{6w^2-12w+9}{(w+1)(w-2)(2w-1)} \equiv \dfrac{1}{w-2}-\dfrac{2}{2w-1}+\dfrac{3}{w+1}$

**Question 6**

Express each of the following into partial fractions.

**a)** $\dfrac{x^2-4x+1}{x(x+1)(1-2x)}$

**b)** $\dfrac{10}{(y+1)(y+3)(2y+1)}$

**c)** $\dfrac{4t^2-5t+3}{(t+1)(t-1)(t-2)}$

**d)** $\dfrac{12w^2-4w+3}{(2w+1)(2w-1)(2w-3)}$

$\boxed{\dfrac{1}{x}+\dfrac{1}{2x-1}-\dfrac{2}{x+1}}$, $\boxed{\dfrac{1}{y+3}-\dfrac{5}{y+1}+\dfrac{8}{2y+1}}$, $\boxed{\dfrac{2}{t+1}-\dfrac{1}{t-1}+\dfrac{3}{t-2}}$, $\boxed{\dfrac{1}{2w+1}-\dfrac{1}{2w-1}+\dfrac{3}{2w-3}}$

(a) $\dfrac{x^2-4x+1}{x(x+1)(1-2x)} \equiv \dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{1-2x}$

$x^2-4x+1 \equiv A(x+1)(1-2x)+Bx(1-2x)+Cx(x+1)$

- If $x=0$, $1=A \Rightarrow A=1$
- If $x=-1$, $6=-3B \Rightarrow B=-2$
- If $x=\frac{1}{2}$, $-\frac{3}{4}=\frac{3}{4}C \Rightarrow C=-1$

$\therefore \dfrac{x^2-4x+1}{x(x+1)(1-2x)} \equiv \dfrac{1}{x}-\dfrac{2}{x+1}-\dfrac{1}{1-2x}$

(b) $\dfrac{10}{(y+1)(y+3)(2y+1)} \equiv \dfrac{A}{y+1}+\dfrac{B}{y+3}+\dfrac{C}{2y+1}$

$10 \equiv A(y+3)(2y+1)+B(y+1)(2y+1)+C(y+1)(y+3)$

- If $y=-1 \Rightarrow 10=-2A \Rightarrow A=-5$
- If $y=-3 \Rightarrow 10=10B \Rightarrow B=1$
- If $y=-\frac{1}{2} \Rightarrow 10=\frac{5}{4}C \Rightarrow C=8$

$\therefore \dfrac{10}{(y+1)(y+3)(2y+1)} \equiv \dfrac{1}{y+3}-\dfrac{5}{y+1}+\dfrac{8}{2y+1}$

(c) $\dfrac{4t^2-5t+3}{(t+1)(t-1)(t-2)} \equiv \dfrac{A}{t+1}+\dfrac{B}{t-1}+\dfrac{C}{t-2}$

$4t^2-5t+3 \equiv A(t-1)(t-2)+B(t+1)(t-2)+C(t+1)(t-1)$

- If $t=1 \Rightarrow 2=-2B \Rightarrow B=-1$
- If $t=2 \Rightarrow 9=3C \Rightarrow C=3$
- If $t=-1 \Rightarrow 12=6A \Rightarrow A=2$

$\therefore \dfrac{4t^2-5t+3}{(t+1)(t-1)(t-2)} \equiv \dfrac{2}{t+1}-\dfrac{1}{t-1}+\dfrac{3}{t-2}$

(d) $\dfrac{12w^2-4w+3}{(2w+1)(2w-1)(2w-3)} \equiv \dfrac{A}{2w+1}+\dfrac{B}{2w-1}+\dfrac{C}{2w-3}$

$12w^2-4w+3 \equiv A(2w-1)(2w-3)+B(2w+1)(2w-3)+C(2w+1)(2w-1)$

- If $w=\frac{1}{2} \Rightarrow 4=-4B \Rightarrow B=-1$
- If $w=-\frac{1}{2} \Rightarrow 8=8A \Rightarrow A=1$
- If $w=\frac{3}{2} \Rightarrow 24=8C \Rightarrow C=3$

$\dfrac{12w^2-4w+3}{(2w+1)(2w-1)(2w-3)} \equiv \dfrac{3}{2w-3}+\dfrac{1}{2w+1}-\dfrac{1}{2w-1}$

**Question 7**

Express each of the following into partial fractions.

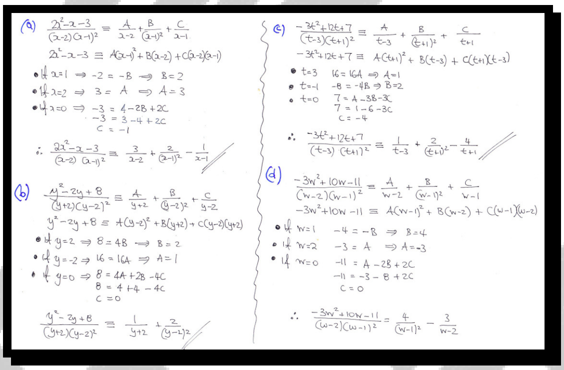
**a)** $\dfrac{2x^2 - x - 3}{(x-2)(x-1)^2}$

**b)** $\dfrac{y^2 - 2y + 8}{(y+2)(y-2)^2}$

**c)** $\dfrac{-3t^2 + 12t + 7}{(t-3)(t+1)^2}$

**d)** $\dfrac{-3w^2 + 10w - 11}{(w-2)(w-1)^2}$

$\boxed{\dfrac{3}{x-2} - \dfrac{1}{x-1} + \dfrac{2}{(x-1)^2}}$, $\boxed{\dfrac{2}{(y-2)^2} + \dfrac{1}{y+2}}$, $\boxed{\dfrac{1}{t-3} - \dfrac{4}{t+1} + \dfrac{2}{(t+1)^2}}$, $\boxed{\dfrac{4}{(w-1)^2} - \dfrac{3}{w-2}}$



(a) $\dfrac{2x^2-x-3}{(x-2)(x-1)^2} \equiv \dfrac{A}{x-2} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-1}$

$2x^2-x-3 \equiv A(x-1)^2 + B(x-2) + C(x-2)(x-1)$

- If $x=1 \Rightarrow -2 = -B \Rightarrow B=2$
- If $x=2 \Rightarrow 3 = A \Rightarrow A=3$
- If $x=0 \Rightarrow -3 = 4 - 2B + 2C$
  $-3 = 3 - 4 + 2C$
  $C = -1$

$\therefore \dfrac{2x^2-x-3}{(x-2)(x-1)^2} \equiv \dfrac{3}{x-2} + \dfrac{2}{(x-1)^2} - \dfrac{1}{x-1}$

(b) $\dfrac{y^2-2y+8}{(y+2)(y-2)^2} \equiv \dfrac{A}{y+2} + \dfrac{B}{(y-2)^2} + \dfrac{C}{y-2}$

$y^2-2y+8 \equiv A(y-2)^2 + B(y+2) + C(y-2)(y+2)$

- If $y=2 \Rightarrow 8 = 4B \Rightarrow B=2$
- If $y=-2 \Rightarrow 16 = 16A \Rightarrow A=1$
- If $y=0 \Rightarrow 8 = 4A + 2B - 4C$
  $8 = 4 + 4 - 4C$
  $C = 0$

$\dfrac{y^2-2y+8}{(y+2)(y-2)^2} \equiv \dfrac{1}{y+2} + \dfrac{2}{(y-2)^2}$

(c) $\dfrac{-3t^2+12t+7}{(t-3)(t+1)^2} \equiv \dfrac{A}{t-3} + \dfrac{B}{(t+1)^2} + \dfrac{C}{t+1}$

$-3t^2+12t+7 \equiv A(t+1)^2 + B(t-3) + C(t+1)(t-3)$

- $t=3$ $\quad 16 = 16A \Rightarrow A=1$
- $t=-1$ $\quad -8 = -4B \Rightarrow B=2$
- $t=0$ $\quad 7 = A - 3B - 3C$
  $7 = 1 - 6 - 3C$
  $C = -4$

$\therefore \dfrac{-3t^2+12t+7}{(t-3)(t+1)^2} \equiv \dfrac{1}{t-3} + \dfrac{2}{(t+1)^2} - \dfrac{4}{t+1}$

(d) $\dfrac{-3w^2+10w-11}{(w-2)(w-1)^2} \equiv \dfrac{A}{w-2} + \dfrac{B}{(w-1)^2} + \dfrac{C}{w-1}$

$-3w^2+10w-11 \equiv A(w-1)^2 + B(w-2) + C(w-1)(w-2)$

- If $w=1$ $\quad -4 = -B \Rightarrow B=4$
- If $w=2$ $\quad -3 = A \Rightarrow A=-3$
- If $w=0$ $\quad -11 = A - 2B + 2C$
  $-11 = -3 - 8 + 2C$
  $C = 0$

$\therefore \dfrac{-3w^2+10w-11}{(w-2)(w-1)^2} = \dfrac{4}{(w-1)^2} - \dfrac{3}{w-2}$

## Question 8

Express each of the following into partial fractions.

**a)** $\dfrac{2x^2-3}{(3-2x)(1-x)^2}$

**b)** $\dfrac{3y^2+17y+4}{y^2(y+4)}$

**c)** $\dfrac{t^2}{(t-2)(t-1)^2}$

**d)** $\dfrac{9w^2}{(2w+1)(w-1)^2}$

$\boxed{\dfrac{6}{3-2x}-\dfrac{2}{1-x}-\dfrac{1}{(1-x)^2}}$, $\boxed{\dfrac{1}{y^2}+\dfrac{4}{y}-\dfrac{1}{y+4}}$, $\boxed{\dfrac{4}{t-2}-\dfrac{1}{(t-1)^2}-\dfrac{3}{t-1}}$, $\boxed{\dfrac{4}{w-1}+\dfrac{3}{(w-1)^2}+\dfrac{1}{2w+1}}$

(a) $\dfrac{2x^2-3}{(3-2x)(1-x)^2} \equiv \dfrac{A}{3-2x}+\dfrac{B}{(1-x)^2}+\dfrac{C}{1-x}$

$\boxed{2x^2-3 \equiv A(1-x)^2+B(3-2x)+C(1-x)(3-2x)}$

- If $x=1 \Rightarrow -1 = B \Rightarrow \boxed{B=-1}$
- If $x=\frac{3}{2} \Rightarrow \frac{3}{2}=\frac{1}{4}A \Rightarrow \boxed{A=6}$
- If $x=0 \Rightarrow \left.\begin{array}{l} -3 = A+3B+3C \\ -3 = 6-3+3C \end{array}\right\} \Rightarrow \boxed{C=-2}$

$\dfrac{2x^2-3}{(3-2x)(1-x)} \equiv \dfrac{6}{3-2x}-\dfrac{1}{(1-x)^2}-\dfrac{2}{1-x}$ //

(b) $\dfrac{3y^2+17y+4}{y^2(y+4)} \equiv \dfrac{A}{y^2}+\dfrac{B}{y+4}+\dfrac{C}{y}$

$\boxed{3y^2+17y+4 \equiv A(y+4)+By^2+Cy(y+4)}$

- If $y=0 \Rightarrow 4=4A \Rightarrow \boxed{A=1}$
- If $y=-4 \Rightarrow -16=16B \Rightarrow \boxed{B=-1}$
- If $y=1 \Rightarrow \left.\begin{array}{l} 24 = 5A+B+5C \\ 24 = 5-1+5C \end{array}\right\} \Rightarrow \boxed{C=4}$

$\therefore \dfrac{3y^2+17y+4}{y^2(y+4)} \equiv \dfrac{1}{y^2}-\dfrac{1}{y+4}+\dfrac{4}{y}$ //

(c) $\dfrac{t^2}{(t-2)(t-1)^2} \equiv \dfrac{A}{t-2}+\dfrac{B}{(t-1)^2}+\dfrac{C}{t-1}$

$t^2 \equiv A(t-1)^2+B(t-2)+C(t-2)(t-1)$

- If $t=1 \Rightarrow 1=-B \Rightarrow \boxed{B=-1}$
- If $t=2 \Rightarrow 4=A \Rightarrow \boxed{A=4}$
- If $t=0 \Rightarrow \left.\begin{array}{l} 0 = 4-2B+2C \\ 0 = 4+2+2C \end{array}\right\} \Rightarrow \boxed{C=-3}$

$\therefore \dfrac{t^2}{(t-2)(t-1)^2} \equiv \dfrac{4}{t-2}-\dfrac{1}{(t-1)^2}-\dfrac{3}{t-1}$ //

(d) $\dfrac{9w^2}{(2w+1)(w-1)^2} \equiv \dfrac{A}{2w+1}+\dfrac{B}{(w-1)^2}+\dfrac{C}{(w-1)}$

$\boxed{9w^2 \equiv A(w-1)^2+B(2w+1)+C(w-1)(2w+1)}$

- If $w=1 \Rightarrow 9=3B \Rightarrow \boxed{B=3}$
- If $w=-\frac{1}{2} \Rightarrow \frac{9}{4}=\frac{9}{4}A \Rightarrow \boxed{A=1}$
- If $w=0 \Rightarrow \left.\begin{array}{l} 0 = A+B-C \\ 0 = 1+3-C \end{array}\right\} \Rightarrow \boxed{C=4}$

$\dfrac{9w^2}{(2w+1)(w-1)} \equiv \dfrac{1}{2w+1}+\dfrac{3}{(w-1)^2}+\dfrac{4}{w-1}$

**Question 9**

Express each of the following into partial fractions.

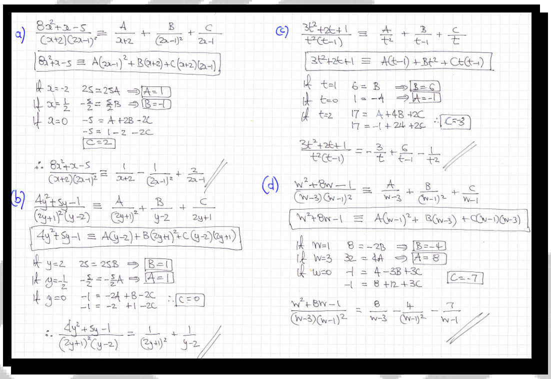
**a)** $\dfrac{8x^2+x-5}{(x+2)(2x-1)^2}$

**b)** $\dfrac{4y^2+5y-1}{(2y+1)^2(y-2)}$

**c)** $\dfrac{3t^2+2t+1}{t^2(t-1)}$

**d)** $\dfrac{w^2+8w-1}{(w-3)(w-1)^2}$

$\boxed{\dfrac{1}{x+2}+\dfrac{2}{2x-1}-\dfrac{1}{(2x-1)^2}}$, $\boxed{\dfrac{1}{(2y+1)^2}+\dfrac{1}{y-2}}$, $\boxed{\dfrac{6}{t-1}-\dfrac{1}{t^2}-\dfrac{3}{t}}$, $\boxed{\dfrac{8}{w-3}-\dfrac{4}{(w-1)^2}-\dfrac{7}{w-1}}$



a) $\dfrac{8x^2+x-5}{(x+2)(2x-1)^2} \equiv \dfrac{A}{x+2}+\dfrac{B}{(2x-1)^2}+\dfrac{C}{2x-1}$

$\boxed{8x^2+x-5 \equiv A(2x-1)^2+B(x+2)+C(x+2)(2x-1)}$

If $x=-2$ $\quad 25=25A \Rightarrow \boxed{A=1}$

If $x=\frac{1}{2}$ $\quad -\frac{5}{2}=\frac{5}{2}B \Rightarrow \boxed{B=-1}$

If $x=0$ $\quad -5=A+2B-2C$

$-5=1-2-2C$

$\boxed{C=2}$

$\therefore \dfrac{8x^2+x-5}{(x+2)(2x-1)^2} \equiv \dfrac{1}{x+2}-\dfrac{1}{(2x-1)^2}+\dfrac{2}{2x-1}$

(b) $\dfrac{4y^2+5y-1}{(2y+1)^2(y-2)} \equiv \dfrac{A}{(2y+1)^2}+\dfrac{B}{y-2}+\dfrac{C}{2y+1}$

$\boxed{4y^2+5y-1 \equiv A(y-2)+B(2y+1)^2+C(y-2)(2y+1)}$

If $y=2$ $\quad 25=25B \Rightarrow \boxed{B=1}$

If $y=-\frac{1}{2}$ $\quad -\frac{5}{2}=-\frac{5}{2}A \Rightarrow \boxed{A=1}$

If $y=0$ $\quad -1=-2A+B-2C$

$-1=-2+1-2C \quad \therefore \boxed{C=0}$

$\therefore \dfrac{4y^2+5y-1}{(2y+1)^2(y-2)} = \dfrac{1}{(2y+1)^2}+\dfrac{1}{y-2}$

(c) $\dfrac{3t^2+2t+1}{t^2(t-1)} \equiv \dfrac{A}{t^2}+\dfrac{B}{t-1}+\dfrac{C}{t}$

$\boxed{3t^2+2t+1 \equiv A(t-1)+Bt^2+Ct(t-1)}$

If $t=1$ $\quad 6=B \Rightarrow \boxed{B=6}$

If $t=0$ $\quad 1=-A \Rightarrow \boxed{A=-1}$

If $t=2$ $\quad 17=A+4B+2C$

$17=-1+24+2C \quad \therefore \boxed{C=-3}$

$\dfrac{3t^2+2t+1}{t^2(t-1)} = -\dfrac{3}{t}+\dfrac{6}{t-1}-\dfrac{1}{t^2}$

(d) $\dfrac{w^2+8w-1}{(w-3)(w-1)^2} \equiv \dfrac{A}{w-3}+\dfrac{B}{(w-1)^2}+\dfrac{C}{w-1}$

$\boxed{w^2+8w-1 \equiv A(w-1)^2+B(w-3)+C(w-1)(w-3)}$

If $w=1$ $\quad 8=-2B \Rightarrow \boxed{B=-4}$

If $w=3$ $\quad 32=4A \Rightarrow \boxed{A=8}$

If $w=0$ $\quad -1=A-3B+3C$

$-1=8+12+3C \quad \boxed{C=-7}$

$\dfrac{w^2+8w-1}{(w-3)(w-1)^2} = \dfrac{8}{w-3}-\dfrac{4}{(w-1)^2}-\dfrac{7}{w-1}$

**Question 10**

Express each of the following into partial fractions.

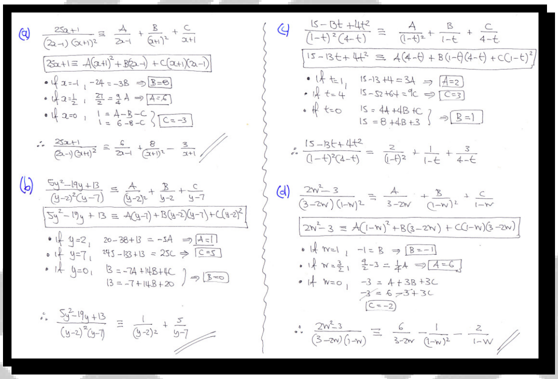
**a)** $\dfrac{25x+1}{(2x-1)(x+1)^2}$

**b)** $\dfrac{5y^2-19y+13}{(y-2)^2(y-7)}$

**c)** $\dfrac{15-13t+4t^2}{(1-t)^2(4-t)}$

**d)** $\dfrac{2w^2-3}{(3-2w)(1-w)^2}$

$\boxed{\dfrac{6}{2x-1}-\dfrac{3}{x+1}+\dfrac{8}{(x+1)^2}}$, $\boxed{\dfrac{1}{(y-2)^2}+\dfrac{5}{y-7}}$, $\boxed{\dfrac{3}{4-t}+\dfrac{1}{1-t}+\dfrac{2}{(1-t)^2}}$, $\boxed{\dfrac{6}{3-2w}-\dfrac{1}{(1-w)^2}-\dfrac{2}{1-w}}$



(a) $\dfrac{25x+1}{(2x-1)(x+1)^2} \equiv \dfrac{A}{2x-1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x+1}$

$\boxed{25x+1 \equiv A(x+1)^2+B(2x-1)+C(x+1)(2x-1)}$

- If $x=-1$, $-24=-3B \Rightarrow \boxed{B=8}$
- If $x=\frac{1}{2}$, $\frac{27}{2}=\frac{9}{4}A \Rightarrow \boxed{A=6}$
- If $x=0$, $1=A-B-C$, $1=6-8-C$ $\}$ $\boxed{C=-3}$

$\therefore \dfrac{25x+1}{(2x-1)(x+1)^2} \equiv \dfrac{6}{2x-1}+\dfrac{8}{(x+1)^2}-\dfrac{3}{x+1}$

(b) $\dfrac{5y^2-19y+13}{(y-2)^2(y-7)} \equiv \dfrac{A}{(y-2)^2}+\dfrac{B}{y-2}+\dfrac{C}{y-7}$

$\boxed{5y^2-19y+13 \equiv A(y-7)+B(y-2)(y-7)+C(y-2)^2}$

- If $y=2$, $20-38+13=-5A \Rightarrow \boxed{A=1}$
- If $y=7$, $245-133+13=25C \Rightarrow \boxed{C=5}$
- If $y=0$, $13=-7A+14B+4C$, $13=-7+14B+20$ $\} \Rightarrow \boxed{B=0}$

$\therefore \dfrac{5y^2-19y+13}{(y-2)^2(y-7)} \equiv \dfrac{1}{(y-2)^2}+\dfrac{5}{y-7}$

(c) $\dfrac{15-13t+4t^2}{(1-t)^2(4-t)} \equiv \dfrac{A}{(1-t)^2}+\dfrac{B}{1-t}+\dfrac{C}{4-t}$

$\boxed{15-13t+4t^2 \equiv A(4-t)+B(1-t)(4-t)+C(1-t)^2}$

- If $t=1$, $15-13+4=3A \Rightarrow \boxed{A=2}$
- If $t=4$, $15-52+64=9C \Rightarrow \boxed{C=3}$
- If $t=0$, $15=4A+4B+C$, $15=8+4B+3$ $\} \Rightarrow \boxed{B=1}$

$\therefore \dfrac{15-13t+4t^2}{(1-t)^2(4-t)} = \dfrac{2}{(1-t)^2}+\dfrac{1}{1-t}+\dfrac{3}{4-t}$

(d) $\dfrac{2w^2-3}{(3-2w)(1-w)^2} \equiv \dfrac{A}{3-2w}+\dfrac{B}{(1-w)^2}+\dfrac{C}{1-w}$

$\boxed{2w^2-3 \equiv A(1-w)^2+B(3-2w)+C(1-w)(3-2w)}$

- If $w=1$, $-1=B \Rightarrow \boxed{B=-1}$
- If $w=\frac{3}{2}$, $\frac{9}{2}-3=\frac{1}{4}A \Rightarrow \boxed{A=6}$
- If $w=0$, $-3=A+3B+3C$, $\cancel{-3}=6\cancel{-3}+3C$, $\boxed{C=-2}$

$\therefore \dfrac{2w^2-3}{(3-2w)(1-w)} \equiv \dfrac{6}{3-2w}-\dfrac{1}{(1-w)^2}-\dfrac{2}{1-w}$

**Question 11**

Express each of the following into partial fractions.

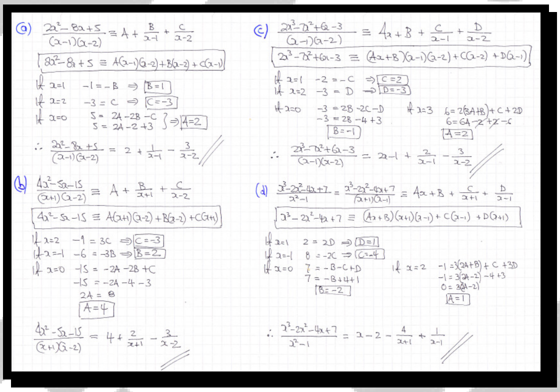
a) $\dfrac{2x^2 - 8x + 5}{(x-1)(x-2)}$

b) $\dfrac{4x^2 - 5x - 15}{(x+1)(x-2)}$

c) $\dfrac{2x^3 - 7x^2 + 6x - 3}{(x-1)(x-2)}$

d) $\dfrac{x^3 - 2x^2 - 4x + 7}{x^2 - 1}$

$\boxed{2 + \frac{1}{x-1} - \frac{3}{x-2}}$, $\boxed{4 + \frac{2}{x+1} - \frac{3}{x-2}}$, $\boxed{2x - 1 + \frac{2}{x-1} - \frac{3}{x-2}}$, $\boxed{x - 2 + \frac{1}{x-1} - \frac{4}{x+1}}$



(a) $\frac{2x^2-8x+5}{(x-1)(x-2)} \equiv A + \frac{B}{x-1} + \frac{C}{x-2}$

$\boxed{2x^2-8x+5 \equiv A(x-1)(x-2)+B(x-2)+C(x-1)}$

If $x=1$ $\quad -1=-B \Rightarrow \boxed{B=1}$

If $x=2$ $\quad -3=C \Rightarrow \boxed{C=-3}$

If $x=0$ $\quad 5=2A-2B-C$, $5=2A-2+3$ $\Rightarrow \boxed{A=2}$

$\therefore \frac{2x^2-8x+5}{(x-1)(x-2)} = 2 + \frac{1}{x-1} - \frac{3}{x-2}$

(b) $\frac{4x^2-5x-15}{(x+1)(x-2)} \equiv A + \frac{B}{x+1} + \frac{C}{x-2}$

$\boxed{4x^2-5x-15 \equiv A(x+1)(x-2)+B(x-2)+C(x+1)}$

If $x=2$ $\quad -9=3C \Rightarrow \boxed{C=-3}$

If $x=-1$ $\quad -6=-3B \Rightarrow \boxed{B=2}$

If $x=0$ $\quad -15=-2A-2B+C$, $-15=-2A-4-3$, $2A=8$, $\boxed{A=4}$

$\frac{4x^2-5x-15}{(x+1)(x-2)} = 4 + \frac{2}{x+1} - \frac{3}{x-2}$

(c) $\frac{2x^3-7x^2+6x-3}{(x-1)(x-2)} \equiv Ax + B + \frac{C}{x-1} + \frac{D}{x-2}$

$\boxed{2x^3-7x^2+6x-3 \equiv (Ax+B)(x-1)(x-2)+C(x-2)+D(x-1)}$

If $x=1$ $\quad -2=-C \Rightarrow \boxed{C=2}$

If $x=2$ $\quad -3=D \Rightarrow \boxed{D=-3}$

If $x=0$ $\quad -3=2B-2C-D$, $-3=2B-4+3$, $\boxed{B=-1}$

If $x=3$ $\quad 6=2(3A+B)+C+2D$, $6=6A-\cancel{2}+\cancel{2}-6$, $\boxed{A=2}$

$\therefore \frac{2x^3-7x^2+6x-3}{(x-1)(x-2)} = 2x-1 + \frac{2}{x-1} - \frac{3}{x-2}$

(d) $\frac{x^3-2x^2-4x+7}{x^2-1} = \frac{x^3-2x^2-4x+7}{(x+1)(x-1)} \equiv Ax + B + \frac{C}{x+1} + \frac{D}{x-1}$

$\boxed{x^3-2x^2-4x+7 \equiv (Ax+B)(x+1)(x-1)+C(x-1)+D(x+1)}$

If $x=1$ $\quad 2=2D \Rightarrow \boxed{D=1}$

If $x=-1$ $\quad 8=-2C \Rightarrow \boxed{C=-4}$

If $x=0$ $\quad 7=-B-C+D$, $7=-B+4+1$, $\boxed{B=-2}$

If $x=2$ $\quad -1=3(2A+B)+C+3D$, $-1=3(2A-2)-4+3$, $0=3(2A-2)$, $\boxed{A=1}$

$\therefore \frac{x^3-2x^2-4x+7}{x^2-1} = x-2 - \frac{4}{x+1} + \frac{1}{x-1}$

## Question 12

Express each of the following into partial fractions.

a) $\dfrac{10x^2+8}{(x+1)(5x+1)}$

b) $\dfrac{2y^2}{y^2-16}$

c) $\dfrac{2t^2-t+11}{(t+2)(2t-3)}$

d) $\dfrac{3w^2-31w+45}{(w-2)(w-7)}$

$\boxed{2+\dfrac{21}{2(5x+1)}-\dfrac{9}{2(x+1)}}$, $\boxed{2+\dfrac{4}{y-4}-\dfrac{4}{y+4}}$, $\boxed{1+\dfrac{4}{2t-3}-\dfrac{3}{t+2}}$, $\boxed{3+\dfrac{1}{w-2}-\dfrac{5}{w-7}}$

(a) $\dfrac{10x^2+8}{(x+1)(5x+1)} = \dfrac{10x^2+8}{5x^2+6x+1}$

$\dfrac{10x^2+8}{(x+1)(5x+1)} \equiv A + \dfrac{B}{x+1} + \dfrac{C}{5x+1}$

$10x^2+8 \equiv A(x+1)(5x+1) + B(5x+1) + C(x+1)$

- If $x=-1$ $\quad 18 = -4B \Rightarrow B = -\frac{9}{2}$
- If $x=-\frac{1}{5}$ $\quad \frac{42}{5} = \frac{4}{5}C \Rightarrow C = \frac{21}{2}$
- If $x=0$ $\quad 8 = A+B+C$, $\quad 8 = A - \frac{9}{2} + \frac{21}{2}$, $\quad A = 2$

$\therefore \dfrac{10x^2+8}{(x+1)(5x+1)} = 2 + \dfrac{21}{2(5x+1)} - \dfrac{9}{2(x+1)}$

(b) $\dfrac{2y^2}{y^2-16} = \dfrac{2y^2}{(y+4)(y-4)} = A + \dfrac{B}{y-4} + \dfrac{C}{y+4}$

$A(y-4)(y+4) + B(y+4) + C(y-4) \equiv 2y^2$

- If $y=4$ $\quad 8B = 32 \Rightarrow B = 4$
- If $y=-4$ $\quad -8C = 32 \Rightarrow C = -4$
- If $y=0$ $\quad \left.\begin{array}{l} -16A+4B-4C=0 \\ 16+16=16A \end{array}\right\} \Rightarrow A=2$

$\therefore \dfrac{2y^2}{y^2-16} \equiv 2 + \dfrac{4}{y-4} - \dfrac{4}{y+4}$

(c) $\dfrac{2t^2-t+11}{(t+2)(2t-3)} \equiv \dfrac{2t^2-t+11}{2t^2+t-6} \equiv A + \dfrac{B}{t+2} + \dfrac{C}{2t-3}$

$2t^2-t+11 \equiv A(t+2)(2t-3) + B(2t-3) + C(t+2)$

- If $t=-2$ $\quad 21 = -7B \Rightarrow B = -3$
- If $t=\frac{3}{2}$ $\quad 14 = \frac{7}{2}C \Rightarrow C = 4$
- If $t=0$ $\quad \left.\begin{array}{l} 11 = -6A-3B+2C \\ 11 = -6A+9+8 \\ 6A = 6 \end{array}\right\} \Rightarrow A=1$

$\therefore \dfrac{2t^2-t+11}{(t+2)(2t-3)} \equiv 1 + \dfrac{4}{2t-3} - \dfrac{3}{t+2}$

(d) $\dfrac{3w^2-31w+45}{(w-2)(w-7)} \equiv \dfrac{3w^2-31w+45}{w^2-9w+14} \equiv A + \dfrac{B}{w-2} + \dfrac{C}{w-7}$

$3w^2-31w+45 \equiv A(w-2)(w-7) + B(w-7) + C(w-2)$

- If $w=2$ $\quad -5 = -5B \Rightarrow B = 1$
- If $w=7$ $\quad -25 = 5C \Rightarrow C = -5$
- If $w=0$ $\quad \left.\begin{array}{l} 45 = 14A-7B-2C \\ 45 = 14A-7+10 \\ 14A = 42 \end{array}\right\} \Rightarrow A=3$

$\therefore \dfrac{3w^2-31w+45}{(w-2)(w-7)} \equiv 3 + \dfrac{1}{w-2} - \dfrac{5}{w-7}$

**Question 13**

Express each of the following into partial fractions.

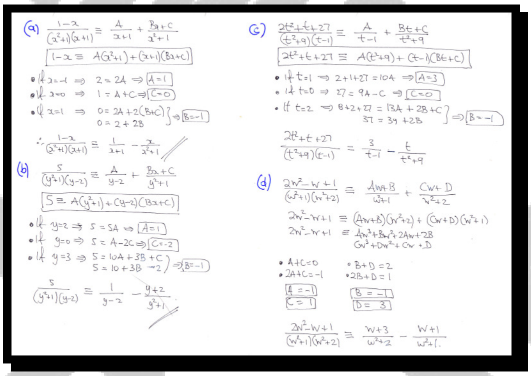
a) $\dfrac{1-x}{\left(x^2+1\right)(x+1)}$

b) $\dfrac{5}{\left(y^2+1\right)(y-2)}$

c) $\dfrac{2t^2+t+27}{\left(t^2+9\right)(t-1)}$

d) $\dfrac{2w^2-w+1}{\left(w^2+1\right)\left(w^2+2\right)}$

$\boxed{\dfrac{1}{x+1}-\dfrac{x}{x^2+1}}$, $\boxed{\dfrac{1}{y-2}-\dfrac{y+2}{y^2+1}}$, $\boxed{\dfrac{3}{t-1}-\dfrac{t}{t^2+9}}$, $\boxed{\dfrac{w+3}{w^2+2}-\dfrac{w+1}{w^2+1}}$



(a) $\dfrac{1-x}{(x^2+1)(x+1)} \equiv \dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$

$\boxed{1-x \equiv A(x^2+1)+(x+1)(Bx+C)}$

- If $x=-1 \Rightarrow 2=2A \Rightarrow \boxed{A=1}$
- If $x=0 \Rightarrow 1=A+C \Rightarrow \boxed{C=0}$
- If $x=1 \Rightarrow 0=2A+2(B+C)$, $0=2+2B \Rightarrow \boxed{B=-1}$

$\therefore \dfrac{1-x}{(x^2+1)(x+1)} \equiv \dfrac{1}{x+1}-\dfrac{x}{x^2+1}$

(b) $\dfrac{5}{(y^2+1)(y-2)} \equiv \dfrac{A}{y-2}+\dfrac{Bx+C}{y^2+1}$

$\boxed{5 \equiv A(y^2+1)+(y-2)(Bx+C)}$

- If $y=2 \Rightarrow 5=5A \Rightarrow \boxed{A=1}$
- If $y=0 \Rightarrow 5=A-2C \Rightarrow \boxed{C=-2}$
- If $y=3 \Rightarrow 5=10A+3B+C$, $5=10+3B-2 \Rightarrow \boxed{B=-1}$

$\dfrac{5}{(y^2+1)(y-2)} \equiv \dfrac{1}{y-2}-\dfrac{y+2}{y^2+1}$

(c) $\dfrac{2t^2+t+27}{(t^2+9)(t-1)} \equiv \dfrac{A}{t-1}+\dfrac{Bt+C}{t^2+9}$

$\boxed{2t^2+t+27 \equiv A(t^2+9)+(t-1)(Bt+C)}$

- If $t=1 \Rightarrow 2+1+27=10A \Rightarrow \boxed{A=3}$
- If $t=0 \Rightarrow 27=9A-C \Rightarrow \boxed{C=0}$
- If $t=2 \Rightarrow 8+2+27=13A+2B+C$, $37=39+2B \Rightarrow \boxed{B=-1}$

$\dfrac{2t^2+t+27}{(t^2+9)(t-1)} = \dfrac{3}{t-1}-\dfrac{t}{t^2+9}$

(d) $\dfrac{2w^2-w+1}{(w^2+1)(w^2+2)} \equiv \dfrac{Aw+B}{w^2+1}+\dfrac{Cw+D}{w^2+2}$

$2w^2-w+1 \equiv (Aw+B)(w^2+2)+(Cw+D)(w^2+1)$

$2w^2-w+1 \equiv Aw^3+Bw^2+2Aw+2B$ $\quad Cw^3+Dw^2+Cw+D$

- $A+C=0$
- $2A+C=-1$

$\boxed{A=-1}$ $\boxed{C=1}$

- $B+D=2$
- $2B+D=1$

$\boxed{B=-1}$ $\boxed{D=3}$

$\dfrac{2w^2-w+1}{(w^2+1)(w^2+2)} \equiv \dfrac{w+3}{w^2+2}-\dfrac{w+1}{w^2+1}.$

**Question 14**

Express each of the following into partial fractions.

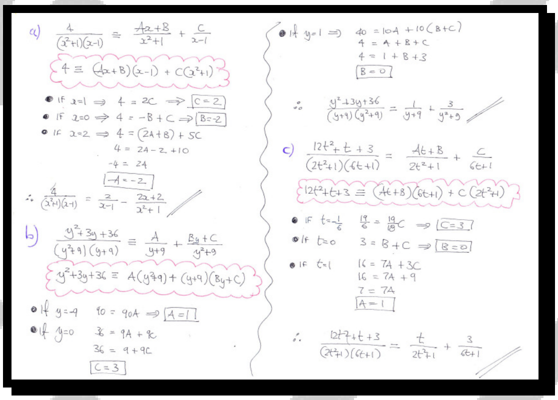
**a)** $\dfrac{4}{\left(x^2+1\right)(x-1)}$

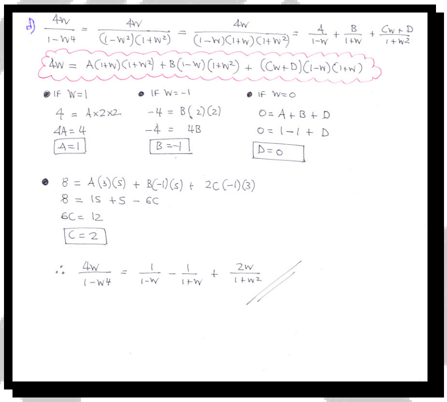
**b)** $\dfrac{y^2+3y+36}{\left(y^2+9\right)(y+9)}$

**c)** $\dfrac{12t^2+t+3}{\left(2t^2+1\right)(6t+1)}$

**d)** $\dfrac{4w}{1-w^4}$

$\boxed{\dfrac{2}{x-1}-\dfrac{2x+2}{x^2+1}}$, $\boxed{\dfrac{1}{y+9}+\dfrac{3}{y^2+9}}$, $\boxed{\dfrac{3}{6t+1}+\dfrac{t}{2t^2+1}}$, $\boxed{\dfrac{1}{1-w}-\dfrac{1}{1+w}+\dfrac{2w}{1+w^2}}$





**Question 15**

Express each of the following into partial fractions.

**a)** $\dfrac{1-4x}{(x^2+3)(x-2)^2}$

**b)** $\dfrac{y^2+3y+36}{(y^2+9)(y+9)}$

**c)** $\dfrac{12t^2+t+3}{(2t^2+1)(6t+1)}$

**d)** $\dfrac{4w}{1-w^4}$

$\boxed{\dfrac{1}{x^2+3}-\dfrac{1}{(x-2)^2}}$, $\boxed{\dfrac{1}{y+9}+\dfrac{1}{y^2+9}}$, $\boxed{\dfrac{3}{6t+1}+\dfrac{t}{2t^2+1}}$, $\boxed{\dfrac{1}{1-w}-\dfrac{1}{1+w}+\dfrac{2w}{1+w^2}}$

**Question 16**

Express each of the following into partial fractions.

**a)** $$\frac{4x^3-3x^2+4}{(2x-1)\left(x^2+1\right)}$$

**b)** $$\frac{y^2+3y+36}{\left(y^2+9\right)(y+9)}$$

**c)** $$\frac{12t^2+t+3}{\left(2t^2+1\right)(6t+1)}$$

**d)** $$\frac{4w}{1-w^4}$$

$$\boxed{2+\frac{3}{2x-1}-\frac{2x+3}{x^2+1}},\ \boxed{\frac{1}{y+9}+\frac{1}{y^2+9}},\ \boxed{\frac{3}{6t+1}+\frac{t}{2t^2+1}},\ \boxed{\frac{1}{1-w}-\frac{1}{1+w}+\frac{2w}{1+w^2}}$$