DISCRIMINANT PRACTICE

26
BASIC
QUESTIONS

Question 1 (**)

The quadratic equation

$$x^2 + 10x + k = 0,$$

where k is a constant, has no real roots.

Find the range of the possible values of k.

$\boxed{k > 25}$

$x^2+10x+k=0$

NO REAL ROOTS $\Rightarrow b^2-4ac<0$

$\Rightarrow 10^2-4\times1\times k<0$

$\Rightarrow 100-4k<0$

$\Rightarrow -4k<-100$

$\Rightarrow k>25$

Question 2 (**)

It is given that

$$f(x) \equiv 25x^2 + 20x + p,$$

where p is a non zero constant.

The quadratic equation $f(x) = 0$ has equal roots.

Find the value of p.

$\boxed{p = 4}$

$25x^2+20x+p=0$

EQUAL ROOTS $\Rightarrow b^2-4ac=0$

$\Rightarrow 20^2-4\times25\times p=0$

$\Rightarrow 400-100p=0$

$\Rightarrow 400=100p$

$\therefore p=4$

Question 3 **(**)**

The quadratic equation

$$mx^2 + 12x + m = 0,$$

where m is a constant, has repeated roots.

Find the possible values of m.

$\boxed{m = \pm 6}$

$mx^2 + 12x + m = 0$

REPEATED ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow 12^2 - 4m \times m = 0$

$\Rightarrow 144 - 4m^2 = 0$

$\Rightarrow 144 = 4m^2$

$\Rightarrow m^2 = 36$

$\Rightarrow m = 6$ or -6

Question 4 **(**)**

The quadratic equation

$$3x^2 + 2x + p = 0,$$

where p is a constant, has two distinct real roots.

Find the range of possible values of p.

$\boxed{p < \frac{1}{3}}$

$3x^2 + 2x + p = 0$

DISTINCT REAL ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow 2^2 - 4 \times 3 \times p > 0$

$\Rightarrow 4 - 12p > 0$

$-12p > -4$

$12p < 4$

$p < \frac{1}{3}$

Question 5 (**)

The quadratic equation

$$x^2 + 3x + m = 0,$$

where m is a constant, has no real roots.

Find the range of possible values of m.

$$\boxed{m > \frac{9}{4}}$$

$x^2+3x+m=0$

NO REAL ROOTS $\Rightarrow b^2-4ac<0$

$\Rightarrow 3^2-4\times1\times m<0$

$\Rightarrow 9-4m<0$

$\Rightarrow -4m<-9$

$\Rightarrow 4m>9$

$\Rightarrow m>\frac{9}{4}$

Question 6 (**)

Find the range of the possible values of the constant p, given that the equation

$$px^2 - 5x + 5 = 0$$

has real roots.

$$\boxed{p \le \frac{5}{4}}$$

$px^2-5x+5=0$

REAL ROOTS (ONE OR TWO) $\Rightarrow b^2-4ac \geqslant 0$

$\Rightarrow (-5)^2-4\times p\times 5 \geqslant 0$

$\Rightarrow 25-20p \geqslant 0$

$\Rightarrow -20p \geqslant -25$

$\Rightarrow 20p \leqslant 25$

$p \leqslant \frac{5}{4}$

Question 7 (**+)

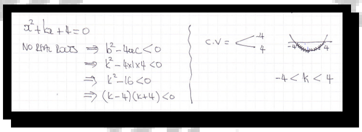
The quadratic equation

$$x^2 + kx + 4 = 0,$$

where k is a constant, has no real roots.

Find the range of possible values of k.

$\boxed{-4 < k < 4}$



$x^2 + kx + 4 = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow k^2 - 4 \times 1 \times 4 < 0$

$\Rightarrow k^2 - 16 < 0$

$\Rightarrow (k-4)(k+4) < 0$

C.V. = -4, 4

$-4 < k < 4$

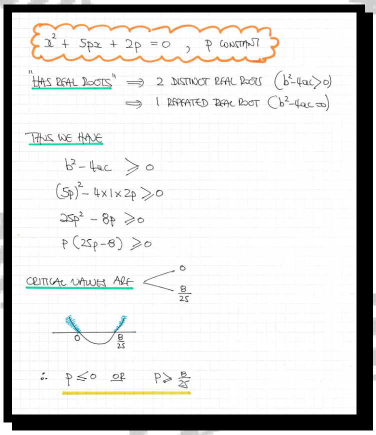
Question 8 (**+)

Find the range of the possible values of the constant p, given that the equation

$$x^2 + 5px + 2p = 0$$

has real roots.

$\boxed{}$, $\boxed{p \le 0 \quad \text{or} \quad p \ge \frac{8}{25}}$



$x^2 + 5px + 2p = 0$, p CONSTANT

"HAS REAL ROOTS" $\Rightarrow$ 2 DISTINCT REAL ROOTS $(b^2 - 4ac > 0)$

$\Rightarrow$ 1 REPEATED REAL ROOT $(b^2 - 4ac = 0)$

THUS WE HAVE

$b^2 - 4ac \geqslant 0$

$(5p)^2 - 4 \times 1 \times 2p \geqslant 0$

$25p^2 - 8p \geqslant 0$

$p(25p - 8) \geqslant 0$

CRITICAL VALUES ARE 0, $\frac{8}{25}$

$\therefore\ p \leqslant 0$ OR $p \geqslant \frac{8}{25}$

Question 9 **(**+)**

$$f(x) \equiv 9x^2 - 6x + c,$$

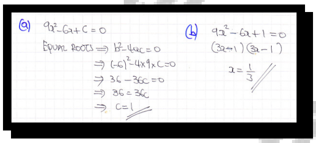
where c is a non zero constant.

The equation $f(x) = 0$ has equal roots.

a) Determine the value of c.

b) Solve the equation $f(x) = 0$ for the value of c found in part (a).

$\boxed{c = 1}$, $\boxed{x = \frac{1}{3}}$



Question 10 **(**+)**

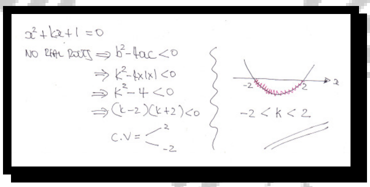
$$f(x) \equiv x^2 + kx + 1,$$

where k is a constant.

The equation $f(x) = 0$ has no real roots.

Determine the range of the possible values of k.

$\boxed{-2 < k < 2}$



Question 11 (**+)

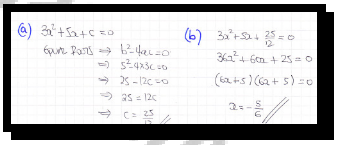
The equation $3x^2 + 5x + c = 0$, where c is a constant, has equal roots.

a) Determine the value of c.

b) Solve the equation

$$3x^2 + 5x + c = 0.$$

$\boxed{c = \frac{25}{12}}$, $\boxed{x = -\frac{5}{6}}$



Question 12 (**+)

It is given that

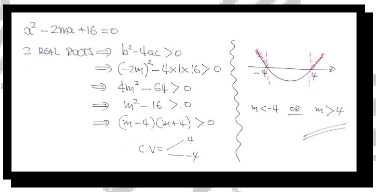
$$f(x) \equiv x^2 - 2mx + 16,$$

where m is a constant.

The equation $f(x) = 0$ has two distinct real roots.

Determine the range of values of m.

$\boxed{m < -4 \text{ or } m > 4}$



Question 13 (**+)

It is given that

$$f(x) \equiv x^2 + kx + k,$$

where k is a constant.

The equation $f(x) = 0$ has two distinct real roots.

Determine the range of the possible values of k.

$\boxed{k < 0 \text{ or } k > 4}$

$x^2 + kx + k = 0$

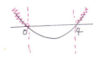
2 DISTINCT REAL ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow k^2 - 4 \times 1 \times k > 0$

$\Rightarrow k^2 - 4k > 0$

$\Rightarrow k(k-4) > 0$

C.V: $\begin{cases} 0 \\ 4 \end{cases}$



$k < 0$ OR $k > 4$

Question 14 (**+)

The quadratic equation

$$x^2 + 3mx + m = 0,$$

where m is a constant, has real roots.

Find the range of possible values of m.

$\boxed{m \le 0 \text{ or } m \ge \frac{4}{9}}$

$x^2 + 3mx + m = 0$

REAL ROOTS (ONE OR TWO) $\Rightarrow b^2 - 4ac \geqslant 0$

$\Rightarrow (3m)^2 - 4 \times 1 \times m \geqslant 0$

$\Rightarrow 9m^2 - 4m \geqslant 0$

$\Rightarrow m(9m-4) \geqslant 0$

C.V $= \begin{cases} 0 \\ \frac{4}{9} \end{cases}$

$m \le 0$ OR $m \geqslant \frac{4}{9}$

Question 15 **(***)**

The quadratic equation

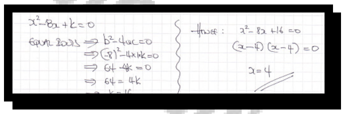
$$x^2 - 8x + k = 0,$$

where k is a constant, has equal roots.

Solve the equation

$$x^2 - 8x + k = 0.$$

$\boxed{x = 4}$



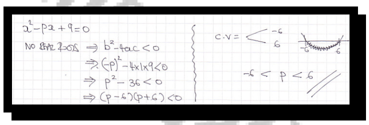
Question 16 **(***)**

Find the range of the values of the constant p, given that the quadratic equation

$$x^2 - px + 9 = 0$$

has no real roots.

$\boxed{-6 < p < 6}$



Question 17 (***)

Find the range of values of the constant p so that the quadratic equation

$$2x^2 - 4x - (2p+1) = 0$$

has no real roots.

$$\boxed{p < -\frac{3}{2}}$$

$2x^2 - 4x - (2p+1) = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow (-4)^2 - 4 \times 2 \times [-(2p+1)] < 0$

$\Rightarrow 16 + 8(2p+1) < 0$

$\Rightarrow 16 + 16p + 8 < 0$

$16p < -24$

$p < -\frac{24}{16}$

$p < -\frac{3}{2}$

Question 18 (***)

Find the range of values of the constant k so that the quadratic equation

$$x^2 + 6kx - 2k = 0$$

has real roots.

$$\boxed{k \le -\frac{2}{9} \text{ or } k \ge 0}$$

$x^2 + 6kx - 2k = 0$

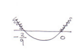
REAL ROOTS (ONE OR TWO) $\Rightarrow b^2 - 4ac \geqslant 0$

$\Rightarrow (6k)^2 - 4 \times 1 \times (-2k) \geqslant 0$

$\Rightarrow 36k^2 + 8k \geqslant 0$

$\Rightarrow 4k(9k+2) \geqslant 0$

C.V. $= 0, -\frac{2}{9}$



$k \le -\frac{2}{9}$ OR $k \geqslant 0$

Question 19 (***)

It is given that

$$f(x) = x^2 - kx + (k+3),$$

where k is a non zero constant.

If the equation $f(x) = 0$ has real roots find the range of the values of k.

$\boxed{}$, $\boxed{k \le -2}$, $\boxed{k \ge 6}$

$x^2 - kx + (k+3) = 0$

REAL ROOTS (ONE OR TWO) $\Rightarrow b^2 - 4ac \geqslant 0$

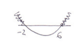
$\Rightarrow (-k)^2 - 4 \times 1 \times (k+3) \geqslant 0$

$\Rightarrow k^2 - 4(k+3) \geqslant 0$

$\Rightarrow k^2 - 4k - 12 \geqslant 0$

$\Rightarrow (k+2)(k-6) \geqslant 0$

C.V. = -2, 6



$k \leqslant -2$ OR $k \geqslant 6$

Question 20 (***)

Find the range of values of the constant p so that the quadratic equation

$$(3p-2)x^2 + 8x + p = 0, \quad p \neq \frac{2}{3}$$

has no real roots.

$\boxed{}$, $\boxed{p < -2 \text{ or } p > \frac{8}{3}}$

$(3p-2)x^2 + 8x + p = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow 8^2 - 4(3p-2)p < 0$

$\Rightarrow 64 - 4p(3p-2) < 0$

$\Rightarrow 64 - 12p^2 + 8p < 0$

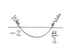
$\Rightarrow -12p^2 + 8p + 64 < 0$

$\Rightarrow 12p^2 - 8p - 64 > 0$

$\Rightarrow 3p^2 - 2p - 16 > 0$

$\Rightarrow (3p-8)(p+2) > 0$

C.V. = -2, $\frac{8}{3}$



$x < -2$ or $x > \frac{8}{3}$

Question 21 **(***)**

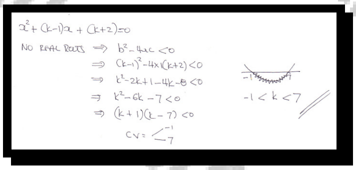
The quadratic equation

$$x^2+(k-1)x+(k+2)=0,$$

where k is a constant, has no real roots.

Find the range of possible values of k.

$\boxed{-1<k<7}$



Question 22 **(***)**

$$f(x)=x^2+(1-p)x+4,$$

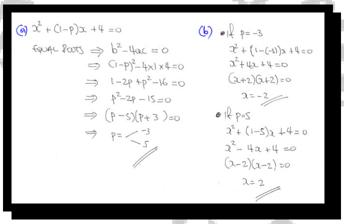
where p is a non zero constant.

The equation $f(x)=0$ has equal roots.

a) Determine the possible values of p.

b) Solve the equation $f(x)=0$ for each of the values of p found in part (**a**).

$\boxed{p=-3,\ 5}$, $\boxed{x=\pm2}$



Question 23 **(***)**

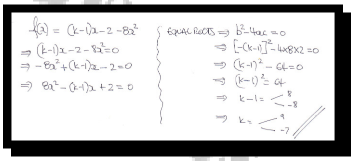
$$f(x) = (k-1)x - 2 - 8x^2,$$

where k is a non zero constant

The equation $f(x) = 0$ has equal roots.

Determine the possible values of k.

$\boxed{}$, $\boxed{k = -7, 9}$



$f(x) = (k-1)x - 2 - 8x^2$

$\Rightarrow (k-1)x - 2 - 8x^2 = 0$

$\Rightarrow -8x^2 + (k-1)x - 2 = 0$

$\Rightarrow 8x^2 - (k-1)x + 2 = 0$

EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-(k-1)]^2 - 4 \times 8 \times 2 = 0$

$\Rightarrow (k-1)^2 - 64 = 0$

$\Rightarrow (k-1)^2 = 64$

$\Rightarrow k - 1 = 8, -8$

$\Rightarrow k = 9, -7$

Question 24 **(***)**

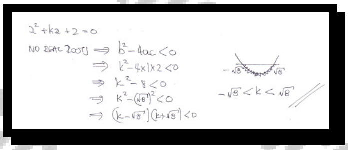
The quadratic equation

$$x^2 + kx + 2 = 0,$$

where k is a constant, has no real roots.

Find, as exact surds, the range of values of k.

$\boxed{-\sqrt{8} < k < \sqrt{8}}$



$x^2 + kx + 2 = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow k^2 - 4 \times 1 \times 2 < 0$

$\Rightarrow k^2 - 8 < 0$

$\Rightarrow k^2 - (\sqrt{8})^2 < 0$

$\Rightarrow (k - \sqrt{8})(k + \sqrt{8}) < 0$

$-\sqrt{8} < k < \sqrt{8}$

Question 25 (***)

The quadratic equation

$$2x^2 + (3k-1)x + (3k^2-1) = 0,$$

where k is a constant, has two different real roots.

Find the range of values of k.

$$\boxed{-1 < k < \frac{3}{5}}$$

$2x^2 + (3k-1)x + (3k^2-1) = 0$

2 DISTINCT REAL ROOTS, $b^2-4ac>0$

HENCE

$\Rightarrow (3k-1)^2 - 4\times 2\times(3k^2-1) > 0$

$\Rightarrow 9k^2 - 6k + 1 - 24k^2 + 8 > 0$

$\Rightarrow -15k^2 - 6k + 9 > 0$ DIVIDE BY -3

$\Rightarrow 5k^2 + 2k - 3 < 0$

$\Rightarrow (5k-3)(k+1) < 0$

C.V. $= -1, \frac{3}{5}$

$-1 < k < \frac{3}{5}$

Question 26 (***)

Find the range of values of the constant m so that the quadratic equation

$$x^2 + (m+3)x + (3m+4) = 0$$

has two distinct real roots.

$$\boxed{m < -1 \text{ or } m > 7}$$

$x^2 + (m+3)x + (3m+4) = 0$

TWO DISTINCT ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow (m+3)^2 - 4\times 1\times(3m+4) > 0$

$\Rightarrow m^2 + 6m + 9 - 12m - 16 > 0$

$\Rightarrow m^2 - 6m - 7 > 0$

$\Rightarrow (m+1)(m-7) > 0$

C.V. $= -1, 7$

$m < -1$ OR $m > 7$

25 STANDARD QUESTIONS

Question 1 (***+)

Find the range of values of the constant k so that the quadratic equation

$$x^2 + (2k+1)x + k^2 = 2$$

has real roots.

$\boxed{k \ge -\frac{9}{4}}$

$x^2 + (2k+1)x + k^2 = 2$

$\Rightarrow x^2 + (2k+1)x + k^2 - 2 = 0$

REAL ROOTS: $b^2 - 4ac \geqslant 0$

$\Rightarrow (2k+1)^2 - 4 \times 1 (k^2 - 2) \geqslant 0$

$\Rightarrow \cancel{4k^2} + 4k + 1 - \cancel{4k^2} + 8 \geqslant 0$

$\Rightarrow 4k \geqslant -9$

$\Rightarrow k \geqslant -\frac{9}{4}$

Question 2 (***+)

Find the range of values of the constant p so that the quadratic equation

$$x^2 + 2px + (2p+8) = 0$$

has real roots.

$\boxed{p \le -2 \text{ or } p \ge 4}$

$x^2 + 2px + (2p+8) = 0$

REAL ROOTS (ONE OR TWO) $\Rightarrow b^2 - 4ac \geqslant 0$

$\Rightarrow (2p)^2 - 4 \times 1 \times (2p+8) \geqslant 0$

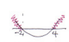
$\Rightarrow 4p^2 - 4(2p+8) \geqslant 0$

$\Rightarrow p^2 - (2p+8) \geqslant 0$

$\Rightarrow p^2 - 2p - 8 \geqslant 0$

$\Rightarrow (p-4)(p+2) \geqslant 0$

C.V. = -2, 4



$p \leqslant -2$ OR $p \geqslant 4$

Question 3 **(***+)**

The quadratic equation

$$mx^2 + 2(m+1)x + 4 = 0,$$

where m is a constant, has equal roots.

Find the possible value of m.

$$\boxed{m = 1}$$

$mx^2 + 2(m+1)x + 4 = 0$

EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [2(m+1)]^2 - 4 \times m \times 4 = 0$

$\Rightarrow 4(m+1)^2 - 16m = 0$

$\Rightarrow (m+1)^2 - 4m = 0$

$\Rightarrow m^2 + 2m + 1 - 4m = 0$

$\Rightarrow m^2 - 2m + 1 = 0$

HENCE

$(m-1)(m-1) = 0$

$m = 1$

Question 4 **(***+)**

The quadratic equation

$$(m+1)x^2 + 12x + (m-4) = 0,$$

where m is a constant, such that $m \neq -1$, has two distinct real roots.

Determine the range of possible values of m.

$$\boxed{-5 < m < 8,\ m \neq -1}$$

$(m+1)x^2 + 12x + (m-4) = 0$

DISTINCT REAL ROOTS. $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow 12^2 - 4(m+1)(m-4) > 0$

$\Rightarrow 144 - 4(m^2 - 3m - 4) > 0$

$\Rightarrow 36 - (m^2 - 3m - 4) > 0$

$\Rightarrow 36 - m^2 + 3m + 4 > 0$

$\Rightarrow -m^2 + 3m + 40 > 0$

$\Rightarrow m^2 - 3m - 40 < 0$

$\Rightarrow (m+5)(m-8) < 0$

C.V. $= -5,\ 8$

$-5 < m < 8$

Question 5 (***+)

Find the possible range of the values of the non zero constant k, so that the quadratic equation

$$kx^2 - x + (3k - 1) = 0$$

has distinct real roots.

$\boxed{-\frac{1}{6} < k < \frac{1}{2},\ k \neq 0}$

$kx^2 - x + (3k-1) = 0$

DISTINCT REAL ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow (-1)^2 - 4k(3k-1) > 0$

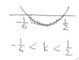
$\Rightarrow 1 - 12k^2 + 4k > 0$

$\Rightarrow -12k^2 + 4k + 1 > 0$

$\Rightarrow 12k^2 - 4k - 1 < 0$

$\Rightarrow (6k+1)(2k-1) < 0$

C.V. $= -\frac{1}{6}, \frac{1}{2}$



$-\frac{1}{6} < k < \frac{1}{2}$

Question 6 (***+)

The quadratic equation

$$x^2 + 2mx + 3m + 4 = 0,$$

where m is a constant, has equal roots.

Find the possible values of m.

$\boxed{}$, $\boxed{m = -1, 4}$

$x^2 + 2mx + (3m+4) = 0$

EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2m)^2 - 4 \times 1 \times (3m+4) = 0$

$\Rightarrow 4m^2 - 4(3m+4) = 0$

$\Rightarrow m^2 - (3m+4) = 0$

$\Rightarrow m^2 - 3m - 4 = 0$

$\Rightarrow (m+1)(m-4) = 0$

$\therefore m = 4, -1$

Question 7 (***+)

The quadratic equation

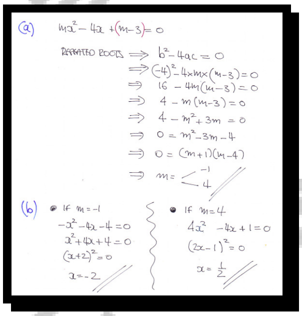
$$mx^2 - 4x + m - 3 = 0,$$

where m is a non zero constant, has repeated roots.

a) Find the possible values of m.

b) Hence solve the equation for each value of m found in part **(a)**.

$\boxed{m = -1, 4}$, $\boxed{x = -2, \frac{1}{2}}$



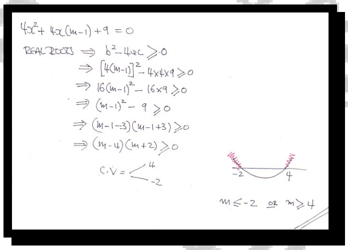
Question 8 (***+)

Find the range of the possible values of the constant m, given that the equation

$$4x^2 + 4x(m-1) + 9 = 0$$

has real roots.

$\boxed{m \le -2 \text{ or } m \ge 4}$



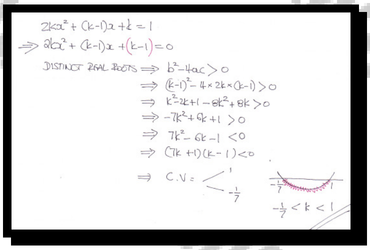
Question 9 **(***+)**

Find the range of values of the non zero constant k, given that the quadratic equation

$$2kx^2 + (k-1)x + k = 1$$

has distinct real roots.

$$\boxed{-\frac{1}{7} < k < 1, \ k \neq 0}$$



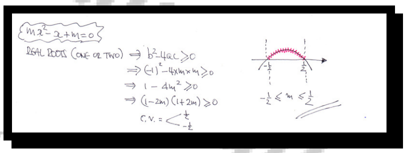
Question 10 **(***+)**

Find the range of values of the constant m so that the quadratic equation

$$mx^2 - x + m = 0$$

has real roots.

$$\boxed{-\frac{1}{2} \le m \le \frac{1}{2}}$$



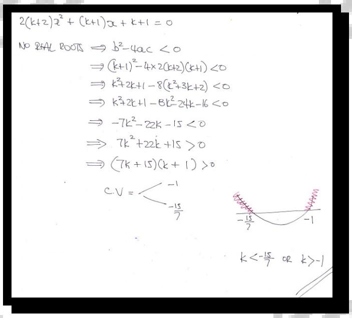
Question 11 (***+)

Find the range of the possible values of the constant k, $k \neq -2$, so that the quadratic equation

$$2(k+2)x^2 + (k+1)x + (k+1) = 0$$

has no real roots.

$$\boxed{k < -\frac{15}{7} \text{ or } k > -1}$$



$2(k+2)x^2 + (k+1)x + k+1 = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow (k+1)^2 - 4 \times 2(k+2)(k+1) < 0$

$\Rightarrow k^2 + 2k + 1 - 8(k^2 + 3k + 2) < 0$

$\Rightarrow k^2 + 2k + 1 - 8k^2 - 24k - 16 < 0$

$\Rightarrow -7k^2 - 22k - 15 < 0$

$\Rightarrow 7k^2 + 22k + 15 > 0$

$\Rightarrow (7k+15)(k+1) > 0$

C.V $= -1, \ -\frac{15}{7}$

$-\frac{15}{7}$ -1

$k < -\frac{15}{7}$ OR $k > -1$

Question 12 (****)

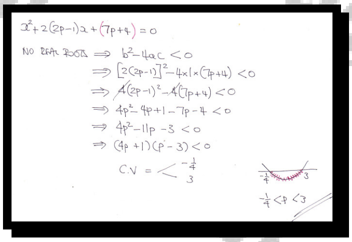
$$f(x) = x^2 + 2(2p-1)x + 7p + 4,$$

where p is a constant

The equation $f(x) = 0$ has no real roots.

Determine the range of the possible values of p.

$$\boxed{}, \ \boxed{-\frac{1}{4} < p < 3}$$



$x^2 + 2(2p-1)x + (7p+4) = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow [2(2p-1)]^2 - 4 \times 1 \times (7p+4) < 0$

$\Rightarrow \cancel{4}(2p-1)^2 - \cancel{4}(7p+4) < 0$

$\Rightarrow 4p^2 - 4p + 1 - 7p - 4 < 0$

$\Rightarrow 4p^2 - 11p - 3 < 0$

$\Rightarrow (4p+1)(p-3) < 0$

C.V $= -\frac{1}{4}, \ 3$

$-\frac{1}{4}$ 3

$-\frac{1}{4} < p < 3$

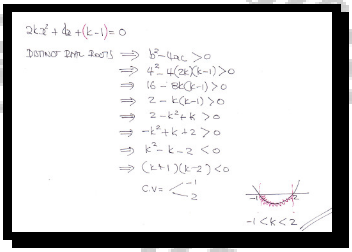
Question 13 (****)

Find the range of values of the non zero constant k so that the quadratic equation

$$2kx^2 + 4x + k - 1 = 0$$

has two distinct real roots.

$$\boxed{-1 < k < 2,\ k \neq 0}$$



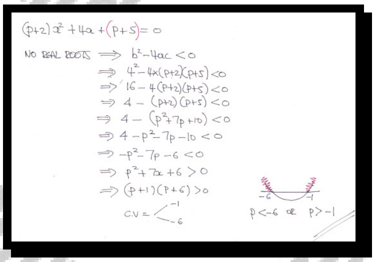
Question 14 (****)

Find the range of values of the constant p, $p \neq -2$, so that the quadratic equation

$$(p+2)x^2 + 4x + p + 5 = 0$$

has no real roots.

$$\boxed{p < -6 \text{ or } p > -1}$$



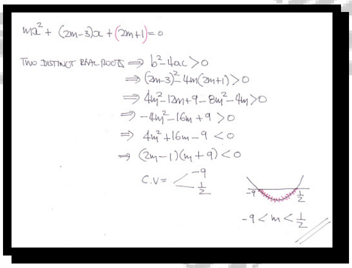
Question 15 **(****)**

Find the range of values of the non zero constant m so that the quadratic equation

$$mx^2 + (2m-3)x + 2m + 1 = 0$$

has two distinct real roots.

$$\boxed{-\frac{9}{2} < m < \frac{1}{2},\ m \neq 0}$$



$mx^2 + (2m-3)x + (2m+1) = 0$

TWO DISTINCT REAL ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow (2m-3)^2 - 4m(2m+1) > 0$

$\Rightarrow 4m^2 - 12m + 9 - 8m^2 - 4m > 0$

$\Rightarrow -4m^2 - 16m + 9 > 0$

$\Rightarrow 4m^2 + 16m - 9 < 0$

$\Rightarrow (2m-1)(m+9) < 0$

C.V. $= -9,\ \frac{1}{2}$

$-9 < m < \frac{1}{2}$

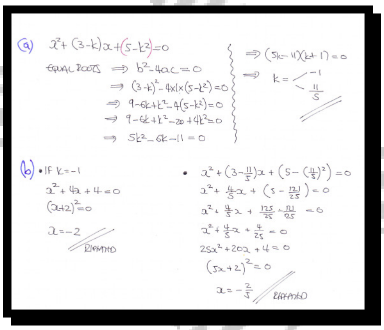
Question 16 **(****)**

$$f(x) = x^2 + (3-k)x + 5 - k^2, \text{ where } k \text{ is a constant.}$$

a) Given that the equation $f(x) = 0$ has equal roots, find the possible values of k.

b) Solve the equation $f(x) = 0$, for each value of k found in part (a)

$$\boxed{k = -1, \frac{11}{5}},\ \boxed{k = -2, \frac{2}{5}}$$



(a) $x^2 + (3-k)x + (5-k^2) = 0$

EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (3-k)^2 - 4 \times 1 \times (5-k^2) = 0$

$\Rightarrow 9 - 6k + k^2 - 4(5-k^2) = 0$

$\Rightarrow 9 - 6k + k^2 - 20 + 4k^2 = 0$

$\Rightarrow 5k^2 - 6k - 11 = 0$

$\Rightarrow (5k-11)(k+1) = 0$

$\Rightarrow k = -1,\ \frac{11}{5}$

(b) • IF $k = -1$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$ REPEATED

• $x^2 + (3 - \frac{11}{5})x + (5 - (\frac{11}{5})^2) = 0$

$x^2 + \frac{4}{5}x + (5 - \frac{121}{25}) = 0$

$x^2 + \frac{4}{5}x + \frac{125}{25} - \frac{121}{25} = 0$

$x^2 + \frac{4}{5}x + \frac{4}{25} = 0$

$25x^2 + 20x + 4 = 0$

$(5x+2)^2 = 0$

$x = -\frac{2}{5}$ REPEATED

Question 17 (****)

$$f(x) = x^2 - 2mx - 5 \text{, where } m \text{ is a constant.}$$

a) **Without** attempting a solution, show that the equation $f(x) = 0$ has two distinct real roots for all possible values of the constant m.

b) Find, in terms of m and in fully simplified form, the roots of the equation

$$f(x) = 0.$$

C1 , $\boxed{x = m \pm \sqrt{m^2 + 5}}$

(a) $x^2 - 2mx - 5 = 0$

$b^2 - 4ac = (-2m)^2 - 4 \times 1 \times (-5) = 4m^2 + 20 \geqslant 20 > 0$

THIS CAN NEVER GO NEGATIVE.

$\therefore$ ALWAYS TWO DISTINCT ROOTS

(b) $x^2 - 2mx - 5 = 0$

METHOD A

$(x-m)^2 - m^2 - 5 = 0$

$(x-m)^2 = m^2 + 5$

$x - m = \pm\sqrt{m^2+5}$

$x = m \pm \sqrt{m^2+5}$

METHOD B

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{+2m \pm \sqrt{4m^2+20}}{2}$

$x = \dfrac{2m \pm \sqrt{4(m^2+5)}}{2} = \dfrac{\cancel{2}m \pm \cancel{2}\sqrt{m^2+5}}{\cancel{2}}$

$x = m \pm \sqrt{m^2+5}$

Question 18 (****)

The quadratic equation

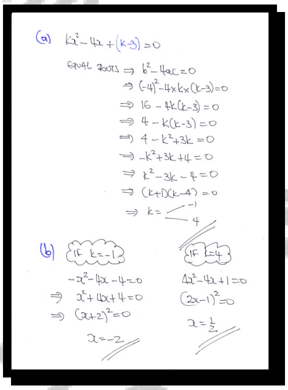
$$kx^2 - 4x + k - 3 = 0,$$

where k is a non zero constant, has equal roots.

a) Determine the possible values of k.

b) Solve the equation for each value of k found in part (a).

$\boxed{k = -1, 4}$, $\boxed{x = -2, \frac{1}{2}}$



(a) $kx^2 - 4x + (k-3) = 0$

EQUAL ROOTS $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-4)^2 - 4 \times k \times (k-3) = 0$

$\Rightarrow 16 - 4k(k-3) = 0$

$\Rightarrow 4 - k(k-3) = 0$

$\Rightarrow 4 - k^2 + 3k = 0$

$\Rightarrow -k^2 + 3k + 4 = 0$

$\Rightarrow k^2 - 3k - 4 = 0$

$\Rightarrow (k+1)(k-4) = 0$

$\Rightarrow k = -1, 4$

(b) IF $k = -1$

$-x^2 - 4x - 4 = 0$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow (x+2)^2 = 0$

$x = -2$

IF $k = 4$

$4x^2 - 4x + 1 = 0$

$(2x-1)^2 = 0$

$x = \frac{1}{2}$

Question 19 (****)

The quadratic equation

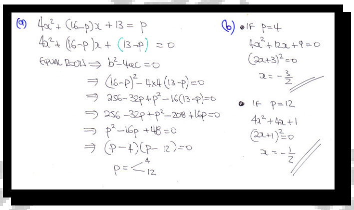
$$4x^2+(16-p)x+13=p,$$

where p is a constant, has equal roots.

a) Determine the possible values of p.

b) Solve the equation for each of the values of p found in part (a).

$\boxed{}$, $\boxed{p=4,\ 12}$, $\boxed{x=-\frac{3}{2},-\frac{1}{2}}$


(a) $4x^2+(16-p)x+13=p$

$4x^2+(16-p)x+(13-p)=0$

EQUAL ROOTS $\Rightarrow b^2-4ac=0$

$\Rightarrow (16-p)^2-4\times4(13-p)=0$

$\Rightarrow 256-32p+p^2-16(13-p)=0$

$\Rightarrow 256-32p+p^2-208+16p=0$

$\Rightarrow p^2-16p+48=0$

$\Rightarrow (p-4)(p-12)=0$

$p=\genfrac{}{}{0pt}{}{4}{12}$

(b) • IF $p=4$

$4x^2+12x+9=0$

$(2x+3)^2=0$

$x=-\frac{3}{2}$

• IF $p=12$

$4x^2+4x+1$

$(2x+1)^2=0$

$x=-\frac{1}{2}$

Question 20 (****)

The quadratic equation

$$3(k+2)x^2 - (5k+7)x + 3k + 1 = 0,$$

where k is a constant, $k \neq -2$, has two distinct real roots.

Show clearly that

$$-\frac{25}{11} < k < 1.$$

$\boxed{}$, $\boxed{\text{proof}}$

$3(k+2)x^2 - (5k+7)x + (3k+1) = 0$

DISTINCT REAL ROOTS $\Rightarrow b^2 - 4ac > 0$

$\Rightarrow [-(5k+7)]^2 - 4 \times 3(k+2)(3k+1) > 0$

$\Rightarrow (5k+7)^2 - 12(k+2)(3k+1) > 0$

$\Rightarrow 25k^2 + 70k + 49 - 12(3k^2 + 7k + 2) > 0$

$\Rightarrow 25k^2 + 70k + 49 - 36k^2 - 84k - 24 > 0$

$\Rightarrow -11k^2 - 14k + 25 > 0$

$\Rightarrow 11k^2 + 14k - 25 < 0$

$\Rightarrow (11k + 25)(k - 1) < 0$

C.V. $= 1, -\frac{25}{11}$

$-\frac{25}{11} < k < 1$

Question 21 (****)

$$f(x) = m(1-x) - x^2, \text{ where } m \text{ is a constant.}$$

The equation $f(x) = 0$ has no real roots.

Determine the range of the possible values of m.

$\boxed{}$, $\boxed{-4 < m < 0}$

$f(x) = m(1-x) - x^2$

$\Rightarrow 0 = m(1-x) - x^2$

$\Rightarrow 0 = m - mx - x^2$

$\Rightarrow x^2 + mx - m = 0$

NO REAL ROOTS $\Rightarrow b^2 - 4ac < 0$

$\Rightarrow m^2 - 4 \times 1(-m) < 0$

$\Rightarrow m^2 + 4m < 0$

$\Rightarrow m(m+4) < 0$

$\Rightarrow$ C.V. $= 0, -4$

$-4 < m < 0$

Question 22 (****)

A curve C has equation

$$y = x^2 + 2mx + (3m+4),$$

where m is a real constant.

The graph of C **touches** the x axis.

a) Determine the possible values of m.

b) For each value of m found in part in part (a), find the x coordinate of the point where the graph of C touches the x axis.

$\boxed{}$, $\boxed{m = -1, 4}$, $\boxed{x = -2, 1}$

(a) • $y = x^2 + 2mx + (3m+4)$

• CROSSES x AXIS, IF $y = 0$

• $x^2 + 2mx + (3m+4) = 0$
HAS EQUAL ROOTS

• $b^2 - 4ac = 0$

$\Rightarrow (2m)^2 - 4 \times 1 \times (3m+4) = 0$

$\Rightarrow 4m^2 - 4(3m+4) = 0$

$\Rightarrow m^2 - 3m - 4 = 0$

$\Rightarrow (m-4)(m+1) = 0$

$\Rightarrow m = 4, -1$ //

(b) • IF $m = 4$

$x^2 + 8x + 16 = 0$

$(x+4)^2 = 0$

$\therefore x = -4$ IE $(-4, 0)$ //

• IF $m = -1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$\therefore x = 1$ $(1, 0)$ //

Question 23 (****)

The quadratic equation

$$3(p+2)x^2+(p+5)x+p=0,$$

where p is a constant, $p \neq -2$, has repeated roots.

Find the possible roots of the equation.

$\boxed{}$, $\boxed{x=-\frac{1}{3},\ x=\frac{5}{3}}$

PROCEED BY FORMING AN EQUATION BASED ON THE DISCRIMINANT

REPEATED ROOTS $\Rightarrow b^2-4ac=0$

$\Rightarrow \underbrace{(p+5)^2}_{b^2}-4\times \underbrace{3(p+2)}_{a}\times \underbrace{p}_{c}=0$

$\Rightarrow p^2+10p+25-12p(p+2)=0$

$\Rightarrow p^2+10p+25-12p^2-24p=0$

$\Rightarrow -11p^2-14p+25=0$

$\Rightarrow 11p^2+14p-25=0$

$\Rightarrow (11p+25)(p-1)=0$

$\boxed{p = 1 \text{ or } -\frac{25}{11}}$

EACH OF THESE TWO VALUES OF p, PRODUCES A QUADRATIC EQUATION IN x, WHICH MUST HAVE REPEATED ROOTS

- IF $p=1$

$3(p+2)x^2+(p+5)x+p=0$

$9x^2+6x+1=0$

$(3x+1)^2=0$

$x=-\frac{1}{3}$

- IF $p=-\frac{25}{11}$

$3(p+2)x^2+(p+5)x+p=0$

$-\frac{9}{11}x^2+\frac{30}{11}x-\frac{25}{11}=0$ $\times 11$

$-9x^2+30x-25=0$ $\times(-1)$

$9x^2-30x+25=0$

$(3x-5)^2=0$

$x=\frac{5}{3}$

Question 24 (****)

The quadratic equation, where m is a constant,

$$x^2+2mx+3x+m^2=0,$$

has equal roots.

Find the value of m.

$\boxed{}$, $\boxed{m=-\frac{3}{4}}$

$x^2+2mx+3x+m^2=0$

$\Rightarrow x^2+(2m+3)x+m^2=0$

EQUAL ROOTS $\Rightarrow b^2-4ac=0$

$\Rightarrow (2m+3)^2-4\times1\times m^2=0$

$\Rightarrow \cancel{4m^2}+12m+9-\cancel{4m^2}=0$

$\Rightarrow 12m=-9$

$\Rightarrow m=-\frac{9}{12}$

$\Rightarrow m=-\frac{3}{4}$

Question 25 (****)

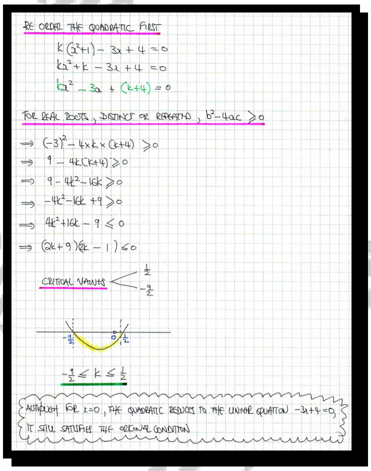
The quadratic equation

$$k\left(x^2+1\right)-3x+4=0,$$

where k is a non zero constant, has real roots.

Find the range of possible values of k.

$\boxed{}$, $\boxed{-\frac{9}{2} \le k \le \frac{1}{2}}$



RE ORDER THE QUADRATIC FIRST

$k(x^2+1)-3x+4=0$

$kx^2+k-3x+4=0$

$kx^2-3x+(k+4)=0$

FOR REAL ROOTS, DISTINCT OR REPEATED, $b^2-4ac \geqslant 0$

$\Rightarrow (-3)^2-4\times k\times(k+4) \geqslant 0$

$\Rightarrow 9-4k(k+4) \geqslant 0$

$\Rightarrow 9-4k^2-16k \geqslant 0$

$\Rightarrow -4k^2-16k+9 \geqslant 0$

$\Rightarrow 4k^2+16k-9 \leqslant 0$

$\Rightarrow (2k+9)(2k-1) \leqslant 0$

CRITICAL VALUES $\frac{1}{2}$, $-\frac{9}{2}$

$-\frac{9}{2} \leqslant k \leqslant \frac{1}{2}$

ALTHOUGH FOR $k=0$, THE QUADRATIC REDUCES TO THE LINEAR EQUATION $-3x+4=0$, IT STILL SATISFIES THE ORIGINAL CONDITION

2
HARD QUESTIONS

Question 1 (****+)

Find the range of values of the non zero constant k, given that the quadratic equation

$$3kx^2 - 2kx - 4x + 3 = 0$$

has two different real roots.

$\boxed{}$, $\boxed{k < 1 \quad \text{or} \quad k > 4, \quad k \neq 0}$

PROCEED AS FOLLOWS

$\Rightarrow 3kx^2 - 2kx - 4x + 3 = 0$

$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$

$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$

TWO DIFFERENT REAL ROOTS $b^2 - 4ac > 0$

$\Rightarrow [-(2k+4)]^2 - 4 \times 3k \times 3 > 0$

$\Rightarrow (2k+4)^2 - 36k > 0$

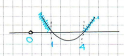
$\Rightarrow 4k^2 + 16k + 16 - 36k > 0$

$\Rightarrow 4k^2 - 20k + 16 > 0$

$\Rightarrow k^2 - 5k + 4 > 0$

$\Rightarrow (k-4)(k-1) > 0$

CRITICAL VALUES $< \begin{matrix} 1 \\ 4 \end{matrix}$



$\therefore k < 1, k \neq 0$ OR $k > 4$

NOTE THAT IF $k = 0$, THE QUADRATIC SEIZES TO EXIST & REDUCES TO A LINEAR EQUATION WITH ONE ROOT ONLY

Question 2 (****+)

It is given that

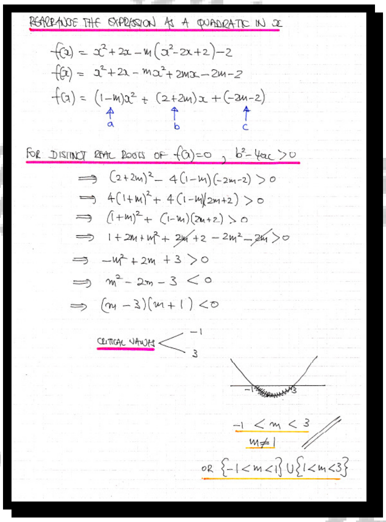
$$f(x) = x^2 + 2x - m\left(x^2 - 2x + 2\right) - 2,$$

where m is a constant such that $m \neq 1$.

The equation $f(x) = 0$ has distinct real roots.

Determine the range of values of m.

$\boxed{-1 < m < 3,\ m \neq 1}$



REARRANGE THE EXPRESSION AS A QUADRATIC IN x

$f(x) = x^2 + 2x - m(x^2 - 2x + 2) - 2$

$f(x) = x^2 + 2x - mx^2 + 2mx - 2m - 2$

$f(x) = \underbrace{(1-m)}_{a}x^2 + \underbrace{(2+2m)}_{b}x + \underbrace{(-2m-2)}_{c}$

FOR DISTINCT REAL ROOTS OF $f(x) = 0$, $b^2 - 4ac > 0$

$\Rightarrow (2+2m)^2 - 4(1-m)(-2m-2) > 0$

$\Rightarrow 4(1+m)^2 + 4(1-m)(2m+2) > 0$

$\Rightarrow (1+m)^2 + (1-m)(2m+2) > 0$

$\Rightarrow 1 + 2m + m^2 + \cancel{2m} + 2 - 2m^2 - \cancel{2m} > 0$

$\Rightarrow -m^2 + 2m + 3 > 0$

$\Rightarrow m^2 - 2m - 3 < 0$

$\Rightarrow (m-3)(m+1) < 0$

CRITICAL VALUES $\left\langle \begin{matrix} -1 \\ 3 \end{matrix} \right.$

$-1 < m < 3$

$m \neq 1$

OR $\{-1 < m < 1\} \cup \{1 < m < 3\}$