STRAIGHT LINE COORDINATE GEOMETRY

GRADIENTS AND INTERCEPTS

Question 1

For each of the following lines find its gradient, its y intercept and its x intercept.

a) $y = 2x + 3$

b) $x = 4 - 2y$

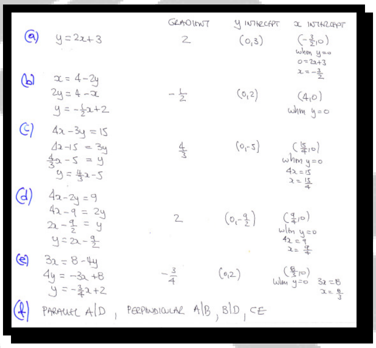
c) $4x - 3y = 15$

d) $4x - 2y = 9$

e) $3x = 8 - 4y$

f) Which of the above lines are parallel or perpendicular to each other?

$$\boxed{\begin{array}{l} m=2 \\ (0,3) \\ \left(-\frac{3}{2},0\right) \end{array}}, \boxed{\begin{array}{l} m=-\frac{1}{2} \\ (0,2) \\ (4,0) \end{array}}, \boxed{\begin{array}{l} m=\frac{4}{3} \\ (0,-5) \\ \left(\frac{15}{4},0\right) \end{array}}, \boxed{\begin{array}{l} m=2 \\ \left(0,-\frac{9}{2}\right) \\ \left(\frac{9}{4},0\right) \end{array}}, \boxed{\begin{array}{l} m=-\frac{3}{4} \\ (0,2) \\ \left(\frac{8}{3},0\right) \end{array}}, \boxed{\begin{array}{l} a \perp b \\ b \perp d \\ a \parallel d \\ c \perp e \end{array}}$$



	GRADIENT	y INTERCEPT	x INTERCEPT
(a) $y = 2x+3$	2	$(0,3)$	$\left(-\frac{3}{2},0\right)$ when $y=0$, $0 = 2x+3$, $x = -\frac{3}{2}$
(b) $x = 4-2y$, $2y = 4-x$, $y = -\frac{1}{2}x+2$	$-\frac{1}{2}$	$(0,2)$	$(4,0)$ when $y=0$
(c) $4x-3y=15$, $4x-15 = 3y$, $\frac{4}{3}x-5 = y$, $y = \frac{4}{3}x-5$	$\frac{4}{3}$	$(0,-5)$	$\left(\frac{15}{4},0\right)$ when $y=0$, $4x=15$, $x=\frac{15}{4}$
(d) $4x-2y=9$, $4x-9=2y$, $2x-\frac{9}{2}=y$, $y=2x-\frac{9}{2}$	2	$\left(0,-\frac{9}{2}\right)$	$\left(\frac{9}{4},0\right)$ when $y=0$, $4x=9$, $x=\frac{9}{4}$
(e) $3x = 8-4y$, $4y = -3x+8$, $y = -\frac{3}{4}x+2$	$-\frac{3}{4}$	$(0,2)$	$\left(\frac{8}{3},0\right)$ when $y=0$, $3x=8$, $x=\frac{8}{3}$

(f) PARALLEL A|D , PERPENDICULAR A|B , B|D , CE

Question 2

For each of the following lines find its gradient, its y intercept and its x intercept.

a) $y = 3x + 2$

b) $y = \frac{1}{3}x - 2$

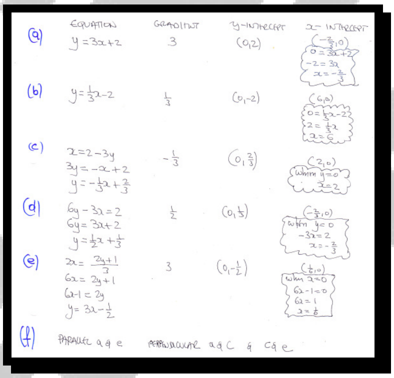
c) $x = 2 - 3y$

d) $6y - 3x = 2$

e) $2x = \frac{2y+1}{3}$

f) Which of the above lines are parallel or perpendicular to each other?

$\boxed{\begin{matrix} m=3 \\ (0,2) \\ \left(-\frac{2}{3},0\right) \end{matrix}}$, $\boxed{\begin{matrix} m=\frac{1}{3} \\ (0,-2) \\ (6,0) \end{matrix}}$, $\boxed{\begin{matrix} m=-\frac{1}{3} \\ \left(0,\frac{2}{3}\right) \\ (2,0) \end{matrix}}$, $\boxed{\begin{matrix} m=\frac{1}{2} \\ \left(0,\frac{1}{3}\right) \\ \left(-\frac{2}{3},0\right) \end{matrix}}$, $\boxed{\begin{matrix} m=3 \\ \left(0,-\frac{1}{2}\right) \\ \left(\frac{1}{6},0\right) \end{matrix}}$, $\boxed{\begin{matrix} a \perp c \\ a \parallel e \\ c \perp e \end{matrix}}$



USING STANDARD FORMULAE

Question 1

Given the points $A(4,6)$ and $B(8,18)$ find …

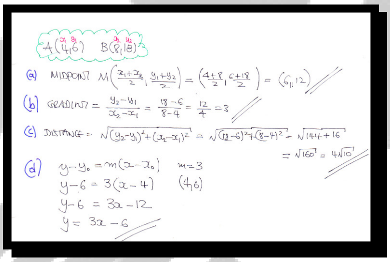
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the exact distance AB.

d) … the equation of the straight line which passes through A and B, giving the answer in the form $y = mx + c$, where m and c are constants.

$\boxed{M(6,12)}$, $\boxed{m=3}$, $\boxed{d=4\sqrt{10}}$, $\boxed{y=3x-6}$



$A(\overset{x_1}{4},\overset{y_1}{6})$ $B(\overset{x_2}{8},\overset{y_2}{18})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = \left(\frac{4+8}{2},\frac{6+18}{2}\right) = (6,12)$

(b) GRADIENT $= \frac{y_2-y_1}{x_2-x_1} = \frac{18-6}{8-4} = \frac{12}{4} = 3$

(c) DISTANCE $= \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(18-6)^2+(8-4)^2} = \sqrt{144+16}$
$= \sqrt{160} = 4\sqrt{10}$

(d) $y - y_0 = m(x - x_0)$ $m=3$
$y - 6 = 3(x-4)$ $(4,6)$
$y - 6 = 3x - 12$
$y = 3x - 6$

Question 2

Given the points $A(5,-1)$ and $B(3,5)$ find …

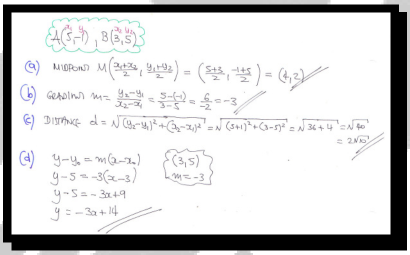
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the exact distance AB.

d) … the equation of the straight line which passes through A and B, giving the answer in the form $y = mx + c$, where m and c are constants.

$\boxed{M(4,2)}$, $\boxed{m=-3}$, $\boxed{d=2\sqrt{10}}$, $\boxed{y=-3x+14}$



$A(5,-1)$, $B(3,5)$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+3}{2}, \frac{-1+5}{2}\right) = (4,2)$

(b) GRADIENT $m = \frac{y_2-y_1}{x_2-x_1} = \frac{5-(-1)}{3-5} = \frac{6}{-2} = -3$

(c) DISTANCE $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(5+1)^2+(3-5)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

(d) $y - y_0 = m(x - x_0)$ $\quad (3,5)$, $m=-3$

$y - 5 = -3(x-3)$

$y - 5 = -3x + 9$

$y = -3x + 14$

Question 3

Given the points $A(3,1)$ and $B(-6,22)$, find …

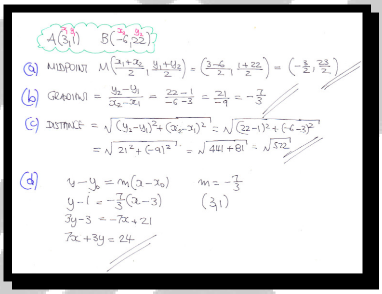
a) …. the exact coordinates of the midpoint of AB.

b) … the exact gradient of AB.

c) … the exact distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by=c$, where a, b and c are integers.

$\boxed{M\left(-\frac{3}{2},\frac{23}{2}\right)}$, $\boxed{m=-\frac{7}{3}}$, $\boxed{d=\sqrt{522}=3\sqrt{58}}$, $\boxed{7x+3y=24}$


$A(\overset{x_1}{3},\overset{y_1}{1})$ $B(\overset{x_2}{-6},\overset{y_2}{22})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3-6}{2},\frac{1+22}{2}\right)=\left(-\frac{3}{2},\frac{23}{2}\right)$

(b) GRADIENT $=\frac{y_2-y_1}{x_2-x_1}=\frac{22-1}{-6-3}=\frac{21}{-9}=-\frac{7}{3}$

(c) DISTANCE $=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(22-1)^2+(-6-3)^2}$
$=\sqrt{21^2+(-9)^2}=\sqrt{441+81}=\sqrt{522}$

(d) $y-y_0=m(x-x_0)$ $m=-\frac{7}{3}$
$y-1=-\frac{7}{3}(x-3)$ $(3,1)$
$3y-3=-7x+21$
$7x+3y=24$

Question 4

Given the points $A(-6,1)$ and $B(2,7)$, find …

a) … the coordinates of the midpoint of AB.

b) … the exact gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M(-2,4)}$, $\boxed{m=\frac{3}{4}}$, $\boxed{d=10}$, $\boxed{3x-4y+22=0}$

$\overset{x_1\,y_1}{A(-6,1)}$ $\overset{x_2\,y_2}{B(2,7)}$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-6+2}{2}, \frac{1+7}{2}\right) = (-2,4)$

(b) GRADIENT $m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{2-(-6)} = \frac{6}{8} = \frac{3}{4}$

(c) DISTANCE $d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(7-1)^2+(2+6)^2} = \sqrt{36+64} = 10$

(d) USING $\left.\begin{array}{l} m=\frac{3}{4} \\ B(2,7) \end{array}\right\}$

$$y-y_0 = m(x-x_0)$$
$$y-7 = \frac{3}{4}(x-2)$$
$$4y-28 = 3x-6$$
$$4y-3x-22=0$$
$$(\text{OR } 3x-4y+22=0)$$

Question 5

Given the points $A(4,9)$ and $B(-4,-11)$, find …

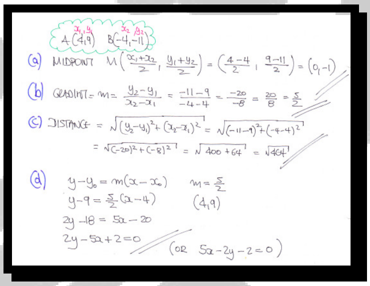
a) … the coordinates of the midpoint of AB.

b) … the exact gradient of AB.

c) … the exact distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M(0,-1)}$, $\boxed{m=\frac{5}{2}}$, $\boxed{d=\sqrt{464}=4\sqrt{29}}$, $\boxed{5x-2y-2=0}$



$A(4,9)$ $B(-4,-11)$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4-4}{2}, \frac{9-11}{2}\right) = (0,-1)$

(b) GRADIENT $= m = \frac{y_2-y_1}{x_2-x_1} = \frac{-11-9}{-4-4} = \frac{-20}{-8} = \frac{20}{8} = \frac{5}{2}$

(c) DISTANCE $= \sqrt{(y_2-y_1)^2+(x_2-x_1)^2} = \sqrt{(-11-9)^2+(-4-4)^2}$
$= \sqrt{(-20)^2+(-8)^2} = \sqrt{400+64} = \sqrt{464}$

(d) $y-y_0 = m(x-x_0)$ $m=\frac{5}{2}$
$y-9 = \frac{5}{2}(x-4)$ $(4,9)$
$2y-18 = 5x-20$
$2y-5x+2=0$ (OR $5x-2y-2=0$)

Question 6

Given the points $A(-1,8)$ and $B(5,-2)$, find …

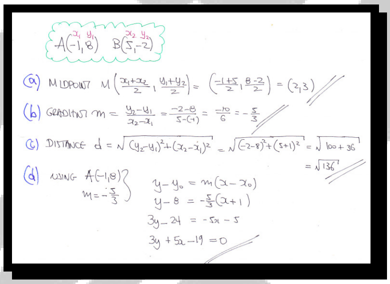
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the exact distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M(2,3)}$, $\boxed{m=-\frac{5}{3}}$, $\boxed{d=\sqrt{136}=2\sqrt{34}}$, $\boxed{5x+3y-19=0}$



$A(\overset{x_1}{-1},\overset{y_1}{8})$ $B(\overset{x_2}{5},\overset{y_2}{-2})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+5}{2},\frac{8-2}{2}\right)=(2,3)$

(b) GRADIENT $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-2-8}{5-(-1)}=\frac{-10}{6}=-\frac{5}{3}$

(c) DISTANCE $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-2-8)^2+(5+1)^2}=\sqrt{100+36}$
$=\sqrt{136}$

(d) USING $\left.\begin{array}{l}A(-1,8)\\ m=-\frac{5}{3}\end{array}\right\}$

$y-y_0=m(x-x_0)$
$y-8=-\frac{5}{3}(x+1)$
$3y-24=-5x-5$
$3y+5x-19=0$

Question 7

Given the points $A(4,6)$ and $B(-1,9)$, find …

a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the exact distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M\left(\frac{3}{2},\frac{15}{2}\right)}$, $\boxed{m=-\frac{3}{5}}$, $\boxed{d=\sqrt{34}}$, $\boxed{3x+5y-42=0}$

$A(\overset{x_1}{4},\overset{y_1}{6})$ $B(\overset{x_2}{-1},\overset{y_2}{9})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{4-1}{2},\frac{6+9}{2}\right)=M\left(\frac{3}{2},\frac{15}{2}\right)$

(b) GRADIENT $m=\frac{y_2-y_1}{x_2-x_1}=\frac{9-6}{-1-4}=\frac{3}{-5}=-\frac{3}{5}$

(c) DISTANCE $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(9-6)^2+(-1-4)^2}=\sqrt{9+25}$
$=\sqrt{34}$

(d) USING $\left.\begin{array}{l} m=-\frac{3}{5} \\ (4,6) \end{array}\right\} \Rightarrow$ $y-y_0=m(x-x_0)$
$y-6=-\frac{3}{5}(x-4)$
$5y-30=-3x+12$
$3x+5y-42=0$

Question 8

Given the points $A(-12,7)$ and $B(-6,-3)$, find …

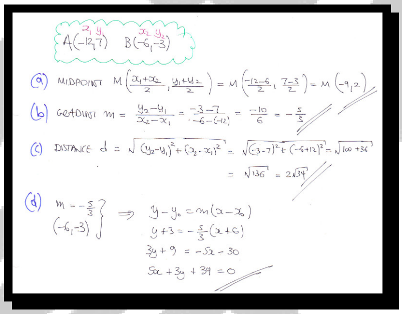
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M(-9,2)}$, $\boxed{m=-\frac{5}{3}}$, $\boxed{d=\sqrt{136}=2\sqrt{34}}$, $\boxed{5x+3y+39=0}$



$A(\overset{x_1}{-12},\overset{y_1}{7})$ $B(\overset{x_2}{-6},\overset{y_2}{-3})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{-12-6}{2},\frac{7-3}{2}\right)=M(-9,2)$

(b) GRADIENT $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-3-7}{-6-(-12)}=\frac{-10}{6}=-\frac{5}{3}$

(c) DISTANCE $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-3-7)^2+(-6+12)^2}=\sqrt{100+36}$

$=\sqrt{136}=2\sqrt{34}$

(d) $\left.\begin{matrix} m=-\frac{5}{3} \\ (-6,-3)\end{matrix}\right\} \Rightarrow$ $y-y_0=m(x-x_0)$

$y+3=-\frac{5}{3}(x+6)$

$3y+9=-5x-30$

$5x+3y+39=0$

Question 9

Given the points $A(-3,2)$ and $B(4,-7)$, find …

a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M\left(\frac{1}{2},-\frac{5}{2}\right)}$, $\boxed{m=-\frac{9}{7}}$, $\boxed{d=\sqrt{130}}$, $\boxed{9x+7y+13=0}$

$A(\overset{x_1}{-3},\overset{y_1}{2})$ $B(\overset{x_2}{4},\overset{y_2}{-7})$

(a) MIDPOINT $= M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3+4}{2},\frac{2-7}{2}\right)=\left(\frac{1}{2},\frac{-5}{2}\right)=\left(\frac{1}{2},-\frac{5}{2}\right)$

(b) GRADIENT $= m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-2}{4-(-3)}=\frac{-9}{7}=-\frac{9}{7}$

(c) DISTANCE $=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-7-2)^2+(4+3)^2}=\sqrt{81+49}=\sqrt{130}$

(d) USING $\left.\begin{array}{l} m=-\frac{9}{4} \\ A(-3,2)\end{array}\right\}$

$y-y_0=m(x-x_0)$

$y-2=-\frac{9}{7}(x+3)$

$7y-14=-9x-27$

$9x+7y+13=0$

Question 10

Given the points $A(-4,9)$ and $B(2,-1)$, find …

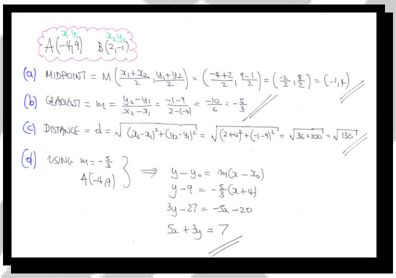
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by=c$, where a, b and c are integers.

$\boxed{M(-1,4)}$, $\boxed{m=-\frac{5}{3}}$, $\boxed{d=\sqrt{136}=2\sqrt{34}}$, $\boxed{5x+3y=7}$



$A(-4,9)$ $B(2,-1)$

(a) MIDPOINT $= M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+2}{2},\frac{9-1}{2}\right)=\left(\frac{-2}{2},\frac{8}{2}\right)=(-1,4)$

(b) GRADIENT $= m = \frac{y_2-y_1}{x_2-x_1}=\frac{-1-9}{2-(-4)}=\frac{-10}{6}=-\frac{5}{3}$

(c) DISTANCE $= d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(2+4)^2+(-1-9)^2}=\sqrt{36+100}=\sqrt{136}$

(d) USING $m=-\frac{5}{3}$, $A(-4,9)$ $\Rightarrow$ $y-y_0=m(x-x_0)$

$y-9=-\frac{5}{3}(x+4)$

$3y-27=-5x-20$

$5x+3y=7$

Question 11

Given the points $A(5,-2)$ and $B(7,5)$, find …

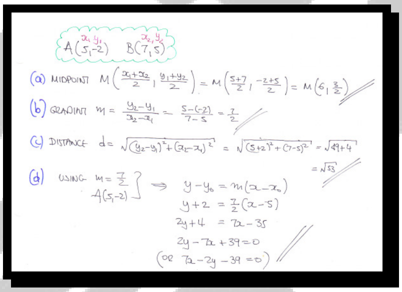
a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by+c=0$, where a, b and c are integers.

$\boxed{M\left(6,\frac{3}{2}\right)}$, $\boxed{m=\frac{7}{2}}$, $\boxed{d=\sqrt{53}}$, $\boxed{7x-2y-39=0}$



$A(\overset{x_1,y_1}{5,-2})$ $B(\overset{x_2,y_2}{7,5})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=M\left(\frac{5+7}{2},\frac{-2+5}{2}\right)=M\left(6,\frac{3}{2}\right)$

(b) GRADIENT $m=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-2)}{7-5}=\frac{7}{2}$

(c) DISTANCE $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(5+2)^2+(7-5)^2}=\sqrt{49+4}$
$=\sqrt{53}$

(d) USING $\left.\begin{array}{l} m=\frac{7}{2} \\ A(5,-2) \end{array}\right\} \Rightarrow$ $y-y_0=m(x-x_0)$
$y+2=\frac{7}{2}(x-5)$
$2y+4=7x-35$
$2y-7x+39=0$
$(\text{OR } 7x-2y-39=0)$

Question 12

Given the points $A(-5,8)$ and $B(15,-8)$, find …

a) … the coordinates of the midpoint of AB.

b) … the gradient of AB.

c) … the distance AB.

d) … an equation of the straight line which passes through A and B, giving the answer in the form $ax+by=c$, where a, b and c are integers.

$$\boxed{M(5,0)},\ \boxed{m=-\frac{4}{5}},\ \boxed{d=\sqrt{656}=4\sqrt{41}},\ \boxed{4x+5y=20}$$

$A(\overset{x_1}{-5},\overset{y_1}{8})$ $B(\overset{x_2}{15},\overset{y_2}{-8})$

(a) MIDPOINT $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-5+15}{2},\frac{8-8}{2}\right)=\left(\frac{10}{2},\frac{0}{2}\right)=(5,0)$

(b) GRADIENT $m=\frac{y_2-y_1}{x_2-x_1}=\frac{-8-8}{15-(-5)}=\frac{-16}{20}=-\frac{4}{5}$

(c) DISTANCE $d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(-8-8)^2+(15+5)^2}=\sqrt{256+400}$
$=\sqrt{656}$

(d) $\left.\begin{array}{l} m=-\frac{4}{5} \\ A(-5,8) \end{array}\right\} \Rightarrow$
$y-y_0=m(x-x_0)$
$y-8=-\frac{4}{5}(x+5)$
$5y-40=-4x-20$
$4x+5y=20$

Question 13

Find an equation of the straight line that passes through the points $A(1,3)$ and $B(4,9)$, giving the answer in the form $y=mx+c$, where m and c are constants.

$$\boxed{y=2x+1}$$

$A(\overset{x_1}{1},\overset{y_1}{3})$ $B(\overset{x_2}{4},\overset{y_2}{9})$

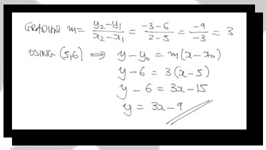
GRADIENT $AB=\frac{y_2-y_1}{x_2-x_1}=\frac{9-3}{4-1}=\frac{6}{3}=2$

HENCE $y-y_0=m(x-x_0)$ $m=2$
$y-3=2(x-1)$ $(1,3)$
$y-3=2x-2$
$y=2x+1$

Question 14

Determine the equation of the straight line that passes through the points $A(5,6)$ and $B(2,-3)$, giving the answer in the form $y = mx + c$, where m and c are constants .

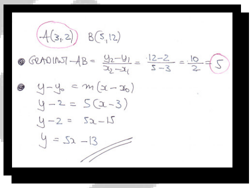
$$\boxed{y = 3x - 9}$$



Question 15

Determine the equation of the straight line that passes through the points $A(3,2)$ and $B(5,12)$, giving the answer in the form $y = mx + c$, where m and c are constants

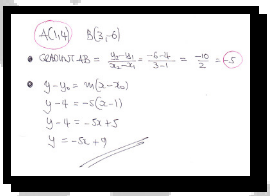
$$\boxed{y = 5x - 13}$$



Question 16

Determine the equation of the straight line that passes through the points $A(1,4)$ and $B(3,-6)$, giving the answer in the form $y = mx + c$, where m and c are constants

$$\boxed{y = -5x + 9}$$



PARALLEL AND PERPENDICULAR LINES

Question 1

The straight line L_1 has equation

$$y = 2 - 3x\,.$$

a) Find an equation of the straight line L_2 which is parallel to L_1 and passes through the point with coordinates $(2,5)$.

b) Find an equation of the straight line L_3 which is perpendicular to L_1 and passes through the point with coordinates $(-3,7)$.

$\boxed{L_2:\ y = -3x + 11}$, $\boxed{L_3:\ 3y = x + 24}$

$L_1:\ y = 2 - 3x$
$\quad\ \ y = -3x + 2$

(a) L_2 HAS GRAD -3, $(2,5)$

$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y - 5 = -3(x - 2)$
$\Rightarrow y - 5 = -3x + 6$
$\Rightarrow y = -3x + 11$

(b) L_3 HAS GRAD $\frac{1}{3}$, $(-3,7)$

$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y - 7 = \frac{1}{3}(x + 3)$
$\Rightarrow 3(y - 7) = x + 3$
$\Rightarrow 3y - 21 = x + 3$
$\Rightarrow 3y = x + 24$

Question 2

The straight line L_1 has equation

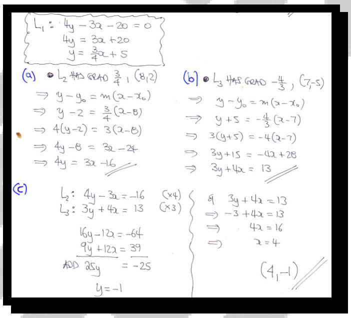
$$4y - 3x - 20 = 0.$$

a) Find an equation of the straight line L_2 which is parallel to L_1 and passes through the point with coordinates $(8,2)$.

b) Find an equation of the straight line L_3 which is perpendicular to L_1 and passes through the point with coordinates $(7,-5)$.

c) Find the coordinates of the point of intersection between L_2 and L_3.

$\boxed{L_2 : \ 3x - 4y - 16 = 0}$, $\boxed{L_3 : \ 3y + 4x = 13}$, $\boxed{(4,-1)}$



$L_1 : 4y - 3x - 20 = 0$
$4y = 3x + 20$
$y = \frac{3}{4}x + 5$

(a) • L_2 HAS GRAD $\frac{3}{4}$, $(8,2)$
$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y - 2 = \frac{3}{4}(x - 8)$
$\Rightarrow 4(y-2) = 3(x-8)$
$\Rightarrow 4y - 8 = 3x - 24$
$\Rightarrow 4y = 3x - 16$

(b) • L_3 HAS GRAD $-\frac{4}{3}$, $(7,-5)$
$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y + 5 = -\frac{4}{3}(x - 7)$
$\Rightarrow 3(y+5) = -4(x-7)$
$\Rightarrow 3y + 15 = -4x + 28$
$\Rightarrow 3y + 4x = 13$

(c)
$L_2 : 4y - 3x = -16 \quad (\times 4)$
$L_3 : 3y + 4x = 13 \quad (\times 3)$

$16y - 12x = -64$
$9y + 12x = 39$
ADD $25y = -25$
$y = -1$

$3y + 4x = 13$
$\Rightarrow -3 + 4x = 13$
$\Rightarrow 4x = 16$
$\Rightarrow x = 4$

$(4,-1)$

Question 3

The straight line L_1 has equation

$$2y = x - 8 .$$

a) Find an equation of the straight line L_2 which is parallel to L_1 and passes through the point with coordinates $(6,1)$.

b) Find an equation of the straight line L_3 which is perpendicular to L_1 and passes through the point with coordinates $(1,-3)$.

c) Find the exact coordinates of the point of intersection between L_2 and L_3.

$\boxed{L_2 :\ 2y = x - 4}$, $\boxed{L_3 :\ y + 2x + 1 = 0}$, $\boxed{\left(\frac{2}{5}, -\frac{9}{5}\right)}$

(a) $L_1:\ 2y = x - 8$
$y = \frac{1}{2}x - 4$

(a) L_2 HAS GRADIENT $\frac{1}{2}$

$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y - 1 = \frac{1}{2}(x - 6)$
$\Rightarrow 2(y-1) = 1(x-6)$
$\Rightarrow 2y - 2 = x - 6$
$\Rightarrow 2y = x - 4$

(b) L_3 HAS GRADIENT -2, $(1,-3)$

$\Rightarrow y - y_0 = m(x - x_0)$
$\Rightarrow y + 3 = -2(x - 1)$
$\Rightarrow y + 3 = -2x + 2$
$\Rightarrow y + 2x + 1 = 0$

(c) $L_2:\ 2y = x - 4$
$L_3:\ y = -2x - 1$

$\Rightarrow 2(-2x-1) = x - 4$
$\Rightarrow -4x - 2 = x - 4$
$\Rightarrow 2 = 5x$
$\Rightarrow x = \frac{2}{5}$

$y = -2x - 1$
$\Rightarrow y = -2\left(\frac{2}{5}\right) - 1$
$\Rightarrow y = -\frac{4}{5} - 1$
$\Rightarrow y = -\frac{9}{5}$

$\therefore \left(\frac{2}{5}, -\frac{9}{5}\right)$

Question 4

Find an equation of the straight line that passes through the point $(1,2)$ and is perpendicular to the straight line with equation $3x + 2y = 5$.

Give the answer in the form $ax + by = c$, where a, b and c are integers.

$\boxed{3y - 2x = 4}$

$3x + 2y = 5$
$2y = -3x + 5$
$y = -\frac{3}{2}x + \frac{5}{2}$

PERPENDICULAR GRADIENT IS $\frac{2}{3}$

HENCE $y - y_0 = m(x - x_0)$
$\Rightarrow y - 2 = \frac{2}{3}(x - 1)$
$\Rightarrow 3y - 6 = 2x - 2$
$\Rightarrow 3y - 2x = 4$

Question 5

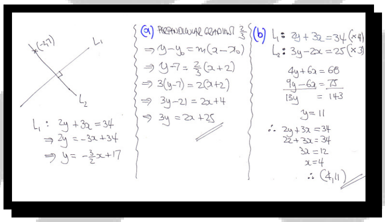
The straight line L_1 has equation

$$2y + 3x = 34.$$

a) Find an equation of the straight line L_2 which is perpendicular to L_1 and passes through the point with coordinates $(-2, 7)$.

b) Find the coordinates of the point of intersection between L_1 and L_2.

$\boxed{L_2 :\ 3y = 2x + 25}$, $\boxed{(4, 11)}$



Question 6

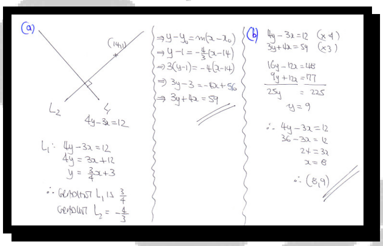
The straight line L_1 has equation

$$4y - 3x = 12.$$

a) Find an equation of the straight line L_2 which is perpendicular to L_1 and passes through the point with coordinates $(14, 1)$.

b) Find the coordinates of the point of intersection between L_1 and L_2.

$\boxed{L_2 :\ 3y + 4x = 59}$, $\boxed{(8, 9)}$



Question 7

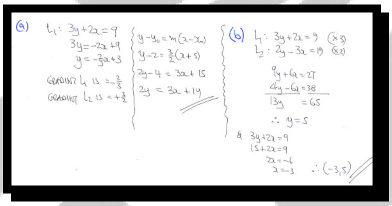
The line straight L_1 has equation

$$3y + 2x = 9 .$$

a) Find an equation of the straight line L_2 which is perpendicular to L_1 and passes through the point with coordinates $(-5, 2)$.

b) Find the coordinates of the point of intersection between L_1 and L_2.

$\boxed{L_2 : \ 2y = 3x + 19}$, $\boxed{(-3, 5)}$

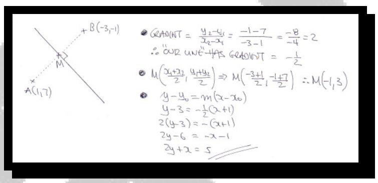


Question 8

The points A and B have coordinates $(1, 7)$ and $(-3, -1)$, respectively.

Find an equation of the straight line that is perpendicular to the straight line AB, and passing through the midpoint of AB.

$\boxed{2y + x = 5}$

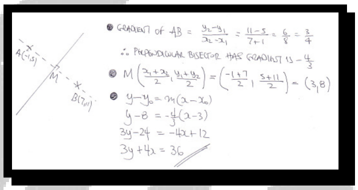


Question 9

The points A and B have coordinates $(-1,5)$ and $(7,11)$, respectively.

Show that the equation of the perpendicular bisector of AB is $4x+3y=36$.

$\boxed{\text{proof}}$

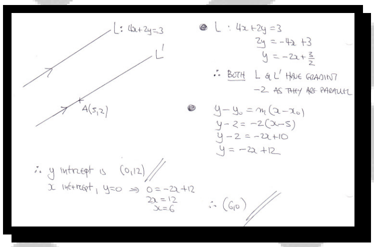


Question 10

A straight line L has equation $4x+2y=3$ and the point A has coordinates $(5,2)$.

Find the coordinates of the points where the straight line that is parallel to L and passing through A, crosses the coordinate axes.

$\boxed{(0,12),(6,0)}$

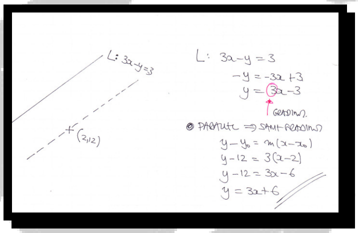


Question 11

A straight line L has equation $3x - y = 3$ and the point A has coordinates $(2,12)$.

Find an equation of the straight line that passes through A and is parallel to L.

$$\boxed{y = 3x + 6}$$

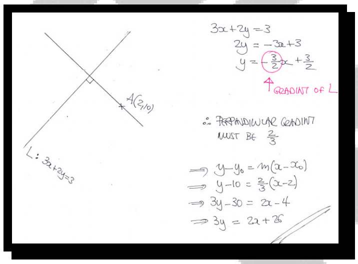


Question 12

A straight line L has equation $3x + 2y = 3$ and the point A has coordinates $(2,10)$.

Find an equation of the straight line that passes through A and is perpendicular to L.

$$\boxed{3y = 2x + 26}$$

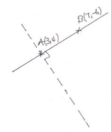


Question 13

The points A and B have coordinates $(3,4)$ and $(7,-6)$, respectively.

Find an equation of the straight line that passes through A and is perpendicular to AB, giving the answer in the form $ax+by+c=0$ where a, b and c are integers.

$$\boxed{2x-5y+14=0}$$



GRADIENT $AB = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{-6-4}{7-3}$

$= -\frac{10}{4} = -\frac{5}{2}$

$\therefore$ PERPENDICULAR GRADIENT $= \frac{2}{5}$

$y-y_0 = m(x-x_0)$

$y-4 = \frac{2}{5}(x-3)$

$5y-20 = 2x-6$

$5y-2x-14=0$

(OR $2x-5y+14$)