

Created by T. Madas

SURDS PRACTICE

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Question 1

Write each of the following surds in its simplest form.

a) $\sqrt{45}$

b) $\sqrt{48}$

c) $\sqrt{80}$

d) $\sqrt{50} + 3\sqrt{8}$

e) $5\sqrt{12} - 2\sqrt{75}$

$3\sqrt{5}$, $4\sqrt{3}$, $4\sqrt{5}$, $11\sqrt{2}$, 0

Handwritten solutions for the surd simplification questions:

- (a) $\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \sqrt{5} = 3\sqrt{5}$
- (b) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \sqrt{3} = 4\sqrt{3}$
- (c) $\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \sqrt{5} = 4\sqrt{5}$
- (d) $\sqrt{50} + 3\sqrt{8} = \sqrt{25 \times 2} + 3\sqrt{4 \times 2} = \sqrt{25} \sqrt{2} + 3\sqrt{4} \sqrt{2} = 5\sqrt{2} + 3 \times 2\sqrt{2} = 5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}$
- (e) $5\sqrt{12} - 2\sqrt{75} = 5\sqrt{4 \times 3} - 2\sqrt{25 \times 3} = 5\sqrt{4} \sqrt{3} - 2\sqrt{25} \sqrt{3} = 5 \times 2\sqrt{3} - 2 \times 5\sqrt{3} = 10\sqrt{3} - 10\sqrt{3} = 0$

Question 2

Write each of the following surds in its simplest form.

a) $\sqrt{150} + \sqrt{54}$

b) $\sqrt{250} - \sqrt{40}$

c) $\sqrt{450} + 2\sqrt{50}$

d) $\sqrt{243} + \sqrt{27}$

e) $\sqrt{343} - \sqrt{28}$

$$\boxed{8\sqrt{6}}, \boxed{3\sqrt{10}}, \boxed{25\sqrt{2}}, \boxed{12\sqrt{3}}, \boxed{5\sqrt{7}}$$

(a)	$\sqrt{150} + \sqrt{54} = \sqrt{45 \times 4} + \sqrt{9 \times 6} = 5\sqrt{6} + 3\sqrt{6} = 8\sqrt{6}$
(b)	$\sqrt{250} - \sqrt{40} = \sqrt{25 \times 10} - \sqrt{4 \times 10} = 5\sqrt{10} - 2\sqrt{10} = 3\sqrt{10}$
(c)	$\sqrt{450} + 2\sqrt{50} = \sqrt{225 \times 2} + 2\sqrt{25 \times 2} = 15\sqrt{2} + 10\sqrt{2} = 25\sqrt{2}$
(d)	$\sqrt{243} + \sqrt{27} = \sqrt{81 \times 3} + \sqrt{9 \times 3} = 9\sqrt{3} + 3\sqrt{3} = 12\sqrt{3}$
(e)	$\sqrt{343} - \sqrt{28} = \sqrt{49 \times 7} - \sqrt{4 \times 7} = 7\sqrt{7} - 2\sqrt{7} = 5\sqrt{7}$

Question 3

Write each of the following surds in its simplest form.

a) $\sqrt{512} + \sqrt{18}$

b) $\sqrt{245} - \sqrt{45}$

c) $\sqrt{216} - \sqrt{24}$

d) $\sqrt{392} + \sqrt{8}$

e) $\sqrt{432} - \sqrt{48} - \sqrt{12}$

$19\sqrt{2}, 4\sqrt{5}, 4\sqrt{6}, 16\sqrt{2}, 6\sqrt{3}$

Handwritten solutions for Question 3:

- (a) $\sqrt{512} + \sqrt{18} = \sqrt{256 \times 2} + \sqrt{9 \times 2} = 16\sqrt{2} + 3\sqrt{2} = 19\sqrt{2}$
- (b) $\sqrt{245} - \sqrt{45} = \sqrt{49 \times 5} - \sqrt{9 \times 5} = 7\sqrt{5} - 3\sqrt{5} = 4\sqrt{5}$
- (c) $\sqrt{216} - \sqrt{24} = \sqrt{36 \times 6} - \sqrt{4 \times 6} = 6\sqrt{6} - 2\sqrt{6} = 4\sqrt{6}$
- (d) $\sqrt{392} + \sqrt{8} = \sqrt{196 \times 2} + \sqrt{4 \times 2} = 14\sqrt{2} + 2\sqrt{2} = 16\sqrt{2}$
- (e) $\sqrt{432} - \sqrt{48} - \sqrt{12} = \sqrt{144 \times 3} - \sqrt{16 \times 3} - \sqrt{4 \times 3} = 12\sqrt{3} - 4\sqrt{3} - 2\sqrt{3} = 6\sqrt{3}$

Question 4

Express each of the following surds in the form $\frac{m}{n}\sqrt{k}$, where k , m and n are positive integers.

a) $\sqrt{4.5}$

b) $\sqrt{12.5}$

c) $\sqrt{22.5}$

d) $\sqrt{24.5}$

e) $\sqrt{62.5}$

$\frac{3}{2}\sqrt{2}$, $\frac{5}{2}\sqrt{2}$, $\frac{3}{2}\sqrt{10}$, $\frac{7}{2}\sqrt{2}$, $\frac{5}{2}\sqrt{10}$

Handwritten solutions for Question 4:

- a) $\sqrt{4.5} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3\sqrt{2}}{2}$
- b) $\sqrt{12.5} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{5\sqrt{2}}{2}$
- c) $\sqrt{22.5} = \sqrt{\frac{45}{2}} = \frac{3\sqrt{5}}{\sqrt{2}} = \frac{3\sqrt{5}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3\sqrt{10}}{2}$
- d) $\sqrt{24.5} = \sqrt{\frac{49}{2}} = \frac{7}{\sqrt{2}} = \frac{7\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{7\sqrt{2}}{2}$
- e) $\sqrt{62.5} = \sqrt{\frac{125}{2}} = \frac{5\sqrt{5}}{\sqrt{2}} = \frac{5\sqrt{5}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{5\sqrt{10}}{2}$

Question 5

Express each of the following surds in the form $\frac{m}{n}\sqrt{k}$, where k , m and n are positive integers.

a) $\sqrt{\frac{8}{3}}$

b) $\sqrt{\frac{5}{6}}$

c) $\sqrt{\frac{5}{8}}$

d) $\sqrt{\frac{3}{10}}$

e) $\sqrt{\frac{8}{27}}$

$$\boxed{\frac{2}{3}\sqrt{6}}, \quad \boxed{\frac{1}{6}\sqrt{30}}, \quad \boxed{\frac{1}{4}\sqrt{10}}, \quad \boxed{\frac{1}{10}\sqrt{30}}, \quad \boxed{\frac{2}{9}\sqrt{6}}$$

Handwritten solutions for Question 5:

- a) $\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}} = \frac{\sqrt{2^3}}{\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{6}}{3} = \frac{2}{3}\sqrt{6}$
- b) $\sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{5}\sqrt{6}}{\sqrt{6}\sqrt{6}} = \frac{\sqrt{30}}{6} = \frac{1}{6}\sqrt{30}$
- c) $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} = \frac{\sqrt{5}\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{10}}{4} = \frac{1}{4}\sqrt{10}$
- d) $\sqrt{\frac{3}{10}} = \frac{\sqrt{3}}{\sqrt{10}} = \frac{\sqrt{3}\sqrt{10}}{\sqrt{10}\sqrt{10}} = \frac{\sqrt{30}}{10} = \frac{1}{10}\sqrt{30}$
- e) $\sqrt{\frac{8}{27}} = \frac{\sqrt{8}}{\sqrt{27}} = \frac{\sqrt{2^3}}{\sqrt{3^3}} = \frac{2\sqrt{2}}{3\sqrt{3}} = \frac{2\sqrt{2}\sqrt{3}}{3\sqrt{3}\sqrt{3}} = \frac{2\sqrt{6}}{9} = \frac{2}{9}\sqrt{6}$

Question 6

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $2\sqrt{2} \times \sqrt{3}$

b) $3\sqrt{2} \times 2\sqrt{3}$

c) $5\sqrt{3} \times \sqrt{3}$

d) $(2\sqrt{2})^2$

e) $(4\sqrt{3})^2$

$2\sqrt{6}$, $6\sqrt{6}$, 15 , 8 , 48

(a) $2\sqrt{2} \times \sqrt{3} = 2 \times \sqrt{2 \times 3} = 2\sqrt{6}$
(b) $3\sqrt{2} \times 2\sqrt{3} = 3 \times \sqrt{2} \times 2 \times \sqrt{3} = 3 \times 2 \times \sqrt{2 \times 3} = 6 \times \sqrt{6} = 6\sqrt{6}$
(c) $5\sqrt{3} \times \sqrt{3} = 5 \times 3 = 15$
(d) $(2\sqrt{2})^2 = 2\sqrt{2} \times 2\sqrt{2} = 2 \times 2 \times \sqrt{2} \times \sqrt{2} = 2 \times 2 \times 2 = 8$
(e) $(4\sqrt{3})^2 = 4\sqrt{3} \times 4\sqrt{3} = 4 \times 4 \times \sqrt{3} \times \sqrt{3} = 16 \times 3 = 48$

Question 7

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $2\sqrt{6} \times \sqrt{24}$

b) $2\sqrt{2} \times \sqrt{18}$

c) $3\sqrt{2} \times \sqrt{6}$

d) $2\sqrt{5} \times 5\sqrt{10}$

e) $2\sqrt{6} \times 4\sqrt{3}$

$$\boxed{24}, \boxed{12}, \boxed{6\sqrt{3}}, \boxed{50\sqrt{2}}, \boxed{24\sqrt{2}}$$

Handwritten solutions for Question 7:

- (a) $2\sqrt{6} \times \sqrt{24} = 2\sqrt{6} \times \sqrt{4 \times 6} = 2\sqrt{6} \times 2\sqrt{6} = 4 \times 6 = 24$
- (b) $2\sqrt{2} \times \sqrt{18} = 2\sqrt{2} \times \sqrt{9 \times 2} = 2\sqrt{2} \times 3\sqrt{2} = 6 \times 2 = 12$
- (c) $3\sqrt{2} \times \sqrt{6} = 3\sqrt{2} \times \sqrt{2 \times 3} = 3 \times 2 \times \sqrt{3} = 6\sqrt{3}$
- (d) $2\sqrt{5} \times 5\sqrt{10} = 2\sqrt{5} \times 5\sqrt{2 \times 5} = 2 \times 5 \times 5 \times \sqrt{2} = 50\sqrt{2}$
- (e) $2\sqrt{6} \times 4\sqrt{3} = 2\sqrt{2 \times 3} \times 4\sqrt{3} = 2 \times 3 \times 4 \times \sqrt{2} = 24\sqrt{2}$

Question 8

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\sqrt{6} \times \sqrt{27}$

b) $\sqrt{10} \times \sqrt{125}$

c) $3\sqrt{8} \times \sqrt{10}$

d) $\sqrt{14} \times \sqrt{42}$

e) $2\sqrt{6} \times 3\sqrt{10} \times \sqrt{30}$

$\boxed{9\sqrt{2}}$, $\boxed{25\sqrt{2}}$, $\boxed{12\sqrt{5}}$, $\boxed{14\sqrt{3}}$, $\boxed{180\sqrt{2}}$

Handwritten solutions for Question 8:

a) $\sqrt{6} \times \sqrt{27} = \sqrt{162} = \sqrt{81 \times 2} = 9\sqrt{2}$
 $\sqrt{6} \times \sqrt{27} = \sqrt{6} \times \sqrt{3 \times 3 \times 3} = \sqrt{6} \times 3\sqrt{3} = 3\sqrt{18} = 3 \times 3\sqrt{2} = 9\sqrt{2}$
 $\sqrt{6} \times \sqrt{27} = \sqrt{2 \times 3} \times \sqrt{3 \times 3 \times 3} = \sqrt{2} \times 3 \times 3 = 9\sqrt{2}$

b) $\sqrt{10} \times \sqrt{125} = \sqrt{1250} = \sqrt{625 \times 2} = 25\sqrt{2}$
 $\sqrt{10} \times \sqrt{125} = \sqrt{10} \times \sqrt{5 \times 5 \times 5} = \sqrt{10} \times 5\sqrt{5} = 5\sqrt{50} = 5 \times 5\sqrt{2} = 25\sqrt{2}$
 $\sqrt{10} \times \sqrt{125} = \sqrt{2 \times 5} \times \sqrt{5 \times 5 \times 5} = \sqrt{2} \times 5 \times 5 = 25\sqrt{2}$

c) $3\sqrt{8} \times \sqrt{10} = 3\sqrt{80} = 3\sqrt{16 \times 5} = 3 \times 4\sqrt{5} = 12\sqrt{5}$
 $3\sqrt{8} \times \sqrt{10} = 3 \times \sqrt{4 \times 2} \times \sqrt{10} = 3 \times 2\sqrt{2} \times \sqrt{10} = 6\sqrt{20} = 6 \times 2\sqrt{5} = 12\sqrt{5}$
 $3\sqrt{8} \times \sqrt{10} = 3 \times \sqrt{2 \times 2 \times 2} \times \sqrt{2 \times 5} = 3 \times 2 \times 2 \times \sqrt{5} = 12\sqrt{5}$

d) $\sqrt{14} \times \sqrt{42} = \sqrt{588} = \sqrt{196 \times 3} = 14\sqrt{3}$
 $\sqrt{14} \times \sqrt{42} = \sqrt{2 \times 7} \times \sqrt{2 \times 3 \times 7} = \sqrt{2} \times \sqrt{2} \times \sqrt{3} \times \sqrt{7} \times \sqrt{7} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}$

e) $2\sqrt{6} \times 3\sqrt{10} \times \sqrt{30} = 6\sqrt{1800} = 6\sqrt{300 \times 6} = 6 \times 30 \times \sqrt{2} = 180\sqrt{2}$
 $2\sqrt{6} \times 3\sqrt{10} \times \sqrt{30} = 6\sqrt{2 \times 3 \times 2 \times 3 \times 5} \times \sqrt{2 \times 3 \times 5} = 6\sqrt{2 \times 3 \times 5} \times 3\sqrt{2 \times 3 \times 5} = 6\sqrt{30} \times 3\sqrt{30} = 180\sqrt{2}$

Question 9

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\sqrt{3}(3+2\sqrt{3})$

b) $2\sqrt{2}(\sqrt{2}-3)$

c) $(2\sqrt{2}-3\sqrt{5})\times\sqrt{5}$

d) $(2\sqrt{6}-3)\times\sqrt{3}$

e) $2\sqrt{5}(\sqrt{10}-3\sqrt{5})$

$$\boxed{6+3\sqrt{3}}, \quad \boxed{4-6\sqrt{2}}, \quad \boxed{-15+2\sqrt{10}}, \quad \boxed{6\sqrt{2}-3\sqrt{3}}, \quad \boxed{-30+10\sqrt{2}}$$

Handwritten solutions for Question 9:

- (a) $\sqrt{3}(3+2\sqrt{3}) = 3\sqrt{3} + \sqrt{3}\times 2\sqrt{3} = 3\sqrt{3} + 2\times 3 = 3\sqrt{3} + 6$
- (b) $2\sqrt{2}(\sqrt{2}-3) = 2\sqrt{2}\sqrt{2} - 2\sqrt{2}\times 3 = 2\times 2 - 6\sqrt{2} = 4 - 6\sqrt{2}$
- (c) $(2\sqrt{2}-3\sqrt{5})\times\sqrt{5} = 2\sqrt{2}\times\sqrt{5} - 3\sqrt{5}\times\sqrt{5} = 2\sqrt{10} - 3\times 5 = 2\sqrt{10} - 15$
- (d) $(2\sqrt{6}-3)\times\sqrt{3} = 2\sqrt{6}\sqrt{3} - 3\sqrt{3} = 2\sqrt{18} - 3\sqrt{3} = 2\times 3\sqrt{2} - 3\sqrt{3} = 6\sqrt{2} - 3\sqrt{3}$
- (e) $2\sqrt{5}(\sqrt{10}-3\sqrt{5}) = 2\sqrt{5}\sqrt{10} - 2\sqrt{5}\times 3\sqrt{5} = 2\sqrt{50} - 6\times 5 = 2\sqrt{25\times 2} - 30 = 2\times 5\sqrt{2} - 30 = 10\sqrt{2} - 30$

Question 10

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(2+\sqrt{2})(1+\sqrt{2})$

b) $(2+\sqrt{3})(3+\sqrt{3})$

c) $(\sqrt{7}+2)(1+\sqrt{7})$

d) $(\sqrt{5}+2)(3-\sqrt{5})$

e) $(\sqrt{11}+2)(5-\sqrt{11})$

$$\boxed{4+3\sqrt{2}}, \boxed{9+5\sqrt{3}}, \boxed{9+3\sqrt{7}}, \boxed{1+\sqrt{5}}, \boxed{-1+3\sqrt{11}}$$

$$\begin{array}{l} \text{(a)} (2+\sqrt{2})(1+\sqrt{2}) = 2+2\sqrt{2}+\sqrt{2}+2 = 4+3\sqrt{2} \\ \text{(b)} (2+\sqrt{3})(3+\sqrt{3}) = 6+2\sqrt{3}+3\sqrt{3}+3 = 9+5\sqrt{3} \\ \text{(c)} (\sqrt{7}+2)(1+\sqrt{7}) = \sqrt{7}+7+2+2\sqrt{7} = 9+3\sqrt{7} \\ \text{(d)} (\sqrt{5}+2)(3-\sqrt{5}) = 3\sqrt{5}-5+6-2\sqrt{5} = 1+\sqrt{5} \\ \text{(e)} (\sqrt{11}+2)(5-\sqrt{11}) = 5\sqrt{11}-11+10-2\sqrt{11} = -1+3\sqrt{11} \end{array}$$

Question 11

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(\sqrt{7} + 3)(2\sqrt{7} - 3)$

b) $(2\sqrt{3} - 1)(3 - 3\sqrt{3})$

c) $(5 - 2\sqrt{5})(2 + 3\sqrt{5})$

d) $(2\sqrt{2} + 3\sqrt{3})(3\sqrt{2} - 2\sqrt{3})$

e) $(3\sqrt{3} - 2\sqrt{2})(2\sqrt{2} + 3\sqrt{3})$

$$\boxed{5 + 3\sqrt{7}}, \quad \boxed{-21 + 9\sqrt{3}}, \quad \boxed{-20 + 11\sqrt{5}}, \quad \boxed{-6 + 5\sqrt{6}}, \quad \boxed{19}$$

a) $(\sqrt{7} + 3)(2\sqrt{7} - 3) = (2 \times 7) - 3\sqrt{7} + 6\sqrt{7} - 9 = 5 + 3\sqrt{7}$
 b) $(2\sqrt{3} - 1)(3 - 3\sqrt{3}) = 6\sqrt{3} - (6 \times 3) - 3 + 3\sqrt{3} = 9\sqrt{3} - 18 - 3 = -21 + 9\sqrt{3}$
 c) $(5 - 2\sqrt{5})(2 + 3\sqrt{5}) = 10 + 15\sqrt{5} - 4\sqrt{5} - (6 \times 5) = 10 + 11\sqrt{5} - 30 = -20 + 11\sqrt{5}$
 d) $(2\sqrt{2} + 3\sqrt{3})(3\sqrt{2} - 2\sqrt{3}) = (6 \times 2) - 4\sqrt{6} + 9\sqrt{6} - (6 \times 3) = 12 + 5\sqrt{6} - 18 = -6 + 5\sqrt{6}$
 e) $(3\sqrt{3} - 2\sqrt{2})(2\sqrt{2} + 3\sqrt{3}) = 6\sqrt{6} + (9 \times 3) - (4 \times 2) - 6\sqrt{6} = 27 - 8 = 19$

Question 12

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(3+\sqrt{3})(2-\sqrt{3})$

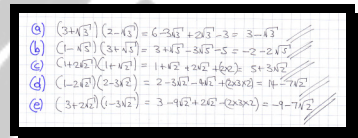
b) $(1-\sqrt{5})(3+\sqrt{5})$

c) $(1+2\sqrt{2})(1+\sqrt{2})$

d) $(1-2\sqrt{2})(2-3\sqrt{2})$

e) $(3+2\sqrt{2})(1-3\sqrt{2})$

$$\boxed{3-\sqrt{3}}, \quad \boxed{-2-2\sqrt{5}}, \quad \boxed{5+3\sqrt{2}}, \quad \boxed{14-7\sqrt{2}}, \quad \boxed{-9-7\sqrt{2}}$$



a) $(3+\sqrt{3})(2-\sqrt{3}) = 6-3\sqrt{3}+2\sqrt{3}-3 = 3-\sqrt{3}$
 b) $(1-\sqrt{5})(3+\sqrt{5}) = 3+\sqrt{5}-3\sqrt{5}-5 = -2-2\sqrt{5}$
 c) $(1+2\sqrt{2})(1+\sqrt{2}) = 1+\sqrt{2}+2\sqrt{2}+4 = 5+3\sqrt{2}$
 d) $(1-2\sqrt{2})(2-3\sqrt{2}) = 2-3\sqrt{2}-4\sqrt{2}+6 = 8-7\sqrt{2}$
 e) $(3+2\sqrt{2})(1-3\sqrt{2}) = 3-9\sqrt{2}+2\sqrt{2}-12 = -9-7\sqrt{2}$

Question 13

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(2 - \sqrt{3})(1 - \sqrt{3})$

b) $(4 - \sqrt{5})(3 + 2\sqrt{5})$

c) $(4 - \sqrt{7})(5 + 2\sqrt{7})$

d) $(3\sqrt{2} + 1)(\sqrt{2} - 1)$

e) $(2 + \sqrt{3})(4 - \sqrt{12})$

$$\boxed{5 - 3\sqrt{3}}, \boxed{2 + 5\sqrt{5}}, \boxed{6 + 3\sqrt{7}}, \boxed{5 - 2\sqrt{2}}, \boxed{2}$$

Handwritten solutions for Question 13:

- a) $(2 - \sqrt{3})(1 - \sqrt{3}) = 2 - 2\sqrt{3} - \sqrt{3} + 3 = 5 - 3\sqrt{3}$
- b) $(4 - \sqrt{5})(3 + 2\sqrt{5}) = 12 + 8\sqrt{5} - 3\sqrt{5} - 2 \times 5 = 2 + 5\sqrt{5}$
- c) $(4 - \sqrt{7})(5 + 2\sqrt{7}) = 20 + 8\sqrt{7} - 5\sqrt{7} - 2 \times 7 = 6 + 3\sqrt{7}$
- d) $(3\sqrt{2} + 1)(\sqrt{2} - 1) = 3 \times 2 - 3\sqrt{2} + \sqrt{2} - 1 = 5 - 2\sqrt{2}$
- e) $(2 + \sqrt{3})(4 - \sqrt{12}) = (2 + \sqrt{3})(4 - 2\sqrt{3}) = 8 - 4\sqrt{3} + 4\sqrt{3} - 2 \times 3 = 8 - 6 = 2$

Question 14

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(2-\sqrt{3})(1-\sqrt{3})$

b) $(4-\sqrt{5})(3+2\sqrt{5})$

c) $(4-\sqrt{7})(5+2\sqrt{7})$

d) $(3\sqrt{2}+1)(\sqrt{2}-1)$

e) $(2+\sqrt{5})(5-\sqrt{20})$

$5-3\sqrt{3}$, $2+5\sqrt{5}$, $6+3\sqrt{7}$, $5-2\sqrt{2}$, $\sqrt{5}$

Question 15

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\sqrt{24} + \sqrt{6}$

b) $\sqrt{98} - \sqrt{50}$

c) $\sqrt{63} + 2\sqrt{28}$

d) $5\sqrt{2} \times 4\sqrt{3} - 6\sqrt{24}$

e) $\frac{18}{\sqrt{3}} - 2\sqrt{27}$

$$\boxed{3\sqrt{6}}, \boxed{2\sqrt{2}}, \boxed{7\sqrt{7}}, \boxed{8\sqrt{6}}, \boxed{0}$$

Handwritten solutions for Question 15:

- a) $\sqrt{24} + \sqrt{6} = 4\sqrt{6} + \sqrt{6} = 5\sqrt{6}$
- b) $\sqrt{98} - \sqrt{50} = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$
- c) $\sqrt{63} + 2\sqrt{28} = 3\sqrt{7} + 4\sqrt{7} = 7\sqrt{7}$
- d) $5\sqrt{2} \times 4\sqrt{3} - 6\sqrt{24} = 20\sqrt{6} - 6\sqrt{6} = 14\sqrt{6}$
- e) $\frac{18}{\sqrt{3}} - 2\sqrt{27} = \frac{18\sqrt{3}}{3} - 2 \times 3\sqrt{3} = 6\sqrt{3} - 6\sqrt{3} = 0$

Question 16

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $2\sqrt{32} + \sqrt{18} - 3\sqrt{8}$

b) $3\sqrt{20} + \frac{10}{\sqrt{5}}$

c) $5\sqrt{8} + \frac{6}{\sqrt{2}}$

d) $\sqrt{48} + \sqrt{27} - \frac{6}{\sqrt{3}}$

$$\boxed{5\sqrt{2}}, \boxed{8\sqrt{5}}, \boxed{13\sqrt{2}}, \boxed{5\sqrt{3}}$$

Handwritten solutions for Question 16:

(a) $2\sqrt{32} + \sqrt{18} - 3\sqrt{8} = 2\sqrt{16 \times 2} + \sqrt{9 \times 2} - 3\sqrt{4 \times 2}$
 $= 2 \times 4\sqrt{2} + 3\sqrt{2} - 3 \times 2\sqrt{2}$
 $= 8\sqrt{2} + 3\sqrt{2} - 6\sqrt{2} = 5\sqrt{2}$

(b) $3\sqrt{20} + \frac{10}{\sqrt{5}} = 3\sqrt{4 \times 5} + \frac{10\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = 3 \times 2\sqrt{5} + \frac{10\sqrt{5}}{5}$
 $= 6\sqrt{5} + 2\sqrt{5} = 8\sqrt{5}$

(c) $5\sqrt{8} + \frac{6}{\sqrt{2}} = 5\sqrt{4 \times 2} + \frac{6\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 5 \times 2\sqrt{2} + \frac{6\sqrt{2}}{2}$
 $= 10\sqrt{2} + 3\sqrt{2} = 13\sqrt{2}$

(d) $\sqrt{48} + \sqrt{27} - \frac{6}{\sqrt{3}} = \sqrt{16 \times 3} + \sqrt{9 \times 3} - \frac{6\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$
 $= 4\sqrt{3} + 3\sqrt{3} - \frac{6\sqrt{3}}{3} = 4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

Question 17

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $2\sqrt{8} + \sqrt{18} - \frac{6}{\sqrt{2}}$

b) $\sqrt{48} - \frac{6}{\sqrt{3}} + \sqrt{6} \times \sqrt{2}$

c) $\frac{14}{\sqrt{2}} - \sqrt{18} - (\sqrt{2})^3$

d) $2\sqrt{5} \times \sqrt{15} - \sqrt{75} - \frac{\sqrt{60}}{\sqrt{5}}$

$$\boxed{4\sqrt{2}}, \boxed{4\sqrt{3}}, \boxed{2\sqrt{2}}, \boxed{3\sqrt{3}}$$

Handwritten solutions for Question 17:

(a) $2\sqrt{8} + \sqrt{18} - \frac{6}{\sqrt{2}} = 2\sqrt{4 \times 2} + \sqrt{9 \times 2} - \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} = 4\sqrt{2} + 3\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$

(b) $\sqrt{48} - \frac{6}{\sqrt{3}} + \sqrt{6} \times \sqrt{2} = \sqrt{16 \times 3} - \frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}} + \sqrt{12} = 4\sqrt{3} - \frac{6\sqrt{3}}{3} + 2\sqrt{3} = 4\sqrt{3} - 2\sqrt{3} + 2\sqrt{3} = 4\sqrt{3}$

(c) $\frac{14}{\sqrt{2}} - \sqrt{18} - (\sqrt{2})^3 = \frac{14\sqrt{2}}{\sqrt{2}\sqrt{2}} - \sqrt{9 \times 2} - \sqrt{2 \times 2 \times 2} = \frac{14\sqrt{2}}{2} - 3\sqrt{2} - 2\sqrt{2} = 7\sqrt{2} - 3\sqrt{2} - 2\sqrt{2} = 2\sqrt{2}$

(d) $2\sqrt{5} \times \sqrt{15} - \sqrt{75} - \frac{\sqrt{60}}{\sqrt{5}} = 2\sqrt{5} \times \sqrt{3 \times 5} - \sqrt{25 \times 3} - \sqrt{4 \times 3 \times 5} = 2 \times 5 \times \sqrt{3} - 5\sqrt{3} - \sqrt{60} = 10\sqrt{3} - 5\sqrt{3} - \sqrt{4 \times 3} = 5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$

Question 18

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\frac{21}{\sqrt{7}}$

b) $(2\sqrt{3})^3 - \frac{12}{\sqrt{3}}$

c) $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}}$

d) $\sqrt{45} + \frac{20}{\sqrt{5}}$

e) $2\sqrt{75} + \frac{3+\sqrt{3}}{3-\sqrt{3}} - \sqrt{2} \times \sqrt{2}$

$$\boxed{3\sqrt{7}}, \boxed{20\sqrt{3}}, \boxed{3\sqrt{7}}, \boxed{7\sqrt{5}}, \boxed{11\sqrt{3}}$$

Handwritten solutions for Question 18:

a) $\frac{21}{\sqrt{7}} = \frac{21}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$

b) $(2\sqrt{3})^3 - \frac{12}{\sqrt{3}} = (2\sqrt{3})(2\sqrt{3})(2\sqrt{3}) - \frac{12\sqrt{3}}{\sqrt{3}\sqrt{3}} = 8 \times 3\sqrt{3} - \frac{12\sqrt{3}}{3}$
 $= 24\sqrt{3} - 4\sqrt{3} = 20\sqrt{3}$

c) $\frac{\sqrt{63}}{3} + \frac{14}{\sqrt{7}} = \frac{\sqrt{9 \times 7}}{3} + \frac{14}{\sqrt{7}} = \frac{3\sqrt{7}}{3} + \frac{14\sqrt{7}}{\sqrt{7}\sqrt{7}} = \sqrt{7} + \frac{14\sqrt{7}}{7}$
 $= \sqrt{7} + 2\sqrt{7} = 3\sqrt{7}$

d) $\sqrt{45} + \frac{20}{\sqrt{5}} = \sqrt{9 \times 5} + \frac{20\sqrt{5}}{\sqrt{5}\sqrt{5}} = 3\sqrt{5} + \frac{20\sqrt{5}}{5}$
 $= 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$

e) $2\sqrt{75} + \frac{3+\sqrt{3}}{3-\sqrt{3}} - \sqrt{2} \times \sqrt{2} = 2\sqrt{25 \times 3} + \frac{(3+\sqrt{3})(3+\sqrt{3})}{(3-\sqrt{3})(3+\sqrt{3})} - 2$
 $= 2 \times 5\sqrt{3} + \frac{9+3\sqrt{3}+3\sqrt{3}+3}{9-3\sqrt{3}\sqrt{3}-3\sqrt{3}\sqrt{3}-3} - 2$
 $= 10\sqrt{3} + \frac{12+6\sqrt{3}}{9-9-3-3} - 2$
 $= 10\sqrt{3} + \frac{12+6\sqrt{3}}{-6} - 2$
 $= 10\sqrt{3} - 2 - \sqrt{3} - 2 = 9\sqrt{3} - 4$

Question 19

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{8}}$

b) $\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}}$

4, $\sqrt{2}$

Handwritten solution for Question 19:

a) $\frac{\sqrt{50} + \sqrt{18}}{\sqrt{8}} = \frac{\sqrt{25 \times 2} + \sqrt{9 \times 2}}{\sqrt{4 \times 2}} = \frac{5\sqrt{2} + 3\sqrt{2}}{2\sqrt{2}} = \frac{8\sqrt{2}}{2\sqrt{2}} = 4$

b) $\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}} = \sqrt{25 \times 2} + \sqrt{3 \times 6} - \frac{14 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = 5\sqrt{2} + 3\sqrt{2} - \frac{14\sqrt{2}}{2} = 8\sqrt{2} - 7\sqrt{2} = \sqrt{2}$

Question 20

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $\frac{7\sqrt{5} - \sqrt{45}}{\sqrt{20}}$

b) $\frac{3 + \sqrt{6}}{\sqrt{3}}$

c) $\frac{5 + 2\sqrt{10}}{\sqrt{5}}$

d) $\frac{\sqrt{75} + \sqrt{48}}{3\sqrt{27}}$

2 , $\sqrt{3} + \sqrt{2}$, $\sqrt{5} + 2\sqrt{2}$, 1

Handwritten solutions for Question 20:

a) $\frac{7\sqrt{5} - \sqrt{45}}{\sqrt{20}} = \frac{7\sqrt{5} - \sqrt{9 \cdot 5}}{\sqrt{4 \cdot 5}} = \frac{7\sqrt{5} - 3\sqrt{5}}{2\sqrt{5}} = \frac{4\sqrt{5}}{2\sqrt{5}} = 2$

b) $\frac{3 + \sqrt{6}}{\sqrt{3}} = \frac{(3 + \sqrt{6})\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3} + \sqrt{18}}{3} = \frac{3\sqrt{3} + \sqrt{9 \cdot 2}}{3} = \frac{3\sqrt{3} + 3\sqrt{2}}{3} = \sqrt{3} + \sqrt{2}$

c) $\frac{5 + 2\sqrt{10}}{\sqrt{5}} = \frac{(5 + 2\sqrt{10})\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{5\sqrt{5} + 2\sqrt{50}}{5} = \frac{5\sqrt{5} + 2\sqrt{25 \cdot 2}}{5} = \frac{5\sqrt{5} + 10\sqrt{2}}{5} = \sqrt{5} + 2\sqrt{2}$

d) $\frac{\sqrt{75} + \sqrt{48}}{3\sqrt{27}} = \frac{\sqrt{25 \cdot 3} + \sqrt{16 \cdot 3}}{3\sqrt{9 \cdot 3}} = \frac{5\sqrt{3} + 4\sqrt{3}}{9\sqrt{3}} = \frac{9\sqrt{3}}{9\sqrt{3}} = 1$

Question 21

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(\sqrt{75} - \sqrt{48})^2$

b) $(\sqrt{8} + \sqrt{2})^2$

c) $(\sqrt{27} - \sqrt{3})^2$

d) $(\sqrt{108} - \sqrt{12})^2$

e) $\frac{7(\sqrt{50} - \sqrt{8})^2}{\sqrt{18} + \sqrt{32}}$

3, 18, 12, 48, $9\sqrt{2}$

(a) $(\sqrt{75} - \sqrt{48})^2 = (\sqrt{25 \times 3} - \sqrt{16 \times 3})^2 = (5\sqrt{3} - 4\sqrt{3})^2 = (\sqrt{3})^2 = 3$
 (b) $(\sqrt{8} + \sqrt{2})^2 = (\sqrt{4 \times 2} + \sqrt{1})^2 = (2\sqrt{2} + \sqrt{2})^2 = (3\sqrt{2})^2 = 9 \times 2 = 18$
 (c) $(\sqrt{27} - \sqrt{3})^2 = (\sqrt{9 \times 3} - \sqrt{1})^2 = (3\sqrt{3} - \sqrt{3})^2 = (2\sqrt{3})^2 = 4 \times 3 = 12$
 (d) $(\sqrt{108} - \sqrt{12})^2 = (\sqrt{36 \times 3} - \sqrt{4 \times 3})^2 = (6\sqrt{3} - 2\sqrt{3})^2 = (4\sqrt{3})^2 = 16 \times 3 = 48$
 (e) $\frac{7(\sqrt{50} - \sqrt{8})^2}{\sqrt{18} + \sqrt{32}} = \frac{7(\sqrt{25 \times 2} - \sqrt{4 \times 2})^2}{\sqrt{9 \times 2} + \sqrt{16 \times 2}} = \frac{7(5\sqrt{2} - 2\sqrt{2})^2}{3\sqrt{2} + 4\sqrt{2}} = \frac{7(3\sqrt{2})^2}{7\sqrt{2}}$
 $= \frac{7 \times 9 \times 2}{7\sqrt{2}} = \frac{18}{\sqrt{2}} = \frac{18 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{18\sqrt{2}}{2} = 9\sqrt{2}$

Question 22

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(4 - \sqrt{5})^2$

b) $(\sqrt{3} - \sqrt{2})^2$

c) $(2 + \sqrt{3})^2$

d) $(2 - 3\sqrt{5})^2$

$\boxed{5 + 3\sqrt{3}}$, $\boxed{22 - 12\sqrt{2}}$, $\boxed{6 + \sqrt{3}}$, $\boxed{\frac{10}{23}}$

Question 23

Find the value of each of the following surd expressions, writing the final answer in its simplest form.

a) $(3 - \sqrt{8})^2$

b) $(2 - 3\sqrt{2})^2$

c) $\frac{(2 - \sqrt{3})^2}{2 + \sqrt{3}}$

d) $(1 + \sqrt{2})^3$

$17 - 12\sqrt{2}$, $22 - 12\sqrt{2}$, $26 - 15\sqrt{3}$, $3 - 5\sqrt{7}$

Question 24

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{22}{4-\sqrt{5}}$

b) $\frac{26}{4+\sqrt{3}}$

c) $\frac{\sqrt{12}+2}{\sqrt{12}-2}$

d) $\frac{44}{2\sqrt{3}-1}$

$$\boxed{8+2\sqrt{5}}, \quad \boxed{8-2\sqrt{3}}, \quad \boxed{2+\sqrt{3}}, \quad \boxed{4+8\sqrt{3}}$$

Handwritten solutions for Question 24:

a) $\frac{22}{4-\sqrt{5}} = \frac{22(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})} = \frac{22(4+\sqrt{5})}{16-5} = \frac{22(4+\sqrt{5})}{11} = 8+2\sqrt{5}$

b) $\frac{26}{4+\sqrt{3}} = \frac{26(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})} = \frac{26(4-\sqrt{3})}{16-3} = \frac{26(4-\sqrt{3})}{13} = 8-2\sqrt{3}$

c) $\frac{\sqrt{12}+2}{\sqrt{12}-2} = \frac{(\sqrt{3}+2)(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} = \frac{3+2\sqrt{3}+2\sqrt{3}+4}{12-3} = \frac{7+4\sqrt{3}}{9}$

d) $\frac{44}{2\sqrt{3}-1} = \frac{44(2\sqrt{3}+1)}{(2\sqrt{3}-1)(2\sqrt{3}+1)} = \frac{44(2\sqrt{3}+1)}{12-1} = \frac{44(2\sqrt{3}+1)}{11} = 8\sqrt{3}+4$

Question 25

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{4}{3-\sqrt{7}}$

b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

c) $\frac{5+\sqrt{7}}{3-\sqrt{7}}$

d) $\frac{\sqrt{7}+1}{\sqrt{7}-2}$

$$\boxed{6+2\sqrt{7}}, \quad \boxed{2+\sqrt{3}}, \quad \boxed{11+4\sqrt{7}}, \quad \boxed{3+\sqrt{7}}$$

Handwritten solutions for the four parts of Question 25:

a) $\frac{4}{3-\sqrt{7}} = \frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{12+4\sqrt{7}}{9-7} = \frac{12+4\sqrt{7}}{2} = 6+2\sqrt{7}$

b) $\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$

c) $\frac{5+\sqrt{7}}{3-\sqrt{7}} = \frac{(5+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{15+5\sqrt{7}+3\sqrt{7}+7}{9-7} = \frac{22+8\sqrt{7}}{2} = 11+4\sqrt{7}$

d) $\frac{\sqrt{7}+1}{\sqrt{7}-2} = \frac{(\sqrt{7}+1)(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{7+2\sqrt{7}+\sqrt{7}+2}{7-4} = \frac{9+3\sqrt{7}}{3} = 3+\sqrt{7}$

Question 26

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{\sqrt{2}}{1+\sqrt{2}}$

b) $\frac{5-\sqrt{3}}{\sqrt{3}+1}$

c) $\frac{2\sqrt{7}-1}{2\sqrt{7}+5}$

d) $\frac{4+\sqrt{28}}{3+\sqrt{7}}$

$2-\sqrt{2}$, $-4+3\sqrt{3}$, $11-4\sqrt{7}$, $-1+\sqrt{7}$

Handwritten solutions for the four parts of Question 26:

(a) $\frac{\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(-1-\sqrt{2})}{(1+\sqrt{2})(-1-\sqrt{2})} = \frac{\sqrt{2}-2}{1-2-2\sqrt{2}-2} = \frac{\sqrt{2}-2}{-1-2\sqrt{2}-2} = 2-\sqrt{2}$

(b) $\frac{5-\sqrt{3}}{\sqrt{3}+1} = \frac{(5-\sqrt{3})(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{5\sqrt{3}-5-3+\sqrt{3}}{3-1} = \frac{6\sqrt{3}-8}{2} = 3\sqrt{3}-4$

(c) $\frac{2\sqrt{7}-1}{2\sqrt{7}+5} = \frac{(2\sqrt{7}-1)(2\sqrt{7}-5)}{(2\sqrt{7}+5)(2\sqrt{7}-5)} = \frac{28-10\sqrt{7}-2\sqrt{7}+5}{28-25} = \frac{33-12\sqrt{7}}{3} = 11-4\sqrt{7}$

(d) $\frac{4+\sqrt{28}}{3+\sqrt{7}} = \frac{4+2\sqrt{7}}{3+\sqrt{7}} = \frac{(4+2\sqrt{7})(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{12-4\sqrt{7}+6\sqrt{7}-14}{9-7} = \frac{-2+2\sqrt{7}}{2} = -1+\sqrt{7}$

Question 27

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{36}{5-\sqrt{7}}$

b) $\frac{\sqrt{5}}{2+\sqrt{5}}$

c) $\frac{1-\sqrt{2}}{3-\sqrt{2}}$

d) $\frac{\sqrt{3}}{6-\sqrt{3}}$

$$\boxed{10+2\sqrt{7}}, \quad \boxed{5-2\sqrt{5}}, \quad \boxed{\frac{1}{7}-\frac{2}{7}\sqrt{2}}, \quad \boxed{\frac{1}{11}+\frac{2}{11}\sqrt{3}}$$

Handwritten solutions for Question 27:

(a) $\frac{36}{5-\sqrt{7}} = \frac{36(5+\sqrt{7})}{(5-\sqrt{7})(5+\sqrt{7})} = \frac{36(5+\sqrt{7})}{25-7} = \frac{36(5+\sqrt{7})}{18} = 10+2\sqrt{7}$

(b) $\frac{\sqrt{5}}{2+\sqrt{5}} = \frac{\sqrt{5}(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})} = \frac{2\sqrt{5}-5}{4-5} = \frac{2\sqrt{5}-5}{-1} = 5-2\sqrt{5}$

(c) $\frac{1-\sqrt{2}}{3-\sqrt{2}} = \frac{(1-\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{3+\sqrt{2}-3\sqrt{2}-2}{9-2} = \frac{1-2\sqrt{2}}{7} = \frac{1}{7}-\frac{2}{7}\sqrt{2}$

(d) $\frac{\sqrt{3}}{6-\sqrt{3}} = \frac{\sqrt{3}(6+\sqrt{3})}{(6-\sqrt{3})(6+\sqrt{3})} = \frac{6\sqrt{3}+3}{36-3} = \frac{3+6\sqrt{3}}{33} = \frac{1}{11}+\frac{2}{11}\sqrt{3}$

Question 28

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{4}{3-\sqrt{5}}$

b) $\frac{5\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

c) $\frac{6+2\sqrt{5}}{3-\sqrt{5}}$

d) $\frac{\sqrt{2}+2}{3\sqrt{2}-4}$

$$\boxed{3+\sqrt{5}}, \quad \boxed{8+\sqrt{21}}, \quad \boxed{7+3\sqrt{5}}, \quad \boxed{7+5\sqrt{2}}$$

a) $\frac{4}{3-\sqrt{5}} = \frac{4(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{12+4\sqrt{5}}{9-5} = \frac{12+4\sqrt{5}}{4} = 3+\sqrt{5}$
 b) $\frac{5\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{(5\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{35+5\sqrt{21}-\sqrt{21}-3}{7+7\sqrt{21}-7\sqrt{21}-3} = \frac{32+4\sqrt{21}}{4} = 8+\sqrt{21}$
 c) $\frac{6+2\sqrt{5}}{3-\sqrt{5}} = \frac{(6+2\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{18+6\sqrt{5}+6\sqrt{5}+10}{9-5} = \frac{28+12\sqrt{5}}{4} = 7+3\sqrt{5}$
 d) $\frac{\sqrt{2}+2}{3\sqrt{2}-4} = \frac{(\sqrt{2}+2)(3\sqrt{2}+4)}{(3\sqrt{2}-4)(3\sqrt{2}+4)} = \frac{6+4\sqrt{2}+6\sqrt{2}+8}{18+12\sqrt{2}-12\sqrt{2}-16} = \frac{14+10\sqrt{2}}{2} = 7+5\sqrt{2}$

Question 29

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{6+\sqrt{30}}{6-\sqrt{30}}$

b) $\frac{2\sqrt{5}+2}{\sqrt{5}-2}$

c) $\frac{2}{3\sqrt{5}+7}$

d) $\frac{10\sqrt{3}-1}{4-\sqrt{3}}$

$11+2\sqrt{30}$, $14+6\sqrt{5}$, $\frac{7}{2}-\frac{3}{2}\sqrt{5}$, $2+3\sqrt{3}$

Handwritten solutions for the four parts of Question 29:

(a) $\frac{6+\sqrt{30}}{6-\sqrt{30}} = \frac{(6+\sqrt{30})(6+\sqrt{30})}{(6-\sqrt{30})(6+\sqrt{30})} = \frac{36+6\sqrt{30}+6\sqrt{30}+30}{36-30} = \frac{66+12\sqrt{30}}{6} = 11+2\sqrt{30}$

(b) $\frac{2\sqrt{5}+2}{\sqrt{5}-2} = \frac{(2\sqrt{5}+2)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)} = \frac{10+4\sqrt{5}+2\sqrt{5}+4}{5-4} = \frac{14+6\sqrt{5}}{1} = 14+6\sqrt{5}$

(c) $\frac{2}{3\sqrt{5}+7} = \frac{2(3\sqrt{5}-7)}{(3\sqrt{5}+7)(3\sqrt{5}-7)} = \frac{6\sqrt{5}-14}{45-49} = \frac{6\sqrt{5}-14}{-4} = \frac{14-6\sqrt{5}}{4} = \frac{7}{2}-\frac{3}{2}\sqrt{5}$

(d) $\frac{10\sqrt{3}-1}{4-\sqrt{3}} = \frac{(10\sqrt{3}-1)(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})} = \frac{40\sqrt{3}+30-4-\sqrt{3}}{16-3} = \frac{26+29\sqrt{3}}{13} = 2+3\sqrt{3}$

Question 30

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{8-\sqrt{7}}{\sqrt{7}-2}$

b) $\frac{6+\sqrt{2}}{2+\sqrt{2}}$

c) $\frac{1+\sqrt{7}}{3-\sqrt{7}}$

d) $\frac{4\sqrt{3}+3\sqrt{7}}{3\sqrt{3}+\sqrt{7}}$

$$\boxed{3+2\sqrt{7}}, \boxed{5-2\sqrt{2}}, \boxed{5+2\sqrt{7}}, \boxed{\frac{3+\sqrt{21}}{4}}$$

Handwritten solutions for the four parts of Question 30:

a) $\frac{8-\sqrt{7}}{\sqrt{7}-2} = \frac{(8-\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{8\sqrt{7}+16-7-2\sqrt{7}}{7-4} = \frac{9+6\sqrt{7}}{3} = 3+2\sqrt{7}$

b) $\frac{6+\sqrt{2}}{2+\sqrt{2}} = \frac{(6+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = \frac{12-6\sqrt{2}+2\sqrt{2}-2}{4-2} = \frac{10-4\sqrt{2}}{2} = 5-2\sqrt{2}$

c) $\frac{1+\sqrt{7}}{3-\sqrt{7}} = \frac{(1+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{3+3\sqrt{7}+3\sqrt{7}+7}{9-7} = \frac{10+6\sqrt{7}}{2} = 5+3\sqrt{7}$

d) $\frac{4\sqrt{3}+3\sqrt{7}}{3\sqrt{3}+\sqrt{7}} = \frac{(4\sqrt{3}+3\sqrt{7})(3\sqrt{3}-\sqrt{7})}{(3\sqrt{3}+\sqrt{7})(3\sqrt{3}-\sqrt{7})} = \frac{36-4\sqrt{21}+9\sqrt{21}-21}{27-7} = \frac{15+5\sqrt{21}}{20} = \frac{3}{4} + \frac{1}{4}\sqrt{21}$

Question 31

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{5+\sqrt{15}}{5-\sqrt{15}}$

b) $\frac{2\sqrt{11}-3}{2+\sqrt{11}}$

c) $\frac{5\sqrt{5}-2}{4+\sqrt{5}}$

d) $\frac{(3-\sqrt{5})^2}{1+\sqrt{5}}$

$$\boxed{4+\sqrt{15}}, \quad \boxed{4-\sqrt{11}}, \quad \boxed{-3+2\sqrt{5}}, \quad \boxed{-11+5\sqrt{5}}$$

Handwritten solutions for Question 31:

a) $\frac{5+\sqrt{15}}{5-\sqrt{15}} = \frac{(5+\sqrt{15})(5+\sqrt{15})}{(5-\sqrt{15})(5+\sqrt{15})} = \frac{25+5\sqrt{15}+5\sqrt{15}+15}{25-15} = \frac{40+10\sqrt{15}}{10} = 4+\sqrt{15}$

b) $\frac{2\sqrt{11}-3}{2+\sqrt{11}} = \frac{(2\sqrt{11}-3)(2-\sqrt{11})}{(2+\sqrt{11})(2-\sqrt{11})} = \frac{4\sqrt{11}-22-6+3\sqrt{11}}{4-11} = \frac{-22+7\sqrt{11}}{-7} = 4-\sqrt{11}$

c) $\frac{5\sqrt{5}-2}{4+\sqrt{5}} = \frac{(5\sqrt{5}-2)(4-\sqrt{5})}{(4+\sqrt{5})(4-\sqrt{5})} = \frac{20\sqrt{5}-25-8+2\sqrt{5}}{16-5} = \frac{-33+22\sqrt{5}}{11} = -3+2\sqrt{5}$

d) $\frac{(3-\sqrt{5})^2}{1+\sqrt{5}} = \frac{9-6\sqrt{5}+5}{(1+\sqrt{5})(1-\sqrt{5})} = \frac{14-6\sqrt{5}}{1-5} = \frac{14-6\sqrt{5}}{-4} = -11+5\sqrt{5}$

Question 32

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a) $\frac{2(\sqrt{3}+2)^2}{\sqrt{3}+1}$

b) $\frac{98}{(3+\sqrt{2})^2}$

c) $\frac{(2+\sqrt{3})^2 - (1-\sqrt{3})^2}{\sqrt{3}}$

d) $\frac{6}{3+\sqrt{7}} - \frac{4}{3-\sqrt{7}}$

$5+3\sqrt{3}$, $22-12\sqrt{2}$, $6+\sqrt{3}$, $3-5\sqrt{7}$

Handwritten solutions for Question 32:

a) $\frac{2(\sqrt{3}+2)^2}{\sqrt{3}+1} = \frac{2(3+4\sqrt{3}+4)}{\sqrt{3}+1} = \frac{(4+8\sqrt{3})(6\sqrt{3}-6)}{(\sqrt{3}+1)(6\sqrt{3}-6)} = \frac{4\sqrt{3}-4+24\sqrt{3}-24}{3-24+36-24} = \frac{28\sqrt{3}-20}{-6} = 5+3\sqrt{3}$

b) $\frac{98}{(3+\sqrt{2})^2} = \frac{98}{9+6\sqrt{2}+2} = \frac{98}{11+6\sqrt{2}} = \frac{98(11-6\sqrt{2})}{(11+6\sqrt{2})(11-6\sqrt{2})} = \frac{98(11-6\sqrt{2})}{121-36(2)} = \frac{98(11-6\sqrt{2})}{121-72} = \frac{98(11-6\sqrt{2})}{49} = 2(11-6\sqrt{2}) = 22-12\sqrt{2}$

c) $\frac{(2+\sqrt{3})^2 - (1-\sqrt{3})^2}{\sqrt{3}} = \frac{(4+4\sqrt{3}+3) - (1-2\sqrt{3}+3)}{\sqrt{3}} = \frac{7+4\sqrt{3}-4+2\sqrt{3}}{\sqrt{3}} = \frac{3+6\sqrt{3}}{\sqrt{3}} = \frac{3+6\sqrt{3}}{\sqrt{3}} = \frac{3}{\sqrt{3}} + \frac{6\sqrt{3}}{\sqrt{3}} = \frac{3}{\sqrt{3}} + 6 = 6 + \sqrt{3}$

d) $\frac{6}{3+\sqrt{7}} - \frac{4}{3-\sqrt{7}} = \frac{6(3-\sqrt{7}) - 4(3+\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{18-6\sqrt{7}-12-4\sqrt{7}}{9-7} = \frac{6-10\sqrt{7}}{2} = 3-5\sqrt{7}$

Question 33

Simplify each of the following surd expressions, giving the final answer in the form $p+q\sqrt{r}$, where p , q and r are integers

a)
$$\frac{1}{5+\sqrt{2}} + \frac{1}{5-\sqrt{2}}$$

b)
$$\frac{5}{\sqrt{3}-1} - \frac{1}{1-\sqrt{3}}$$

c)
$$\frac{2+\sqrt{5}}{3-\sqrt{5}} + \frac{5}{3+\sqrt{5}}$$

d)
$$\frac{1+\sqrt{7}}{3-\sqrt{7}} - \frac{8-\sqrt{7}}{-2+\sqrt{7}}$$

$$\frac{10}{23}, 3+3\sqrt{3}, \frac{13}{2}, 2$$

Question 34

Solve each of the following equations, giving the answer as an exact simplified surd.

a) $\sqrt{3}(x - \sqrt{3}) = x + \sqrt{3}$

b) $\frac{2+y}{y} = \sqrt{2}$

c) $z\sqrt{8} - 6 = \frac{2z}{\sqrt{2}}$

d) $\frac{1+w}{w} = \sqrt{2}$

$x = 3 + 2\sqrt{3}$, $y = 2 + 2\sqrt{2}$, $z = 3\sqrt{2}$, $w = 1 + \sqrt{2}$

Handwritten solutions for Question 34:

a) $\sqrt{3}(x - \sqrt{3}) = x + \sqrt{3}$
 $\Rightarrow \sqrt{3}x - 3 = x + \sqrt{3}$
 $\Rightarrow \sqrt{3}x - x = 3 + \sqrt{3}$
 $\Rightarrow x(\sqrt{3} - 1) = 3 + \sqrt{3}$
 $\Rightarrow x = \frac{3 + \sqrt{3}}{\sqrt{3} - 1}$
 $\Rightarrow x = \frac{(3 + \sqrt{3})(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$
 $\Rightarrow x = \frac{3\sqrt{3} + 3 + \sqrt{3} + \sqrt{3}}{3 - 1}$
 $\Rightarrow x = \frac{4\sqrt{3} + 3}{2}$
 $\Rightarrow x = 3 + 2\sqrt{3}$

b) $\frac{2+y}{y} = \sqrt{2}$
 $\Rightarrow 2 + y = \sqrt{2}y$
 $\Rightarrow 2 = \sqrt{2}y - y$
 $\Rightarrow 2 = y(\sqrt{2} - 1)$
 $\Rightarrow y = \frac{2}{\sqrt{2} - 1}$
 $\Rightarrow y = \frac{2(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$
 $\Rightarrow y = \frac{2\sqrt{2} + 2}{2 - 1}$
 $\Rightarrow y = 2\sqrt{2} + 2$

c) $z\sqrt{8} - 6 = \frac{2z}{\sqrt{2}}$
 $\Rightarrow 2\sqrt{2}z - 6 = 2z$
 $\Rightarrow 2\sqrt{2}z - 2z = 6$
 $\Rightarrow z(2\sqrt{2} - 2) = 6$
 $\Rightarrow z = \frac{6}{2(\sqrt{2} - 1)}$
 $\Rightarrow z = \frac{3}{\sqrt{2} - 1}$
 $\Rightarrow z = \frac{3(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$
 $\Rightarrow z = \frac{3\sqrt{2} + 3}{2 - 1}$
 $\Rightarrow z = 3\sqrt{2} + 3$

d) $\frac{1+w}{w} = \sqrt{2}$
 $\Rightarrow 1 + w = \sqrt{2}w$
 $\Rightarrow 1 = \sqrt{2}w - w$
 $\Rightarrow 1 = w(\sqrt{2} - 1)$
 $\Rightarrow w = \frac{1}{\sqrt{2} - 1}$
 $\Rightarrow w = \frac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$
 $\Rightarrow w = \frac{\sqrt{2} + 1}{2 - 1}$
 $\Rightarrow w = \sqrt{2} + 1$

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