

Created by T. Madas

# **2<sup>nd</sup> ORDER O.D.E.s**

## **PRACTICE**

Created by T. Madas

## Question 1

Find a general solution for each of the following differential equations.

a)  $2\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 3y = 0$

b)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$

c)  $\frac{d^2y}{dx^2} + 4y = 0$

d)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0$

$$\boxed{y = Ae^{-3x} + Be^{-\frac{1}{2}x}}, \quad \boxed{y = Ae^{-2x} + Bxe^{-2x}}, \quad \boxed{y = A\cos 2x + B\sin 2x},$$

$$\boxed{y = e^{-2x} (A\cos 2x + B\sin 2x)}$$

Handwritten solutions for the four differential equations in Question 1:

- a)  $2\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 3y = 0$   
 • Aux equation:  
 $2\lambda^2 + 7\lambda + 3 = 0$   
 $(2\lambda + 1)(\lambda + 3) = 0$   
 $\lambda = -\frac{1}{2}, -3$  (Check if real)  
 $y = Ae^{-\frac{1}{2}x} + Be^{-3x}$
- b)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$   
 • Aux equation:  
 $\lambda^2 + 4\lambda + 4 = 0$   
 $(\lambda + 2)^2 = 0$   
 $\lambda = -2$  (Complex)  
 $y = Ae^{-2x} + Bxe^{-2x}$
- c)  $\frac{d^2y}{dx^2} + 4y = 0$   
 • Aux equation:  
 $\lambda^2 + 4 = 0$   
 $\lambda^2 = -4$   
 $\lambda = \pm 2i$  (Complex imaginary)  
 $y = A\cos 2x + B\sin 2x$
- d)  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0$   
 • Aux equation:  
 $\lambda^2 + 4\lambda + 8 = 0$   
 $(\lambda + 2)^2 + 4 = 0$   
 $(\lambda + 2)^2 = -4$   
 $\lambda + 2 = \pm 2i$   
 $\lambda = -2 \pm 2i$  (Complex)  
 $y = e^{-2x} (A\cos 2x + B\sin 2x)$

## Question 2

Find a general solution for each of the following differential equations.

a)  $4 \frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 5y = 0$

b)  $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 0$

c)  $4 \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + y = 0$

d)  $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 13y = 0$

$$y = e^x \left( A \cos \frac{1}{2}x + B \sin \frac{1}{2}x \right), \quad y = Ae^{-x} + Be^{2x}, \quad y = Ae^{\frac{1}{2}x} + Bxe^{\frac{1}{2}x},$$

$$y = e^{-2x} (A \cos 3x + B \sin 3x)$$

Handwritten solutions for the differential equations in Question 2:

a)  $4 \frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 5y = 0$   
 Aux equation:  
 $4\lambda^2 - 8\lambda + 5 = 0$   
 $\lambda^2 - 2\lambda + \frac{5}{4} = 0$   
 $(\lambda - 1)^2 - (1 - \frac{5}{4}) = 0$   
 $(\lambda - 1)^2 = -\frac{1}{4}$   
 $\lambda - 1 = \pm \frac{1}{2}i$   
 $\lambda = 1 \pm \frac{1}{2}i$   
 $y = e^x (A \cos \frac{1}{2}x + B \sin \frac{1}{2}x)$

b)  $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 2y = 0$   
 Aux equation:  
 $\lambda^2 - \lambda - 2 = 0$   
 $(\lambda + 1)(\lambda - 2) = 0$   
 $\lambda = -1, 2$   
 $y = Ae^{-x} + Be^{2x}$

c)  $4 \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + y = 0$   
 Aux equation:  
 $4\lambda^2 - 4\lambda + 1 = 0$   
 $(2\lambda - 1)^2 = 0$   
 $2\lambda - 1 = 0$   
 $\lambda = \frac{1}{2}$   
 $y = Ae^{\frac{1}{2}x} + Bxe^{\frac{1}{2}x}$

d)  $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 13y = 0$   
 Aux equation:  
 $\lambda^2 + 4\lambda + 13 = 0$   
 $(\lambda + 2)^2 - 4 + 13 = 0$   
 $(\lambda + 2)^2 = -9$   
 $\lambda + 2 = \pm 3i$   
 $\lambda = -2 \pm 3i$   
 $y = e^{-2x} (A \cos 3x + B \sin 3x)$

### Question 3

Find a general solution for each of the following differential equations.

a)  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3x$

b)  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 36x$

c)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36$

$$y = Ae^{-3x} + Bxe^{-3x} + \frac{1}{3}x - \frac{2}{9}, \quad y = Ae^x + Be^{-6x} - 6x - 5,$$

$$y = e^{-x}(A \cos 4x + B \sin 4x) + x + 2$$

a)  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3x$

Assume  $y = Ax^2 + Bx + C$

$2A + 6(2Ax + B) + 9(Ax^2 + Bx + C) = 3x$

$9Ax^2 + (12A + 9B)x + (6B + 9C) = 3x$

Equating coefficients:

$9A = 0 \Rightarrow A = 0$

$12A + 9B = 3 \Rightarrow 9B = 3 \Rightarrow B = \frac{1}{3}$

$6B + 9C = 0 \Rightarrow 6(\frac{1}{3}) + 9C = 0 \Rightarrow 2 + 9C = 0 \Rightarrow C = -\frac{2}{9}$

C.F.:  $y = \frac{1}{3}x - \frac{2}{9}$

P.I.:  $y = \frac{1}{3}x - \frac{2}{9}$

General solution:  $y = Ae^{-3x} + Bxe^{-3x} + \frac{1}{3}x - \frac{2}{9}$

b)  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = 36x$

Assume  $y = Ax^2 + Bx + C$

$2A + 5(2Ax + B) - 6(Ax^2 + Bx + C) = 36x$

$-4Ax^2 + (10A + 5B - 6C)x + (5B - 6C) = 36x$

Equating coefficients:

$-4A = 0 \Rightarrow A = 0$

$10A + 5B - 6C = 36 \Rightarrow 5B - 6C = 36$

$5B - 6C = 36$

$5B = 36 + 6C$

$B = \frac{36}{5} + \frac{6C}{5}$

$5(\frac{36}{5} + \frac{6C}{5}) - 6C = 36$

$36 + 6C - 6C = 36$

$36 = 36$

C.F.:  $y = \frac{36}{5} + \frac{6C}{5}$

P.I.:  $y = \frac{36}{5} + \frac{6C}{5}$

General solution:  $y = Ae^x + Be^{-6x} - 6x - 5$

## Question 4

Find a general solution for each of the following differential equations.

a)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10\sin x$

b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36$

c)  $\frac{d^2y}{dx^2} + 4y = 12\cos x$

$y = e^{-x}(A\cos 2x + B\sin 2x) + 2\sin x - \cos x$ ,  $y = e^{-x}(A\cos 4x + B\sin 4x) + x + 2$ ,

$y = A\cos 2x + B\sin 2x + 4\cos x$

## Question 5

Find a general solution for each of the following differential equations.

a)  $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 6y = \sin x$

b)  $\frac{d^2 y}{dx^2} + 4y = 8x^2 + 9\sin x$

c)  $\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 3y = 10\sin x$

$$y = Ae^{3x} + Be^{-2x} + \frac{1}{50}(\cos x - 7\sin x), \quad y = A\cos 2x + B\sin 2x + 3\sin x + 2x^2 - 1,$$

$$y = Ae^{-x} + Be^{-3x} + \sin x - 2\cos x$$

a)  $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 6y = \sin x$

• Aux equation  
 $\lambda^2 - \lambda - 6 = 0$   
 $(\lambda - 3)(\lambda + 2) = 0$   
 $\lambda = 3, -2$

C.F. :  $y = Ae^{3x} + Be^{-2x}$

• Try  $y = P\cos x + Q\sin x$   
 $\frac{dy}{dx} = -P\sin x + Q\cos x$   
 $\frac{d^2 y}{dx^2} = -P\cos x - Q\sin x$

Sub into the O.D.E.  
 $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 6y = \sin x$   
 $-P\cos x - Q\sin x - (-P\sin x + Q\cos x) - 6(P\cos x + Q\sin x) = \sin x$   
 $-P\cos x - Q\sin x + P\sin x - Q\cos x - 6P\cos x - 6Q\sin x = \sin x$   
 $(-P - Q - 6P)\cos x + (P - Q - 6Q)\sin x = \sin x$   
 $-7P - Q = 0 \quad P - 7Q = 1$   
 $-7P - Q = 0 \Rightarrow Q = -7P$   
 $-7P - (-7P) = 1 \Rightarrow 0 = 1$  (Contradiction)  
 $\therefore$  S.O.P. = 1  
 $P = \frac{1}{10}, Q = -\frac{7}{10}$   
 $\therefore$  P.I. :  $y = \frac{1}{10}\cos x - \frac{7}{10}\sin x$   
 $\therefore$  Gen Sol.  $y = Ae^{3x} + Be^{-2x} + \frac{1}{10}\cos x - \frac{7}{10}\sin x$

b)  $\frac{d^2 y}{dx^2} + 4y = 8x^2 + 9\sin x$

• Aux equation  
 $\lambda^2 + 4 = 0$   
 $\lambda^2 = -4$   
 $\lambda = \pm 2i$

C.F. :  $y = A\cos 2x + B\sin 2x$

• Try  $y = P^2 + Qx + R + T\sin x$   
 $\frac{dy}{dx} = 2Px + Q + T\cos x$   
 $\frac{d^2 y}{dx^2} = 2P - T\sin x$

Sub into the O.D.E.  
 $\frac{d^2 y}{dx^2} + 4y = 8x^2 + 9\sin x$   
 $2P - T\sin x + 4(P^2 + Qx + R + T\sin x) = 8x^2 + 9\sin x$   
 $4P^2 + 4Qx + 4R + 4T\sin x - T\sin x = 8x^2 + 9\sin x$   
 $4P^2 + 4Qx + 4R + 3T\sin x = 8x^2 + 9\sin x$   
 $4P^2 = 8 \Rightarrow P^2 = 2 \Rightarrow P = \sqrt{2}$   
 $4Q = 0 \Rightarrow Q = 0$   
 $4R = 0 \Rightarrow R = 0$   
 $3T = 9 \Rightarrow T = 3$   
 $\therefore$  P.I. :  $y = 2x^2 - 1 + 3\sin x$   
 $\therefore$  Gen Sol.  $y = A\cos 2x + B\sin 2x + 2x^2 - 1 + 3\sin x$

c)  $\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 3y = 10\sin x$

• Aux equation  
 $\lambda^2 + 4\lambda + 3 = 0$   
 $(\lambda + 3)(\lambda + 1) = 0$   
 $\lambda = -3, -1$

C.F. :  $y = Ae^{-x} + Be^{-3x}$

• Try  $y = P\cos x + Q\sin x$   
 $\frac{dy}{dx} = -P\sin x + Q\cos x$   
 $\frac{d^2 y}{dx^2} = -P\cos x - Q\sin x$

Sub into the O.D.E.  
 $\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 3y = 10\sin x$   
 $-P\cos x - Q\sin x + 4(-P\sin x + Q\cos x) + 3(P\cos x + Q\sin x) = 10\sin x$   
 $-P\cos x - Q\sin x - 4P\sin x + 4Q\cos x + 3P\cos x + 3Q\sin x = 10\sin x$   
 $(-P + 4Q + 3P)\cos x + (-Q - 4P + 3Q)\sin x = 10\sin x$   
 $2P + 4Q = 0 \Rightarrow P + 2Q = 0 \Rightarrow P = -2Q$   
 $-Q - 4(-2Q) + 3Q = 10 \Rightarrow -Q + 8Q + 3Q = 10 \Rightarrow 10Q = 10 \Rightarrow Q = 1$   
 $P = -2$   
 $\therefore$  P.I. :  $y = \sin x - 2\cos x$   
 $\therefore$  Gen Sol.  $y = Ae^{-x} + Be^{-3x} + \sin x - 2\cos x$

Question 6

Find the solution of each of the following differential equations.

a)  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \cos 3x$ ,

subject to the conditions  $y = \frac{1}{2}$ ,  $\frac{dy}{dx} = 0$  at  $x = 0$ .

b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10$ ,

subject to the conditions  $y = 0$ ,  $\frac{dy}{dx} = 0$  at  $x = 0$ .

c)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 8e^{3x}$ ,

subject to the conditions  $y = 1$ ,  $\frac{dy}{dx} = 2$  at  $x = 0$ .

$$y = \frac{1}{2}e^{-3x} + \frac{4}{3}xe^{-3x} + \frac{1}{18}\sin 3x, \quad y = 2 - e^{-x}(2\cos 2x - \sin 2x),$$

$$y = \frac{1}{2}e^{3x} + \frac{1}{2}e^{-x} + xe^{-x}$$

a)  $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \cos 3x$

• Aux equation  
 $\lambda^2 + 6\lambda + 9 = 0$   
 $(\lambda + 3)^2 = 0$   
 $\lambda = -3$  (Repeating)

• Try  $y = P\cos 3x + Q\sin 3x$

$\frac{dy}{dx} = -3P\sin 3x + 3Q\cos 3x$   
 $\frac{d^2y}{dx^2} = -9P\cos 3x - 9Q\sin 3x$

Sub into the O.D.E

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \cos 3x$   
 $-9P\cos 3x - 9Q\sin 3x + 6(-3P\sin 3x + 3Q\cos 3x) + 9(P\cos 3x + Q\sin 3x) = \cos 3x$   
 $-9P\cos 3x - 9Q\sin 3x - 18P\sin 3x + 18Q\cos 3x + 9P\cos 3x + 9Q\sin 3x = \cos 3x$   
 $-18P\sin 3x + 9Q\sin 3x = 0$   
 $-9P\sin 3x = 0$   
 $P = 0$

P.T.  $y = Q\sin 3x$

Gen solution  $y = A\cos 3x + B\sin 3x + \frac{1}{18}\sin 3x$

• Apply cond.  $y = \frac{1}{2}$   
 $\frac{1}{2} = A + 0 + 0$   
 $A = \frac{1}{2}$

•  $y = \frac{1}{2}\cos 3x + B\sin 3x + \frac{1}{18}\sin 3x$   
 $\Rightarrow \frac{dy}{dx} = -\frac{3}{2}\sin 3x + 3B\cos 3x + \frac{1}{6}\sin 3x$

Apply cond.  $\frac{dy}{dx} = 0$   
 $0 = -\frac{3}{2} + 0 + 0$   
 $B = \frac{1}{3}$

Ths  $y = \frac{1}{2}\cos 3x + \frac{1}{3}\sin 3x + \frac{1}{18}\sin 3x$

b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10$

• Aux equation  
 $\lambda^2 + 2\lambda + 5 = 0$   
 $(\lambda + 1)^2 - 4 = 0$   
 $\lambda + 1 = \pm 2$   
 $\lambda = -1 \pm 2$

• Try  $y = P$

$\frac{dy}{dx} = 0$   
 $\frac{d^2y}{dx^2} = 0$

Sub into the O.D.E

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10$   
 $0 + 0 + 5P = 10$   
 $P = 2$

P.T.  $y = 2$

• Gen solution:  $y = e^{-(\lambda_1 x + \lambda_2 x)} + 2$

Apply cond.  $y = 0$   
 $0 = A + 2$   
 $A = -2$

•  $y = e^{-(\lambda_1 x + \lambda_2 x)} + 2$   
 $\frac{dy}{dx} = -e^{-(\lambda_1 x + \lambda_2 x)}(\lambda_1 + \lambda_2) + 0$   
 $\frac{dy}{dx} = -e^{-(\lambda_1 x + \lambda_2 x)}(-1 + 2) = -e^{-(\lambda_1 x + \lambda_2 x)}$

Apply cond.  $\frac{dy}{dx} = 0$   
 $0 = -1 + 2$   
 $2 = 1$

Ths  $y = e^{-(\lambda_1 x + \lambda_2 x)} + 2$   
 $y = 2 - e^{-(\lambda_1 x + \lambda_2 x)}$

c)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 8e^{3x}$

• Aux equation  
 $\lambda^2 + 2\lambda + 1 = 0$   
 $(\lambda + 1)^2 = 0$   
 $\lambda = -1$  (Repeating)

• Try  $y = P e^{3x}$

$\frac{dy}{dx} = 3P e^{3x}$   
 $\frac{d^2y}{dx^2} = 9P e^{3x}$

Sub into the O.D.E

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 8e^{3x}$   
 $9P e^{3x} + 2(3P e^{3x}) + P e^{3x} = 8e^{3x}$   
 $16P = 8$   
 $P = \frac{1}{2}$

P.T.  $y = \frac{1}{2}e^{3x}$

• Gen solution:  $y = A e^{3x} + B e^{-x} + \frac{1}{2}e^{3x}$

• Apply cond.  $y = 1$   
 $1 = A + B + \frac{1}{2} \Rightarrow A + B = \frac{1}{2}$

•  $\frac{dy}{dx} = 2$   
 $2 = 3A - B + \frac{3}{2}$   
 $2 = \frac{3}{2} - B + \frac{3}{2}$   
 $2 = 3 - B$   
 $B = 1$

•  $y = \frac{1}{2}e^{3x} + 1e^{-x} + \frac{1}{2}e^{3x}$

## Question 7

Find the solution of each of the following differential equations.

a)  $\frac{d^2 y}{dx^2} - 4y = 10e^{3x},$

subject to the conditions  $y = 3, \frac{dy}{dx} = 8$  at  $x = 0.$

b)  $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} - 3y = 15e^{2x},$

subject to the conditions  $y = 9, \frac{dy}{dx} = 4$  at  $x = 0.$

c)  $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + y = 8e^{3x},$

subject to the conditions  $x = 0, y = 1, \frac{dy}{dx} = 2$

$$y = e^{2x} + 2e^{3x}, \quad y = 4e^x + 2e^{-3x} + 3e^{2x}, \quad y = 2e^{3x} - e^{-x} - 5xe^{-x}$$

## Question 8

Find the solution of each of the following differential equations.

a)  $2 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 2y = 2x + 9,$

subject to the conditions  $y = 3, \frac{dy}{dx} = -1$  at  $x = 0$ .

b)  $\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = 27x,$

subject to the conditions  $y = 2, \frac{dy}{dx} = 6$  at  $x = 0$ .

c)  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = 2x + 3,$

subject to the conditions  $y = 2, \frac{dy}{dx} = -5$  at  $x = 0$ .

$$\boxed{y = e^{-2x} + x + 2}, \quad \boxed{y = 3x(1 + e^{3x}) + 2}, \quad \boxed{y = x^2 + x - 4 + 6e^{-x}}$$

## Question 9

Find the solution of each of the following differential equations.

$$\text{a) } 2 \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} - 4y = 8 \sin x - 19 \cos x,$$

subject to the conditions  $y = 0, \frac{dy}{dx} = 11$  at  $x = 0$ .

$$\text{b) } 2 \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} - 4y = 8 \sin x - 19 \cos x,$$

subject to the conditions  $y = 0, \frac{dy}{dx} = 11$  at  $x = 0$ .

$$\text{c) } 2 \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} - 4y = 8 \sin x - 19 \cos x,$$

subject to the conditions  $y = 0, \frac{dy}{dx} = 11$  at  $x = 0$ .

$$\boxed{y = e^{-2x} + x + 2}, \quad \boxed{y = 3x(1 + e^{3x}) + 2}, \quad \boxed{y = 2e^{4x} - 4e^{\frac{1}{2}x} + \sin x + 2\cos x}$$



**Question 11 (harder PIs)**

Find the solution of each of the following differential equations.

a)  $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y = 10e^x,$

subject to the conditions  $y = 1, \frac{dy}{dx} = 3$  at  $x = 0$ .

b)  $\frac{d^2y}{dx^2} + 16y = 8\cos 4x,$

subject to the conditions  $y = 2, \frac{dy}{dx} = 0$  at  $x = 0$ .

c)  $\frac{d^2y}{dx^2} + 100y = 2\cos 10x,$

subject to the conditions  $y = 0, \frac{dy}{dx} = 0$  at  $x = 0$ .

$$y = e^x(2x+1), \quad y = 2\cos 4x + x\sin 4x, \quad y = \frac{1}{10}x\sin 10x$$

**Question 12 (harder PIs)**

Find the solution of each of the following differential equations.

a)  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x,$

subject to the conditions  $y = 1, \frac{dy}{dx} = 2$  at  $x = 0$ .

b)  $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x},$

subject to the conditions  $y = 1, \frac{dy}{dx} = -1$  at  $x = 0$ .

c)  $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} = 100x,$

subject to the conditions  $y = 2, \frac{dy}{dx} = 1$  at  $x = 0$ .

$$\boxed{y = \left(\frac{1}{2}x^2 + x + 1\right)e^x}, \quad \boxed{y = \left(\frac{1}{2}x^2 + 1\right)e^{-x}}, \quad \boxed{y = e^{5x} + 1 - 4x - 10x^2}$$

# **2<sup>nd</sup> ORDER O.D.E. EULER TYPE**

## Question 1

Find the solution of each of the following differential equations.

a)  $x^2 \frac{d^2 y}{dx^2} - 6x \frac{dy}{dx} + 12y = 0,$

subject to the conditions  $x = 1, y = 1, \frac{dy}{dx} = 0$

b)  $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = 0,$

subject to the conditions  $x = 1, y = 3, \frac{dy}{dx} = 7$

c)  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 4y = 0,$

subject to the conditions  $x = 1, y = 5, \frac{dy}{dx} = 8$

$$\boxed{y = 4x^3 - 3x^4}, \quad \boxed{y = \frac{1}{x} + 2x^4}, \quad \boxed{y = 4x + x^4}$$

