

Created by T. Madas

# KINEMATIC GRAPHS

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# SPEED – TIME GRAPHS

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**Question 1 (\*\*)**

A runner is running along a straight horizontal road.

He starts from rest at point  $A$ , accelerating uniformly for  $6\text{ s}$ , reaching a top speed of  $7\text{ ms}^{-1}$ . This speed is maintained for  $T\text{ s}$ .

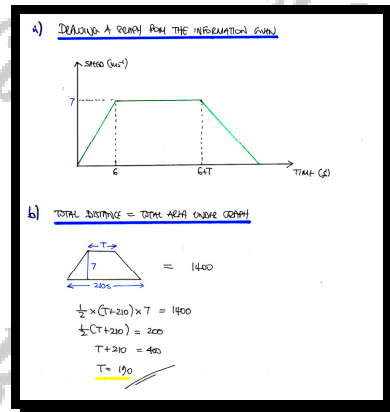
The runner then decelerates uniformly coming to a stop at point  $B$ .

- a) Sketch a speed time graph to show the motion of the runner from  $A$  to  $B$ .

The distance  $AB$  is  $1400\text{ m}$  and the runner took  $3\frac{1}{2}$  minutes to travel from  $A$  to  $B$ .

- b) Determine the value of  $T$ .

$T = 190$



**Question 2 (\*\*)**

A car is travelling along a straight horizontal road. At time  $t = 0$  s it goes past a point  $A$  on this road with speed  $28 \text{ ms}^{-1}$ .

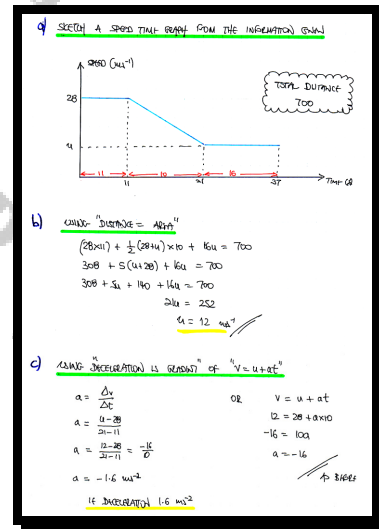
The car maintains this speed until  $t = 11$  s. The car then decelerates uniformly for 10 s reaching a speed of  $u \text{ ms}^{-1}$ .

The car maintains the speed of  $u \text{ ms}^{-1}$  until it goes past a point  $B$  on this road.

The distance  $AB$  is 700 m and the car took 37 s to travel from  $A$  to  $B$ .

- Sketch a speed time graph to show the motion of the car from  $A$  to  $B$ .
- Determine the value of  $u$ .
- Find the deceleration of the car during the motion described above.

$u = 12$ ,  $|a| = 1.6 \text{ ms}^{-2}$



**Question 3 (\*\*)**

A car starts from rest at point A and accelerates at constant rate for 15 s reaching a speed of  $V \text{ ms}^{-1}$ .

The car maintains this speed for 60 s. The car then decelerates at constant rate for 10 s, coming to rest at point B.

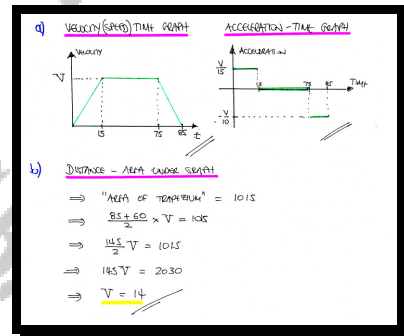
a) Given that the motion of the car takes place on a straight horizontal road, sketch for the motion of the car from A to B,

- i. ... a velocity time graph.
- ii. ... an acceleration time graph.

The distance AB is 1015 m.

b) Find the value of  $V$ .

,  $V = 14$



**Question 4** (\*\*\*)

A train is travelling at  $20 \text{ ms}^{-1}$  on a straight horizontal track when the driver sees a red signal 315 m ahead of the front of the train.

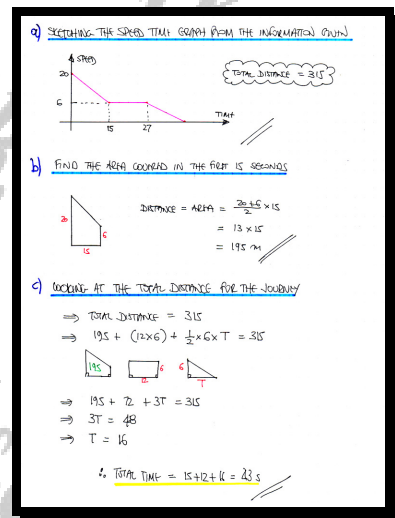
The driver immediately applies the brakes and the train decelerates uniformly for 15 s to a speed of  $6 \text{ ms}^{-1}$ .

The train then maintains this speed for another 12 s.

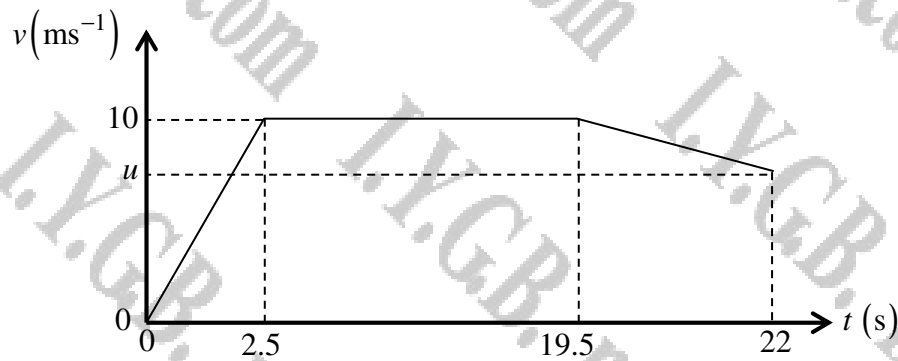
The driver then reapplies the brakes and the front of the train comes to a stop, level with the red signal.

- Sketch a speed time graph which describes the above information.
- Find the distance travelled by the train from the moment the brakes were first applied to the moment its speed first reached  $6 \text{ ms}^{-1}$ .
- Calculate the total time from the moment the brakes were first applied to the moment the train came to rest.

, distance = 195 m ,  t = 43 s



Question 5 (\*\*\*)



The figure above shows the speed time graph  $(t, v)$  of a sprinter running a 200 m race in a straight horizontal track.

The sprinter starts from rest and accelerates uniformly until he reaches his top speed of  $10 \text{ ms}^{-1}$  in 2.5 s. He maintains this speed for 17 s when he experiences a cramp.

The sprinter then decelerates uniformly crossing the finishing line with speed  $u \text{ ms}^{-1}$ , 22 s after the start of the race.

- Calculate the distance covered by the sprinter until the moment he experienced a cramp.
- Determine the value of  $u$ .
- Find the deceleration of the sprinter at the last part of the race.

$\boxed{S-P}$ ,  $\boxed{\text{distance} = 182.5 \text{ m}}$ ,  $\boxed{u = 4}$ ,  $\boxed{|a| = 2.4 \text{ ms}^{-2}}$

Q) LOOKING AT THE SPEED TIME GRAPH

AREA OF TRAPEZIUM

$$\text{DISTANCE} = \frac{17 + 19.5}{2} \times 10$$

$$= 18.25 \times 10$$

$$= 182.5 \text{ m}$$

b) IS THIS IS A 200 METRE RACE...

$$200 - 182.5 = 17.5$$

$$\frac{10 + u}{2} \times 2.5 = 17.5$$

$$(10 + u) \times 2.5 = 35$$

$$10 + u = 14$$

$$u = 4 \text{ ms}^{-1}$$

c) USING THE ACCELERATION = GRADIENT

$$a = \frac{u - 10}{22 - 19.5} = \frac{4 - 10}{2.5} = \frac{-6}{2.5} = -2.4$$

DECELERATION OF  $2.4 \text{ ms}^{-2}$

**Question 6** (\*\*\*)

A car is travelling along a straight horizontal road. At time  $t = 0$  s it goes past a point  $A$  on this road with speed  $16 \text{ ms}^{-1}$ .

As the car passes  $A$  it begins decelerate uniformly reaching a speed of  $10 \text{ ms}^{-1}$  covering a distance of  $104 \text{ m}$ .

The car then immediately begins to accelerate uniformly for  $12 \text{ s}$  reaching a speed  $u \text{ ms}^{-1}$ , where  $u > 16$ .

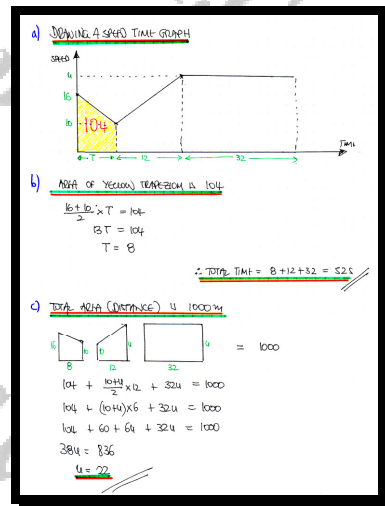
This new speed is maintained for another  $32 \text{ s}$  when the car goes past a point  $B$  on this road.

- Sketch a speed time graph to show the motion of the car from  $A$  to  $B$ .
- Determine the total time taken for the car to move from  $A$  to  $B$ .

The distance  $AB$  is  $1 \text{ km}$ .

- Calculate the value of  $u$ .

,  $t = 52$  ,  $u = 22$





**Question 7 (\*\*\*)**

A car is travelling along a straight horizontal road. It starts from rest at point  $A$  and accelerates uniformly at  $a \text{ ms}^{-2}$ , reaching a speed of  $18 \text{ ms}^{-1}$ .

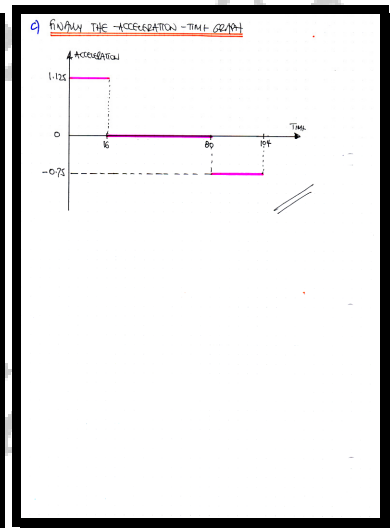
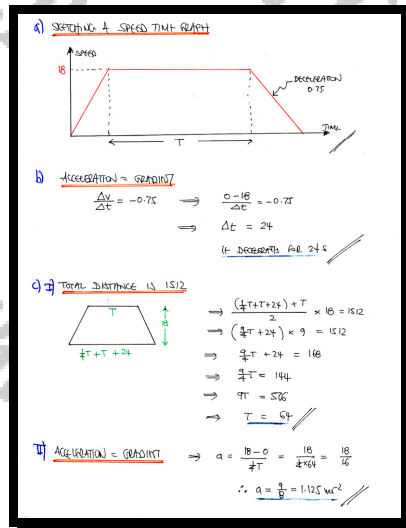
The car then travels at constant speed for  $T \text{ s}$ . Finally the car begins to decelerate uniformly at  $0.75 \text{ ms}^{-2}$  coming to rest at point  $B$ .

- a) Sketch a speed time graph to show the motion of the car from  $A$  to  $B$ .
- b) Determine the time for which the car decelerates.

It is further given that the car accelerates for  $\frac{1}{4}T \text{ s}$  and the distance  $AB$  is  $1512 \text{ m}$ .

- c) Calculate ...
  - i. ... the value of  $T$ .
  - ii. ... the value of  $a$ .
- d) Sketch an acceleration time graph to show the motion of the car from  $A$  to  $B$ .

 ,  $t = 24$ ,  $T = 64$ ,  $a = 1.125$



**Question 8 (\*\*\*)**

A lift is moving upwards.

The lift accelerates from rest with uniform acceleration  $0.36 \text{ ms}^{-2}$  until it reaches a speed of  $1.62 \text{ ms}^{-1}$ .

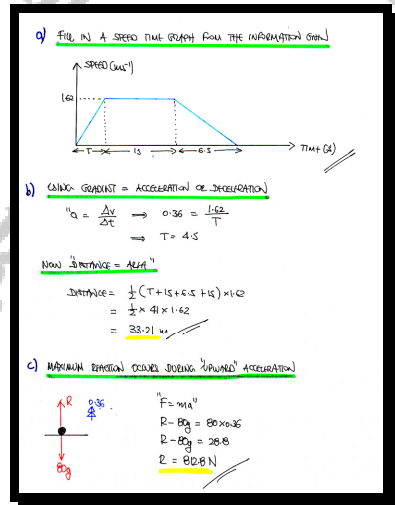
It then travels at constant speed for 15 s before decelerating uniformly to rest in 6.5 s.

- a) Sketch a speed time graph for the lift's journey.
- b) Determine the distance covered by the lift during the journey.

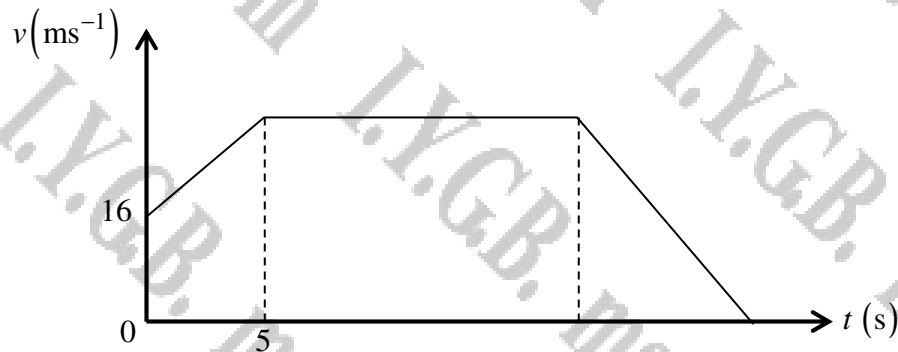
A man of mass 80 kg is standing in the lift during this journey.

- c) Calculate the greatest value of the reaction exerted by the floor of the lift on the man during the journey.

, distance = 33.21 m ,  $R_{\text{max}} = 812.8 \text{ N}$



Question 9 (\*\*\*)



The figure above shows the speed time graph  $(t, v)$  of a particle in a straight line between two points, A and B.

At  $t = 0$ , the particle is observed passing through A with speed  $16 \text{ ms}^{-1}$  and constant acceleration of  $1.6 \text{ ms}^{-2}$ , which it maintains until  $t = 5$ .

For  $t \geq 5$ , the particle moves with constant speed which it maintains until it is 820 m from A.

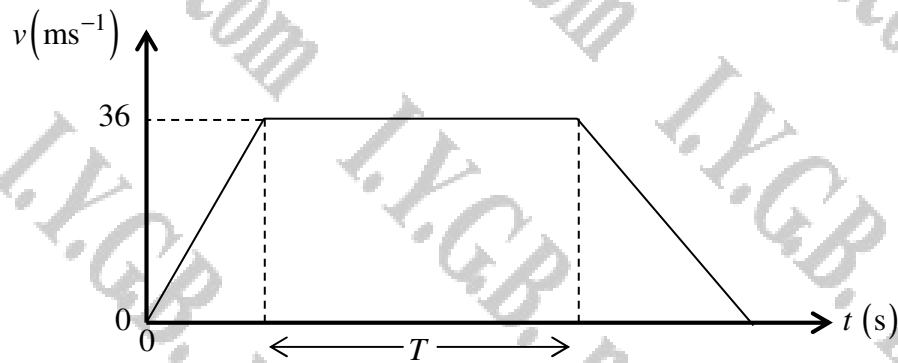
As soon the particle is reaches a distance of 820 m from A, it decelerates uniformly at  $4 \text{ ms}^{-2}$ , coming to rest at B.

Determine in any order the distance AB and the total time taken for the particle to travel from A to B.

,  $|AB| = 892 \text{ m}$  ,  $T = 41 \text{ s}$

<p><u>RECORDING THE INFORMATION GIVEN</u></p> <p><u>LOOKING AT THE "FIRST" SECTION</u></p> <p>Acceleration = GRADIENT = <math>\frac{\Delta v}{\Delta t}</math></p> <p><math>1.6 = \frac{v - 16}{5}</math></p> <p><math>\Rightarrow v = 17 - 16</math></p> <p><math>\Rightarrow v = 26</math></p>	<p><u>LOOKING AT THE FIRST BEAM</u></p> <p><math>\int_0^5 (16 + 1.6t) dt = 820</math></p> <p><math>16 \times 5 + 24t = 820</math></p> <p><math>100 + 24t = 820</math></p> <p><math>24t = 720</math></p> <p><math>t = 30</math></p> <p><u>LOOKING AT THE LAST SECTION</u></p> <p>Deceleration = GRADIENT = <math>\frac{\Delta v}{\Delta t}</math></p> <p><math>-4 = \frac{0 - 26}{t_2}</math></p> <p><math>t_2 = 6.5</math></p> <p><math>\therefore</math> TOTAL TIME = <math>5 + t_1 + t_2</math></p> <p><math>= 5 + 30 + 6.5</math></p> <p><math>= 41.5</math></p>	<p><u>FINISH THE DISTANCE COVERED IN THE LAST SECTION</u></p> <p><math>26 \times 6.5 = \frac{1}{2} \times 6 \times 26 = 78 \text{ m}</math></p> <p><math>\therefore</math> TOTAL DISTANCE IS <math>820 + 78 = 898 \text{ m}</math></p>
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Question 10 (\*\*\*)



The figure above shows the speed time graph  $(t, v)$  of a train travelling along a straight horizontal track between two stations which are 6.3 km apart.

The train starts from rest at the first station and accelerates uniformly for 360 m, reaching a speed of  $36 \text{ ms}^{-1}$ . This speed is maintained for  $T$  s, before the train decelerates uniformly at  $1.2 \text{ ms}^{-2}$ , coming to rest as it reaches the second station.

- a) Calculate the acceleration of the train.
- b) Determine the value of  $T$ .

A high speed train also completes the same journey at exactly the same total time as the first train.

Starting from the first station, it accelerates uniformly to a top speed of  $u \text{ ms}^{-1}$  and on reaching this speed it immediately decelerates uniformly coming to rest as it reaches the second station.

- c) Sketch on the above diagram a speed time graph for journey of the second train and use it to determine the value of  $u$ .

,  $|a| = 1.8 \text{ ms}^{-2}$  ,  $T = 150$  ,  $u = 63 \text{ ms}^{-1}$

a) ACCELERATION = GRADIENT & DISTANCE = AREA  
 $\frac{1}{2} \times 36 \times 20 = 360$   
 $18a = 360$   
 $a = 20$   
 $a = 1.8 \text{ ms}^{-2}$

b) DECELERATION = GRADIENT  
 $a = \frac{\Delta v}{\Delta t} \Rightarrow -1.2 = \frac{-36}{t}$   
 $\Rightarrow t = 30$

Now THE TOTAL AREA IS 6300 m  
 $\frac{1}{2} \times (20 + 30) \times 36 = 6300$

c) SKETCHING THE SPEED TIME GRAPH  
 $\frac{1}{2} \times (20 + 30 \times T) \times 36 = 6300$   
 $\Rightarrow 18(20 + 30T) = 6300$   
 $\Rightarrow 36T + 900 = 6300$   
 $\Rightarrow 36T = 5400$   
 $\Rightarrow T = 150$

Now THE TOTAL AREA IS 6300 m  
 $\frac{1}{2} \times 200 \times u = 6300$   
 $100u = 6300$   
 $u = 63 \text{ ms}^{-1}$

**Question 11** (\*\*\*)

A car is observed travelling along a straight horizontal road between two points on this road, A and B, where  $AB = 1362$  m.

At time  $t = 0$  s the car goes past A with speed  $30 \text{ ms}^{-1}$ .

The car maintains this speed for 17 s.

It then decelerates uniformly to a speed of  $12 \text{ ms}^{-1}$ .

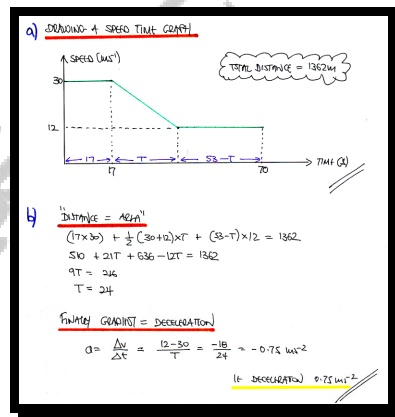
The car finally maintains the speed of  $12 \text{ ms}^{-1}$  until it goes past B.

- a) Sketch a speed time graph to show the motion of the car from A to B.

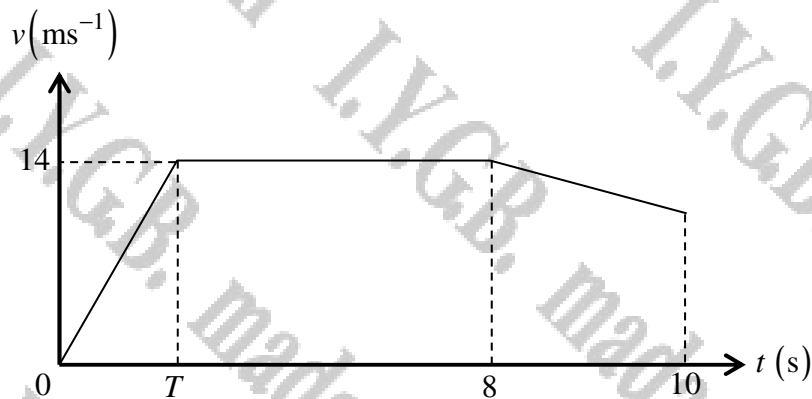
The car took 70 s to travel from A to B.

- b) Calculate the deceleration of the car during the motion described above.

,  $|a| = 0.75 \text{ ms}^{-2}$



## Question 12 (\*\*\*)



The figure above shows the velocity time graph  $(t, v)$  of a sprinter running a 100 m race in a straight horizontal track.

The sprinter starts from rest and accelerates uniformly until he reaches a speed of  $14 \text{ ms}^{-1}$ , when  $t = T \text{ s}$ . He then maintains his speed until  $t = 8 \text{ s}$ . The sprinter then decelerates uniformly at  $0.5 \text{ ms}^{-2}$  until he finishes the race when  $t = 10$ .

- Show by calculation that  $T = 5\frac{4}{7}$ .
- Determine the acceleration of the sprinter at the start of the race.

$$\boxed{a \approx 2.51 \text{ ms}^{-2}}$$

Looking at the decelerating part

$$u = 14 \quad v = u + at \Rightarrow v = 14 + (-0.5)t$$

$$a = -0.5 \quad v = 0$$

$$t = ?$$

$$v = ?$$

Now area is 100

$$\left(\frac{1}{2} \times T \times 14\right) + (8 - T) \times 14 + \frac{1}{2} (14 + 0) \times 2 = 100$$

$$7T + 112 - 14T + 21 = 100$$

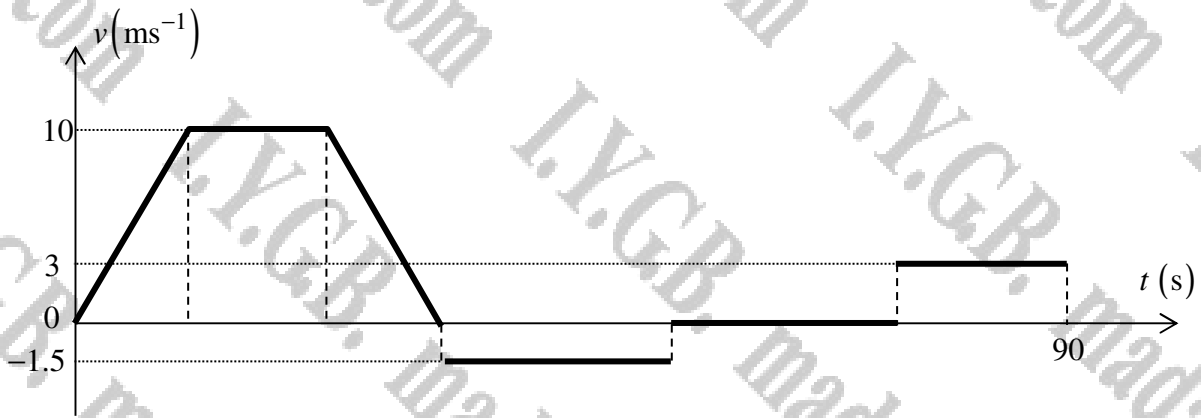
$$31 = 7T$$

$$T = \frac{31}{7} = 5\frac{4}{7}$$

b) Acceleration = gradient

$$a = \frac{\Delta v}{\Delta t} = \frac{14}{\frac{31}{7}} = \frac{98}{31} = 2.51 \text{ ms}^{-2}$$

Question 13 (\*\*\*)



The figure above shows the velocity time graph  $(t, v)$  of Andy riding a bike.

Andy starts cycling from rest with constant acceleration of  $1.25 \text{ ms}^{-2}$  until he reaches a speed  $10 \text{ ms}^{-1}$  which he maintains for 18 s.

Andy hears his friend Maya calling him so he decelerates to rest in 4 s.

He leaves his bike where he stopped and walks back at constant speed of  $1.5 \text{ ms}^{-1}$  to meet Maya, reaching her after a further period of 10 s.

The two children talk to each other for  $T$  s. Andy then runs back to his bike at constant speed of  $3 \text{ ms}^{-1}$ . It takes a total time of 90 s since Andy started his ride.

- Find the time for which Andy is accelerating.
- Find the value of Andy's deceleration.
- Determine the value of  $T$ .

,  $t = 8 \text{ s}$  , deceleration =  $2.5 \text{ ms}^{-2}$  ,  $T = 45 \text{ s}$

a) Acceleration = Gradient  
 $1.25 = \frac{10}{x}$   
 $x = 8$   
 8 seconds

b) Deceleration = Gradient  
 $a = \frac{-10}{4}$   
 $a = 2.5 \text{ ms}^{-2}$

c) The children talk to each other for  $T$  s. Andy then runs back to his bike at constant speed of  $3 \text{ ms}^{-1}$ . It takes a total time of 90 s since Andy started his ride.

The children talk to each other for  $T$  s. Andy then runs back to his bike at constant speed of  $3 \text{ ms}^{-1}$ . It takes a total time of 90 s since Andy started his ride.

2 + 18 + 4 + 10 + T + 4 = 90  
 38 + T = 90  
 $T = 52$

**Question 14** (\*\*\*)

A car is observed travelling along a straight horizontal road between two points on this road,  $A$  and  $B$ , where  $AB = 2125$  m.

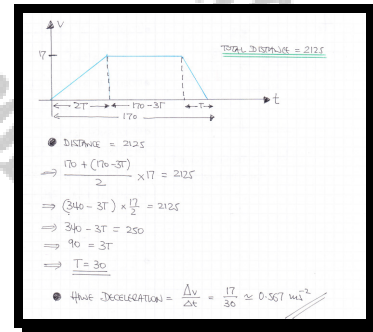
The car starts from rest at  $A$  and accelerates uniformly to a speed of  $17 \text{ ms}^{-1}$ .

It maintains this speed until it decelerates uniformly coming to rest at  $B$ .

The car took  $170$  s to travel from  $A$  to  $B$ .

Calculate the deceleration of the car during the motion described above, given that the magnitude of the deceleration is twice the magnitude of its acceleration.

$$|a| \approx 0.567 \text{ ms}^{-2}$$





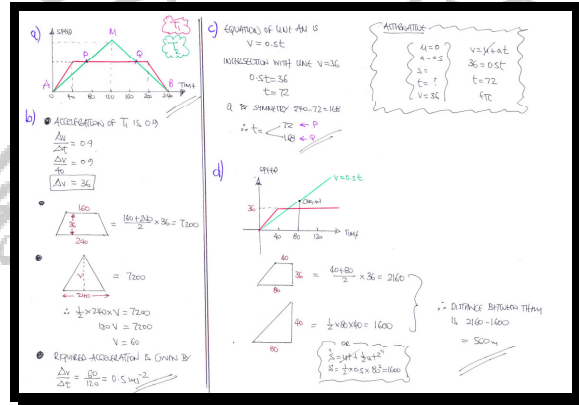
**Question 15** (\*\*\*\*)

Two trains,  $T_1$  and  $T_2$ , start together from rest, at time  $t=0$ , at a station  $A$  and move along parallel straight horizontal tracks.

Both trains come to rest, at the next station  $B$ , after 240 s.

- $T_1$  moves with constant acceleration  $0.9 \text{ ms}^{-2}$  for 40 s, then moves at constant speed for 160 s, and then moves with constant deceleration for the last 40 s.
  - $T_2$  moves with constant acceleration for 120 s, and then moves with constant deceleration for the last 120 s.
- a) Sketch, on the same axes, the speed–time graphs for the motion of the two trains between the two stations.
  - b) Find the acceleration of  $T_2$  for the first 120 s of its journey.
  - c) Determine the times when  $T_1$  and  $T_2$  are moving with the same speed.
  - d) Calculate the distance between  $T_1$  and  $T_2$ , 80 s after they start.

$a = 0.5 \text{ ms}^{-2}$ ,  $t = 72 \text{ s}$  or  $t = 168 \text{ s}$ ,  $d = 560 \text{ m}$



**Question 16** (\*\*\*\*)

A car and a motorbike are moving on a straight horizontal road.

A car starts from rest at some point  $A$  and accelerates uniformly at  $1.5 \text{ ms}^{-2}$  for  $20 \text{ s}$ .

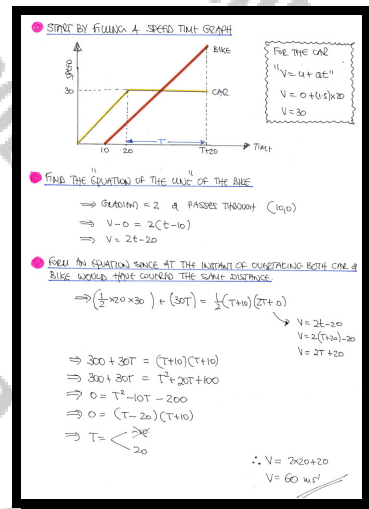
The car then continues at constant speed.

A motorbike also starts from rest from point  $A$ ,  $10 \text{ s}$  **after** the car left  $A$ .

The motorbike accelerates uniformly at  $2 \text{ ms}^{-2}$  overtaking the car with speed  $V$ .

Determine the value of  $V$ .

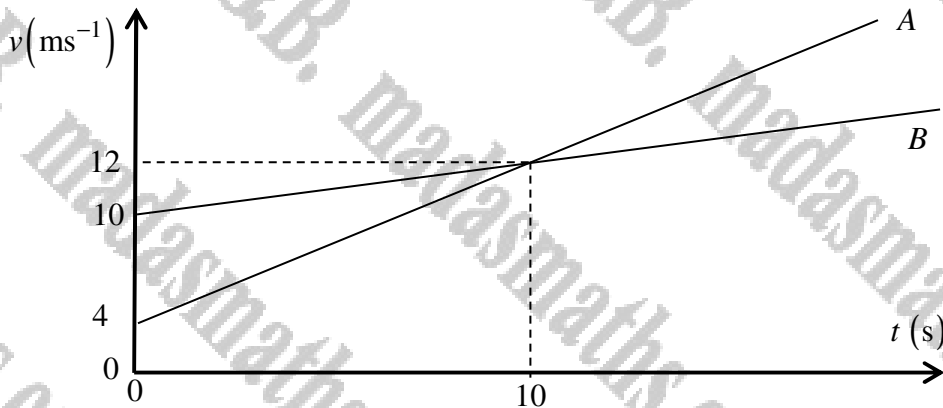
,  $V = 60 \text{ ms}^{-1}$



**Question 17** (\*\*\*\*)

Two trains, A and B, each of length 130 m, are moving on adjacent straight horizontal tracks which are parallel to each other.

At time  $t=0$ , the front ends of both trains pass a signal. The subsequent motion of each of the trains is shown in the speed time graph  $(t, v)$ , below.



Find the value of  $t$  when the front end of B is 110 m behind the back end of A.

$T = 40$  s

- Gradient A =  $\frac{\Delta v}{\Delta t} = \frac{12-4}{10} = 0.8$  ← Acceleration of A
- Gradient B =  $\frac{\Delta v}{\Delta t} = \frac{12-10}{10} = 0.2$  ← Acceleration of B
- Equation of A:  $v = 4 + 0.8t$
- Equation of B:  $v = 10 + 0.2t$
- Next suppose that at  $t=T$  the front end of B is 110m behind the back end of A

Distance covered by A is given by  $\frac{4 + (4 + 0.8T)}{2} \times T = \frac{(8 + 0.8T)T}{2} = 4T + 0.4T^2$

Distance covered by B is given by  $\frac{10 + (10 + 0.2T)}{2} \times T = \frac{(20 + 0.2T)T}{2} = 10T + 0.1T^2$

Finally we need to allow for the length of the trains

$$\Rightarrow (10T + 0.1T^2) + 110 + 130 = 4T + 0.4T^2$$

$$\Rightarrow 10T + 0.1T^2 + 240 = 4T + 0.4T^2$$

$$\Rightarrow 0 = 0.3T^2 - 6T - 240$$

$$\Rightarrow 3T^2 - 60T - 2400 = 0$$

$$\Rightarrow T^2 - 20T - 800 = 0$$

$$\Rightarrow (T - 40)(T + 20) = 0$$

$$\Rightarrow T = \frac{-40}{40}$$

**Question 18** (\*\*\*\*)

Adam and Ben decide to travel from their village to a nearby village by combining walking and cycling.

They both start from the same point in their village at 09:00, and both follow the same route to the other village.

Adam starts the journey by cycling at constant speed of  $16 \text{ km h}^{-1}$  and Ben by walking at constant speed of  $4 \text{ km h}^{-1}$ .

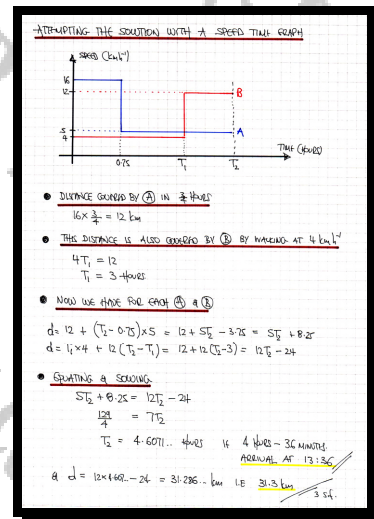
After cycling for 4 of an hour, Adam leaves the bike at the side of the road and continues his journey by walking at constant speed of  $5 \text{ km h}^{-1}$ .

When Ben finds the bike at the point where Adam left it he begins to cycle at constant speed of  $4 \text{ km h}^{-1}$ .

They both arrive at the nearby village at exactly the same time.

By using a speed time graph, or otherwise, determine their arrival time and calculate the distance between the two villages.

,  ,



**Question 19** (\*\*\*\*)

A car and a motorbike are moving on a straight horizontal road.

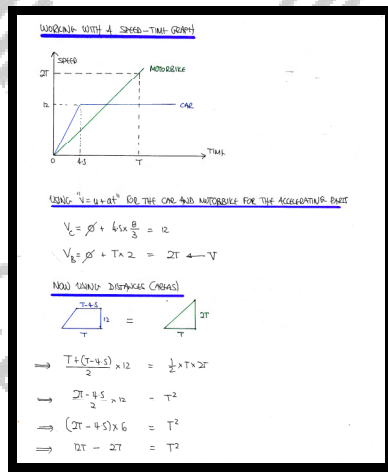
At time  $t = 0$ , the car starts from rest at some point  $A$  and accelerates uniformly at  $2\frac{2}{3} \text{ ms}^{-2}$  for  $4\frac{1}{2} \text{ s}$ .

The car then continues at this constant speed.

At time  $t = 0$ , the motorbike also starts from rest from  $A$ , and accelerates uniformly at  $2 \text{ ms}^{-2}$  overtaking the car with speed  $V$  at some point  $B$ .

Determine the value of  $V$  and the distance  $AB$ .

,  $V = 18 \text{ ms}^{-1}$  ,  $|AB| = 81 \text{ m}$



$$\Rightarrow T^2 - 2T + 27 = 0$$

$$\Rightarrow (T - 9)(T - 3) = 0$$

$$\Rightarrow T = \begin{cases} 9 \\ 3 \end{cases} \quad T > 4.5$$

Finally we have

$$V = 2T = 18 \text{ ms}^{-1}$$

$$|AB| = T^2 = 81 \text{ m}$$

**Question 20** (\*\*\*\*)

A car and a motorbike are moving on a straight horizontal road.

At time  $t = 0$ , the car starts from rest at some point  $A$  and accelerates uniformly for 8 s up to a speed of  $12 \text{ ms}^{-1}$ .

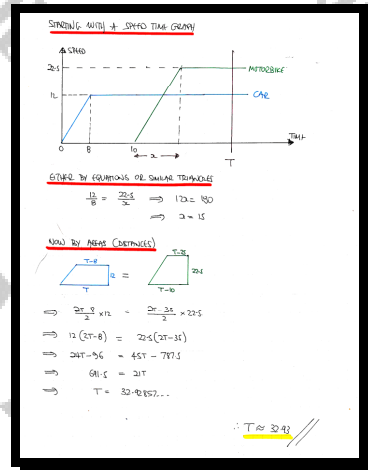
The car then continues at this constant speed.

At time  $t = 10$ , the motorbike also starts from rest from  $A$ , and accelerates uniformly with the same acceleration as the car, reaching a top speed of  $22.5 \text{ ms}^{-1}$ .

The motorbike overtakes the car at time  $t = T$ .

Determine the value of  $T$ , correct to two decimal places.

$$\square, T = \frac{461}{14} \approx 32.93$$



**Question 21** (\*\*\*\*+)

A train is moving on a straight horizontal rail tracks, and is modelled as a particle, by considering the displacement of the front of the train.

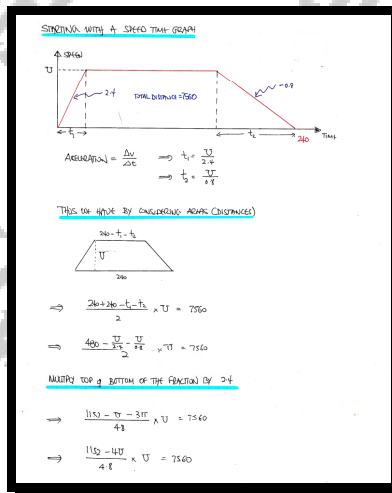
At time  $t = 0$ , the train starts from rest at some station  $A$  and accelerates uniformly at  $2.4 \text{ ms}^{-2}$  reaching a top speed of  $U \text{ ms}^{-1}$ .

The train then continues at this constant speed, until it begins to decelerate uniformly at  $0.8 \text{ ms}^{-2}$ , coming to rest at some point  $B$ .

The train completes the journey  $AB$  in 4 minutes.

Given further that the distance  $AB$  is 7.56 km, determine the value of  $U$ .

$\square$ ,  $\boxed{U = 36}$



$\Rightarrow \frac{1152U - 4U^2}{4} = 7560$   
 $\Rightarrow 1152U - 4U^2 = 36288$   
 $\Rightarrow 288U - U^2 = 9072$   
 $\Rightarrow 0 = U^2 - 288U + 9072$

BY THE QUADRATIC FORMULA OR OTHERWISE

$\rightarrow (U - 36)(U - 252) = 0$   
 $\Rightarrow U = \begin{cases} 36 \\ 252 \end{cases}$

NO! if  $U = 252$  YES! if  $U = 36$   
 $240 - \frac{U}{0.8} = 180 < 0$   $240 - \frac{U}{2.4} = 180 > 0$

$\therefore U = 36$

**Question 22** (\*\*\*\*+)

The motions of two cars,  $A$  and  $B$ , is observed as they travel between two sets of traffic lights, on a straight road, which are 600 m apart.

At time  $t = 0$ ,  $A$  passes through the first set of lights, accelerating uniformly from a speed of  $4 \text{ ms}^{-1}$  to a speed  $20 \text{ ms}^{-1}$ .

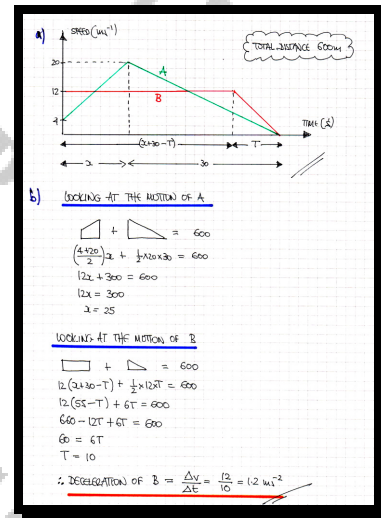
As soon as this speed is reached it begins to decelerate uniformly for 30 s, coming to rest at the next set of traffic lights.

At time  $t = 0$ ,  $B$  passes through the first set of lights, with constant speed of  $12 \text{ ms}^{-1}$ .

It maintains this speed until it begins to decelerate uniformly, coming to rest at the second set of lights at the **same time** as  $A$ .

- Sketch, on the same speed time graph the motions of  $A$  and  $B$ .
- Find the deceleration of  $B$ , just as it arrives at the second set of lights.

,  $|a| = 1.2 \text{ ms}^{-2}$





**Question 23** (\*\*\*\*+)

A cyclist is travelling along a straight horizontal road at constant speed  $12 \text{ ms}^{-1}$  as it passes past a set of traffic lights at time  $t=0$  seconds. The cyclist continues its journey at that constant speed.

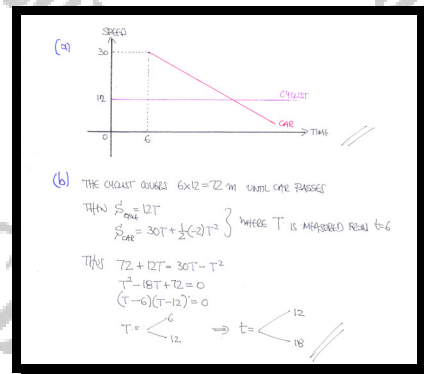
When  $t=6$  a car passes past the same set of traffic lights with speed  $30 \text{ ms}^{-1}$ , decelerating uniformly at  $2 \text{ ms}^{-2}$ .

- a) Sketch on the same speed time graph the motion of both the car and the cyclist for  $t \geq 0$ .

In the consequent motion, the car overtakes the cyclist at some point  $A$  and at a later time the cyclist overtakes the car again at some point  $B$ .

- b) Find the value of  $t$  at  $A$  and  $B$ .

$$t_A = 12, t_B = 18$$



**Question 24** (\*\*\*\*+)

Two cars, A and B, are each travelling with constant speed along the same straight horizontal road.

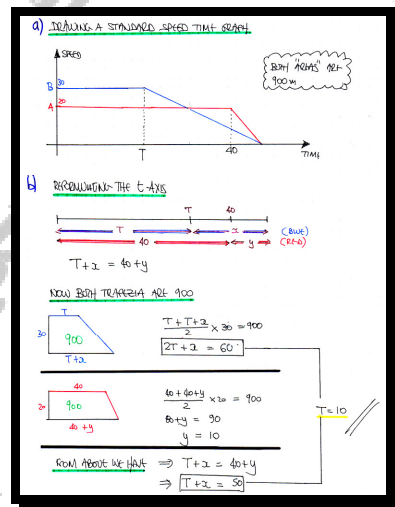
The speed of A is  $20 \text{ ms}^{-1}$  and the speed of B is  $30 \text{ ms}^{-1}$ .

At time  $t = 0 \text{ s}$ , B overtakes A.

At time  $t = T \text{ s}$ , B begins to decelerate uniformly coming to rest at the end of a traffic queue. At time  $t = 40 \text{ s}$ , A begins to decelerate uniformly coming to rest at the end of a traffic queue, next to B. Both cars come to rest at the same time, 900 m from the point where B first overtook A.

- a) Sketch, on the same set of axes, a speed time graph to show the motion of the two cars from  $t = 0$  until both come to rest.
- b) Determine the value of  $T$ , showing a fully detailed method.

,  $T = 10$



**Question 25** (\*\*\*\*+)

A car is travelling on a straight horizontal road, starting from rest from a set of traffic lights at  $A$ , accelerating uniformly to a speed  $20 \text{ ms}^{-1}$ , in  $30 \text{ s}$ .

The car maintains this speed for  $74 \text{ s}$  when it uniformly decelerates at  $1.25 \text{ ms}^{-2}$ , coming to rest at the next set of traffic lights at  $B$ .

- a) Sketch a speed time graph for the motion of the car between  $A$  and  $B$ .
- b) Find the distance from  $A$  to  $B$ .

A motorcycle leaves  $A$ ,  $3.5 \text{ s}$  after the car left  $A$ .

The motorcycle is accelerating uniformly at  $a \text{ ms}^{-2}$  overtaking the car at the point  $M$ , where  $M$  is the midpoint of  $AB$ .

- c) Determine the value of  $a$ , correct to three significant figures.

,  $|AB| = 1940 \text{ m}$  ,  $a \approx 0.539$

**a) SKETCHING THE SPEED TIME GRAPH FROM THE INFO GIVEN**

Speed vs Time graph showing acceleration to  $20 \text{ ms}^{-1}$  in  $30 \text{ s}$ , constant speed for  $74 \text{ s}$ , and deceleration to  $0$  in  $12.8 \text{ s}$ .

**b) DECELERATION = GRADIENT**

$$a = \frac{\Delta v}{\Delta t} \Rightarrow 1.25 = \frac{20}{\Delta t} \Rightarrow \Delta t = 16$$

$\therefore$  TOTAL TIME IS  $120$

**DISTANCE = AREA**

$$\frac{1}{2}(20+0) \times 30 + 20 \times 74 + \frac{1}{2}(20+0) \times 12.8 = 1940 \text{ m}$$

**c) LOOKING AT THE JUNCTION**

BOTH AXES ARE  $1940 = 970$

$$\frac{1}{2} \times 20 \times T = 970 \Rightarrow T = 97$$

$$30 + 20T = 970 \Rightarrow 20T = 670 \Rightarrow T = 33.5$$

BY INSPECTION  $a = 60$   
( $30 + T = 33.5 + 2$ )

**FINDING THE ACCELERATION OF  $a$  FOUND**

Velocity vs Time graph for motorcycle showing  $v = at$  and  $v = 20$  at  $t = 33.5$ .

$$\frac{1}{2} \times 60 \times v = 970 \Rightarrow 30v = 970 \Rightarrow v = \frac{97}{3}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{97/3}{60} = \frac{97}{180} \approx 0.539 \text{ ms}^{-2}$$

**Question 26** (\*\*\*\*+)

Two 100 m relay sprinters are about to exchange the baton.

At time  $t = 0$ , sprinter A is running at constant speed of  $12 \text{ ms}^{-1}$  when he enters the start line of the baton exchange area of his lane.

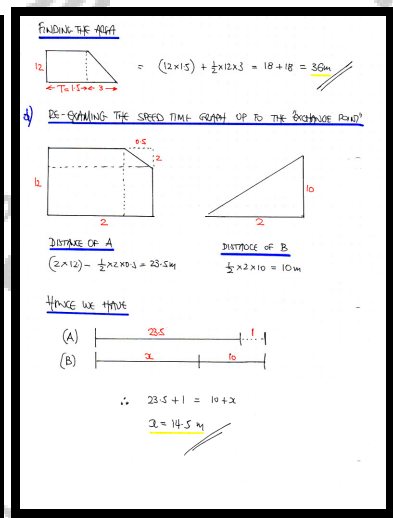
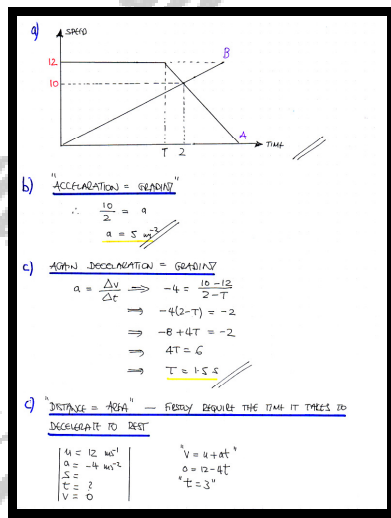
At time  $t = 0$ , sprinter B starts running from rest with constant acceleration  $a \text{ ms}^{-2}$ , from until he reaches a speed of  $12 \text{ ms}^{-1}$ .

When  $t = T$  sprinter A begins to decelerate at  $4 \text{ ms}^{-2}$ , until eventually comes to rest.

When  $t = 2$  the baton is exchanged, when both sprinters have a speed of  $10 \text{ ms}^{-1}$  in the same direction and B is 1 m ahead of A.

- Draw a speed time graph ( $v, t$ ) for  $t \geq 0$ .
- Find the value of  $a$ .
- Determine the value of  $T$ .
- Calculate the total distance A covers from  $t = 0$  until he comes to rest.
- Find the distance between A and B at the instant B begins to run.

,  $a = 5 \text{ ms}^{-2}$  ,  $T = 1.5$  ,  $d = 36 \text{ m}$  ,  $|AB|_0 = 14.5 \text{ m}$



**Question 27** (\*\*\*)

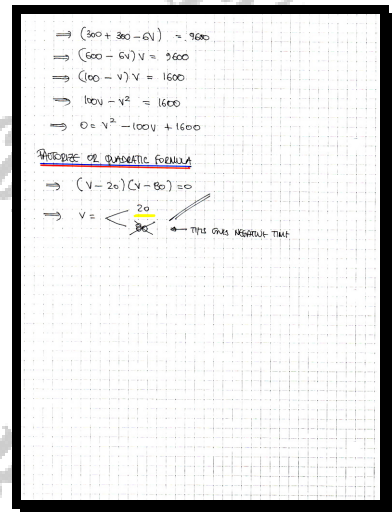
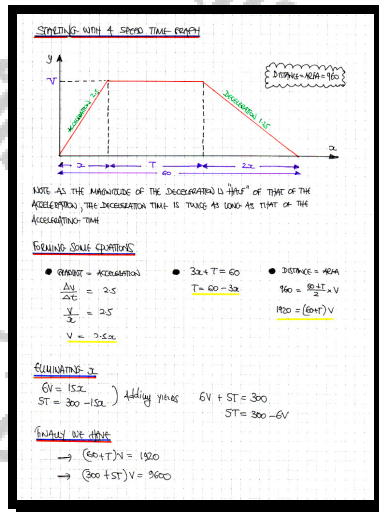
A car travels along a straight horizontal road between two points, A and B, that are 960 m apart.

The car starts from rest at A and moves with constant acceleration of  $2.5 \text{ ms}^{-2}$  until it reaches a speed  $V \text{ ms}^{-1}$ . It then travels at this constant speed before it decelerates uniformly at  $1.25 \text{ ms}^{-2}$ , coming to rest at B.

The car takes 60 seconds for the journey from A and B.

Determine the value of  $V$ .

$V = \boxed{20}$  ,  $V = \boxed{20}$



**Question 28** (\*\*\*\*+)

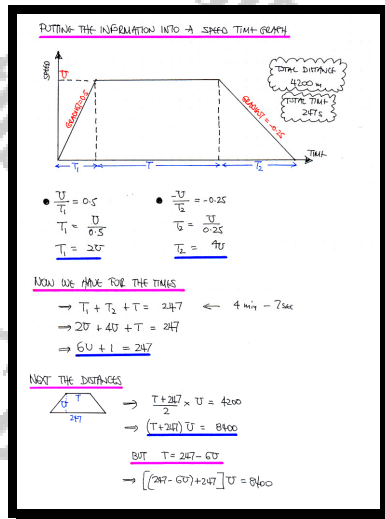
A train travels along a straight horizontal track between two underground stations, A and B, that are 4.2 km apart.

The train starts from rest at A and moves with constant acceleration  $0.5 \text{ ms}^{-2}$  until it reaches a speed  $U \text{ ms}^{-1}$ . The train then travels at this constant speed before it decelerates uniformly at  $0.25 \text{ ms}^{-2}$ , coming to rest at B.

The train takes 4 minutes and 7 seconds for the journey from A and B.

Determine the value of  $U$ .

$U = 24$



$\rightarrow (494 - 6U)U = 8400$   
 $\rightarrow 494U - 6U^2 = 8400$   
 $\rightarrow 0 = 6U^2 - 494U + 8400$   
 $\rightarrow 3U^2 - 247U + 4200 = 0$

BY THE QUADRATIC FORMULA

$U = \frac{247 \pm \sqrt{(-247)^2 - 4 \times 3 \times 4200}}{2 \times 3}$   
 $\rightarrow U = \frac{247 \pm \sqrt{10609}}{6}$   
 $\rightarrow U = \frac{178}{3}$

CHECKING AGAINST TIME,  $T = 247 - 6U$

- IF  $U = \frac{178}{3}$ ,  $T = -103$
- IF  $U = 24$ ,  $T = 103$

$\therefore U = 24$

**Question 29** (\*\*\*\*+)

A motorcycle is travelling along a straight horizontal road.

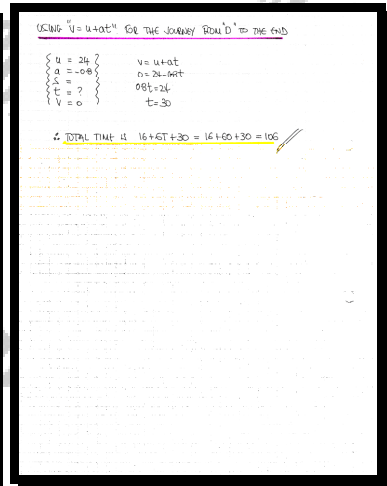
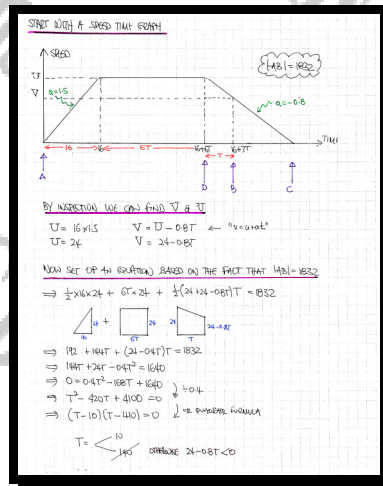
The motorcycle starts from rest at the point  $A$  and accelerates uniformly at  $1.5 \text{ ms}^{-2}$  for  $16 \text{ s}$ , reaching a speed which it then maintains for  $6T \text{ s}$ .

It then decelerates uniformly at  $0.8 \text{ ms}^{-2}$  for  $T \text{ s}$ , as it passes through the point  $B$ .

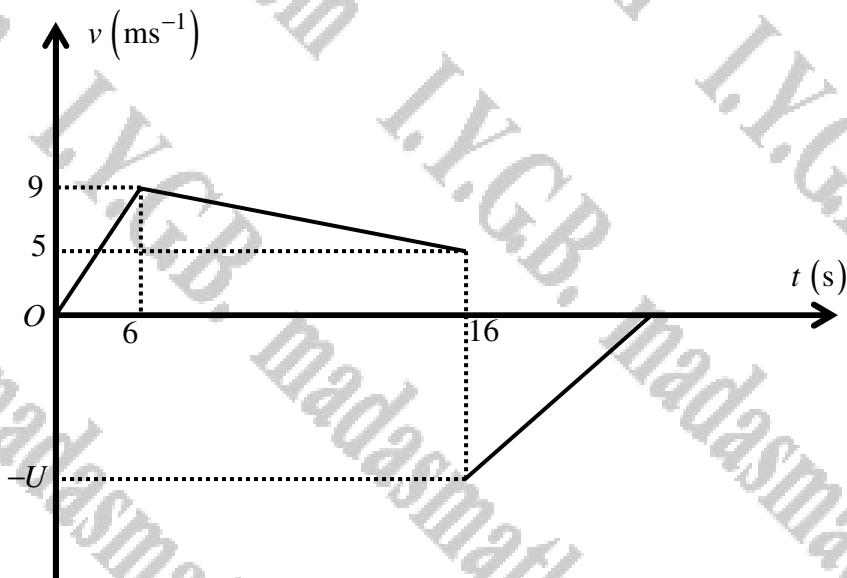
As the motorcycle passes through  $B$  it continues to decelerate at  $0.8 \text{ ms}^{-2}$ , coming to rest at the point  $C$ .

Determine the total time for the entire journey from  $A$  to  $C$ , given further that the distance  $AB$  is  $1832 \text{ m}$ .

$t = 106 \text{ s}$



## Question 30 (\*\*\*\*\*)



A particle of mass 2 kg is released from rest from a point  $A$  on an incline plane and begins to move down a line of greatest slope of the plane.

The plane has a different coefficient of friction at different sections so the resistance to the motion of the particle has different values at different sections of the plane, as the particle slides down.

The particle accelerates uniformly to a speed of  $9 \text{ ms}^{-1}$  in 6 s as it reaches point  $B$ .

The coefficient of friction increases at  $B$  so the particle continues to slide down with constant deceleration for 10 s achieving a speed of  $5 \text{ ms}^{-1}$  as it reaches point  $C$ .

At  $C$  the particle is instantaneously projected with speed  $U \text{ ms}^{-1}$ , **up** a line of greatest slope of the plane, coming to rest at  $B$ .

If the **normal** reaction between the plane and the particle has a magnitude of 15.68 N, determine the value of  $U$ , correct to 2 decimal places.

$$\boxed{\phantom{0000}}, \quad U = \sqrt{1702.4} \approx 41.26$$

[solution overleaf]



IN THIS QUESTION IT TALKS A BIT ABOUT THE DIFFERENTIATION OF VELOCITY  
SO IN A FEW MOMENTS IT IS TRYING TO GIVE YOU THE DISTANCE (NOT SPEED)

LOOKING AT THE JOURNEY A to B, i.e. THE ACCELERATING SECTION

$15.6B = 210000$   
 $15.6B = 15.6(1000)$   
 $(1000) = 0.9$   
 ∴ CONSEQUENTLY  $0.9B = 0.6$

LOOKING AT THE DECELERATING SECTION FROM B to C

$a = \frac{\Delta v}{\Delta t} = \frac{0 - 15.6}{10} = -1.56$   
 $\Rightarrow F = ma$   
 $\Rightarrow 2g \sin \theta - R \cos \theta = 2(-1.56)$   
 $\Rightarrow 2(9.81 \cos \theta) - R \cos \theta = -3.12$   
 $\Rightarrow R \cos \theta = 19.56$

NEXT LOOKING AT THE SPEED TIME GRAPH

$\therefore \text{DISTANCE B to C is } 70$

LOOKING AT THE JOURNEY BACK UP FROM C to B

$\Rightarrow F = ma$   
 $\Rightarrow -15.6 - 2g \sin \theta = 2a'$   
 $\Rightarrow -15.6 - 2(9.81) = 2a'$   
 $\Rightarrow -24.32 = 2a'$   
 $\Rightarrow a' = -12.16 \text{ m/s}^2$

FINALLY DETERMINES THE JOURNEY B to C

$u = ?$   
 $a = -12.16$   
 $t = 70$   
 $v = 0$

$v^2 = u^2 + 2as$   
 $0 = u^2 + 2(-12.16) \times 70$   
 $u^2 = 1702.4$   
 $u = 41.26 \text{ m/s}$

i.e.  $U = 41.26$

**Question 31** (\*\*\*\*\*)

Two cars, A and B, are travelling on a straight horizontal road.

At time  $t=0$ , A drives past a fixed point  $O$  on the road with speed  $4.95 \text{ ms}^{-1}$ , accelerating uniformly at  $2 \text{ ms}^{-2}$ .

After passing through  $O$ , A continues accelerating for 4 s, reaching a speed which it maintains thereafter.

At time  $t=2$ , B starts from rest from  $O$ , accelerating uniformly at  $1.5 \text{ ms}^{-2}$ , until it reaches the point  $P$ , where it draws level with A.

Determine the distance  $OP$ .

,  $d = 243 \text{ m}$

**ATTEMPTING A SOLUTION BY A SPEED-TIME GRAPH**

Graph showing Speed (v) vs Time (t). Car A starts at  $t=0$  with speed  $4.95$  and accelerates to  $u$  at  $t=4$ , then remains constant. Car B starts at  $t=2$  from rest and accelerates to meet A at  $t=T$ .

LET THE MAXIMUM SPEEDS OF A & B BE  $u$  &  $v$  RESPECTIVELY

THUS FOR A " $v = u + at$ " AND FOR B " $v = u + at$ "  
 $\Rightarrow u = 4.95 + 2 \times 4$        $\Rightarrow v = 0 + 1.5(T-2)$   
 $\Rightarrow u = 12.95$        $\Rightarrow v = \frac{3}{2}(T-2)$

WHEN B DRAWS LEVEL THE DISTANCES WOULD BE IDENTICAL

$\Rightarrow 4 \times \frac{4.95 + u}{2} + u(T-4) = \frac{1}{2} \times \frac{3}{2}(T-2)^2$   
 $\Rightarrow \frac{20.8 + 4.95}{2} \times 4 + 12.95(T-4) = \frac{3}{4} \times \frac{3}{2}(T-2)^2$   
 $\Rightarrow 30.8 + 12.95(T-4) = \frac{9}{8}(T-2)^2$   
 $\Rightarrow 74 + 29(T-4) = \frac{9}{8}(T-2)^2$   
 $\Rightarrow 74 + 29T - 116 = 1.125(T^2 - 4T + 4)$   $\times 20$

$\Rightarrow -320 + 29T = 1.125T^2 - 4.5T + 4.5$   
 $\Rightarrow 0 = 1.125T^2 - 31.5T + 320$

BY THE QUADRATIC FORMULA

$T = \frac{31.5 \pm \sqrt{(31.5)^2 - 4 \times 1.125 \times 320}}{2 \times 1.125} = \frac{31.5 \pm 281}{30}$

$T = \frac{31.5}{30} \approx 1.05$  (NOT  $T > 4$ )

THUS THE REQUIRED VALUE IS  $\frac{3}{2}(T-2)^2$  WITH  $T=20$

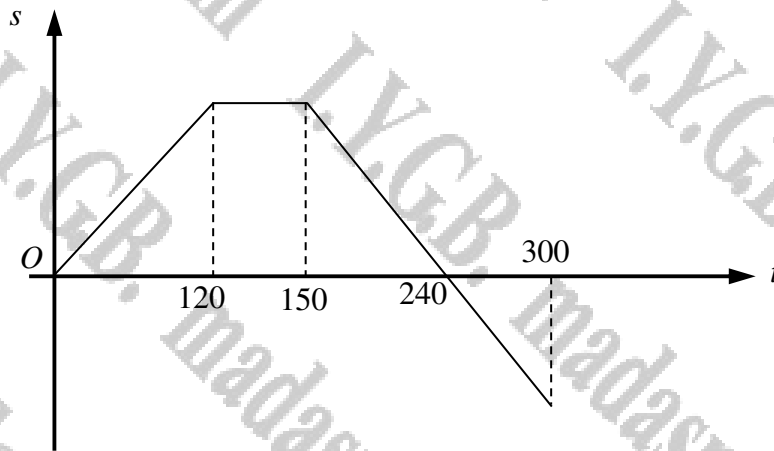
$\therefore$  DISTANCE COVERED =  $\frac{3}{2} \times 16^2 = 243 \text{ m}$

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# OTHER GRAPHS

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Question 1 (\*\*\*)



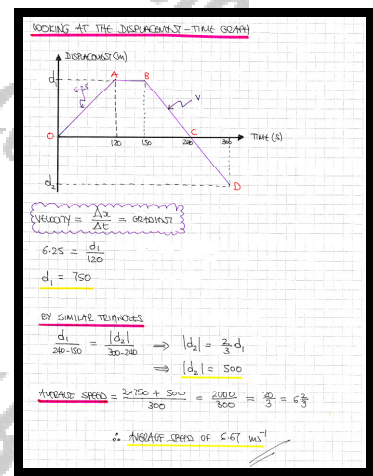
The figure above shows the graph of the **displacement**,  $s$  m, plotted against time,  $t$  s, of a particle travelling in a straight path. The graph consists of straight lines only.

The particle passes through a fixed point  $O$  with constant speed of  $6.25 \text{ ms}^{-1}$  and maintains this speed for 120 s.

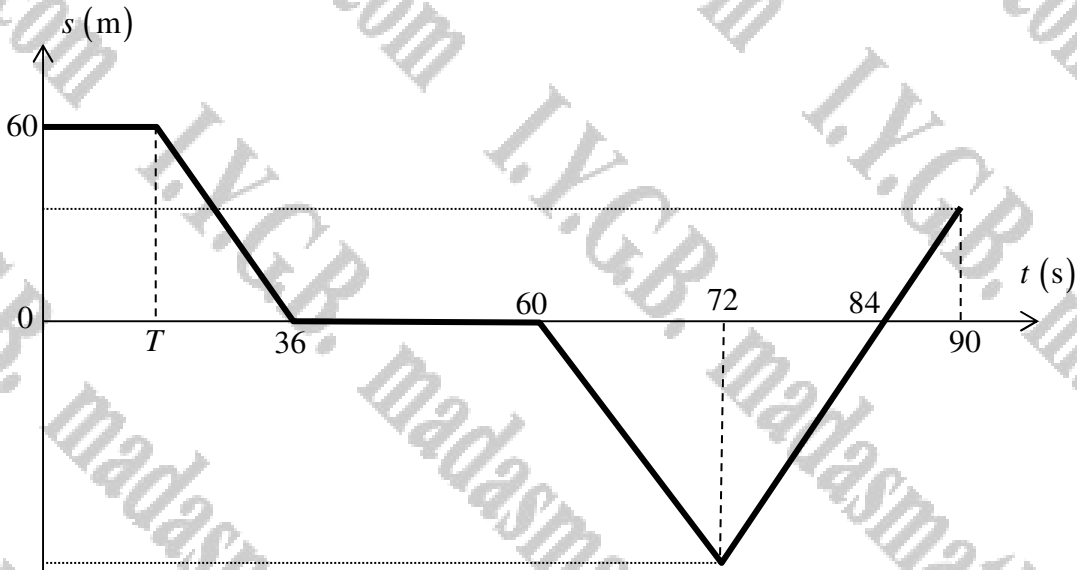
The particle is then at rest for a period of 30 s before resuming its motion for a further period of 150 s.

Given that the displacement of the particle is zero when  $t = 240$ , determine the **average speed** of the particle.

$$\boxed{6.67 \text{ ms}^{-1}}, \quad \frac{20}{3} \approx 6.67 \text{ ms}^{-1}$$



Question 2 (\*\*\*)



The figure above shows the displacement time graph  $(t, s)$  of a particle moving in a straight line. The displacement of the particle is measured relative to a fixed origin  $O$ .

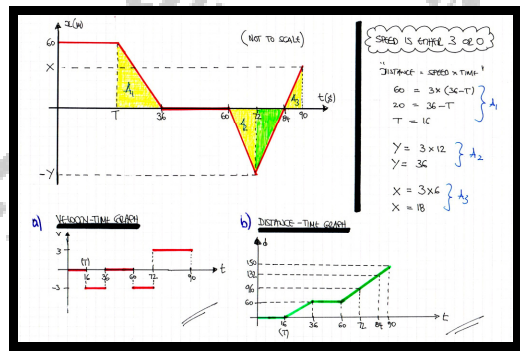
The particle, for  $0 \leq t \leq 90$ , is either at rest or has speed  $3 \text{ ms}^{-1}$ .

For  $0 \leq t \leq 90$ , draw a detailed ...

- a) ... velocity time graph  $(v, t)$ , showing clearly the velocity values at  $t = 0, T, 36, 60, 72, 84$  and  $90$ .
- b) ... distance time graph  $(d, t)$ , showing clearly the total distance covered by the particle up and including  $t = 0, T, 36, 60, 72, 84$  and  $90$ .

State the value of  $T$  explicitly in these graphs.

graph



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