ELASTIC STRINGS & SPRINGS

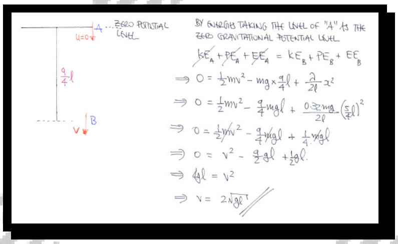
Question 1 **(**)**

A particle of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $\frac{8}{25}mg$. The other end of the string is attached to a fixed point A on a horizontal ceiling. The particle is held level at A and released from rest.

When the particle has fallen a distance $\frac{9}{4}l$ from A, its speed is v.

Find v in terms of g and l.

$$\boxed{v = 2\sqrt{gl}}$$



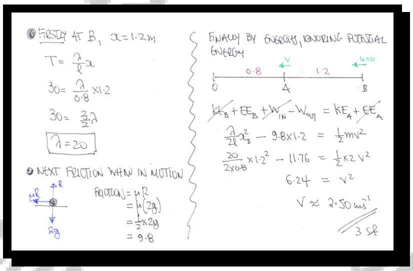
Question 2 (**)

A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.8 m. The other end of the string is attached to a fixed point O on a rough horizontal floor. The coefficient of friction between the particle and the floor is 0.5.

When the particle is held at rest at a point B on the plane, where $OB = 2$ m, the tension in the string is 30 N. The particle is then released from rest, from B.

Calculate the speed of the particle as the sting becomes slack.

$$\boxed{v \approx 2.50 \text{ ms}^{-1}}$$



FIRSTLY AT B, $x = 1.2$ m

$T = \frac{\lambda}{l}x$

$30 = \frac{\lambda}{0.8} \times 1.2$

$30 = \frac{3}{2}\lambda$

$\boxed{\lambda = 20}$

NEXT FRICTION WHEN IN MOTION

FRICTION $= \mu R$
$= \mu(2g)$
$= \frac{1}{2} \times 2g$
$= 9.8$

FINALLY BY ENERGIES, IGNORING POTENTIAL ENERGY

$\cancel{KE_B} + EE_B + \cancel{W_{IN}} - W_{OUT} = KE_A + \cancel{EE_A}$

$\frac{\lambda}{2l}x_B^2 - 9.8 \times 1.2 = \frac{1}{2}mv^2$

$\frac{20}{2 \times 0.8} \times 1.2^2 - 11.76 = \frac{1}{2} \times 2v^2$

$6.24 = v^2$

$v \approx 2.50 \text{ ms}^{-1}$ 3 sf

Question 3 (+)**

Two identical elastic strings AB and BC are fastened together at B. The natural length and modulus of elasticity of each of the strings are a and $2mg$, respectively.

The end A of the composite string is attached to a ceiling and the end C is attached to a floor, so that ABC lies in a vertical line where $|AC| = ka$.

Finally a particle of mass m is attached to B so that when the particle is in equilibrium $|BC| = \frac{7}{4}a$.

Determine the value of k.

$\boxed{k=4}$

STARTING WITH A DIAGRAM

$T_1 = T_2 + mg$

BY HOOKE'S LAW

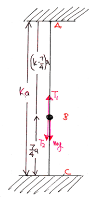
$\Rightarrow \frac{(2mg)\left(k-\frac{7}{4}\right)a}{a} = \frac{(2mg)\left(\frac{7}{4}a\right)}{a} + mg$

$\Rightarrow 2\left(k-\frac{7}{4}\right) = 2 \times \frac{7}{4} + 1$

$\Rightarrow 2k - \frac{7}{2} = \frac{7}{2} + 1$

$\Rightarrow 2k = 8$

$\Rightarrow k = 4$



Question 4 (+)**

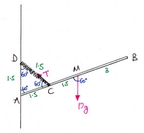
A uniform rod AB, of length 6 m and mass 12 kg is smoothly hinged at A on a vertical wall. An elastic string connects a point C on the rod to a point D on the wall which is 1.5 m vertically above A. The distance AC is 1.5 m.

The rod lies undisturbed in equilibrium so that $\measuredangle DAB = 60°$.

Given further that the natural length of the string is 1 m, determine the modulus of elasticity of the string.

$\boxed{\text{FM1}}$, $\boxed{\lambda = 48g \;\approx\; 470\,\text{N}}$

LOOKING AT A DIAGRAM $\hat{ACD}$ IS EQUILATERAL



IGNORING THE "REACTION FORCE(S)" AT A & TAKING MOMENTS ABOUT A

$1.5 \times T\sin 60 = 12g \sin 60 \times 3$

$1.5T = 36g$

$T = 24g$

BY HOOKE'S LAW

$\Rightarrow T = \lambda \frac{x}{a}$

$\Rightarrow 24g = \lambda\left(\frac{1.5-1}{1}\right)$

$\Rightarrow 24g = \lambda \times \frac{1}{2}$

$\Rightarrow \lambda = 48g$

$\Rightarrow \lambda = 470.4\text{ N}$ IE 470 N

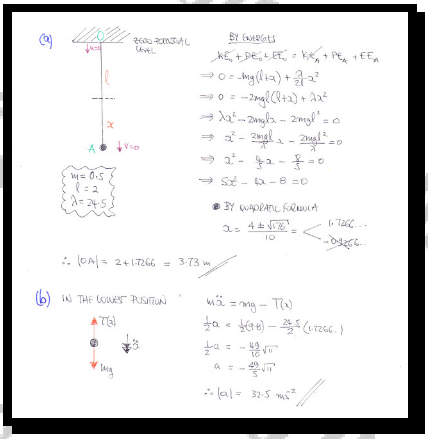
Question 5 (**+)

A particle of mass 0.5 kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point O on a horizontal ceiling. The particle is held level with O and released from rest. In the subsequent motion the particle reaches the lowest position at the point A, which lies vertically below O. It is assumed that there is no air resistance during the motion.

a) Calculate the distance OA.

b) Determine the acceleration of the particle at A.

$\boxed{|OA| \approx 3.73 \text{ m}}$, $\boxed{|a|_A = 32.5 \text{ ms}^{-2}}$



Question 6 (**+)

A particle P, of mass 2 kg rests on a fixed rough plane inclined at $\arctan\frac{4}{3}$ to the horizontal.

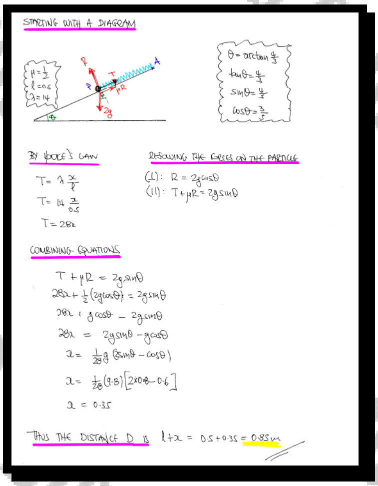
The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The point A is a fixed point at the top of the plane and lies D m above P along a line of greatest slope.

The particle is attached to A by a light elastic string of natural length 0.5 m and modulus of elasticity 14 N.

Given that P is at the point of slipping down the plane determine the value of D.

$\boxed{}$, $\boxed{D = 0.85}$


STARTING WITH A DIAGRAM

$\mu = \frac{1}{2}$, $l = 0.5$, $\lambda = 14$

$\theta = \arctan\frac{4}{3}$, $\tan\theta = \frac{4}{3}$, $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

BY HOOKE'S LAW

$T = \lambda \frac{x}{l}$

$T = 14 \frac{x}{0.5}$

$T = 28x$

RESOLVING THE FORCES ON THE PARTICLE

(I): $R = 2g\cos\theta$

(II): $T + \mu R = 2g\sin\theta$

COMBINING EQUATIONS

$T + \mu R = 2g\sin\theta$

$28x + \frac{1}{2}(2g\cos\theta) = 2g\sin\theta$

$28x + g\cos\theta = 2g\sin\theta$

$28x = 2g\sin\theta - g\cos\theta$

$x = \frac{1}{28}g(2\sin\theta - \cos\theta)$

$x = \frac{1}{28}(9.8)[2\times0.8 - 0.6]$

$x = 0.35$

THUS THE DISTANCE D IS $l + x = 0.5 + 0.35 = 0.85$ m

Question 7 (***)

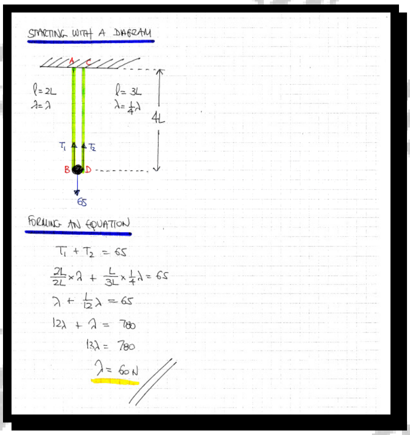
A light elastic string AB has natural length $2L$ m and modulus of elasticity λ N.

A different light elastic string CD has natural length $3L$ m and modulus of elasticity $\frac{1}{4}\lambda$ N.

The two strings are joined together at their ends, with A joined to C and with B joined to D. The "A to C" end is fixed to a horizontal ceiling. A particle of weight 65 N is attached to the "B to D" end, and hangs in equilibrium, without touching the ground.

Given that when the particle hangs in equilibrium the length of the string AB is twice is natural length, determine the value of λ.

$\boxed{\lambda = 60 \text{ N}}$



Question 8 (***)

A particle of mass 0.25 kg is attached to one end of a light elastic spring of natural length 0.8 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point O on a horizontal ceiling. The point A lies 0.3 m vertically below O.

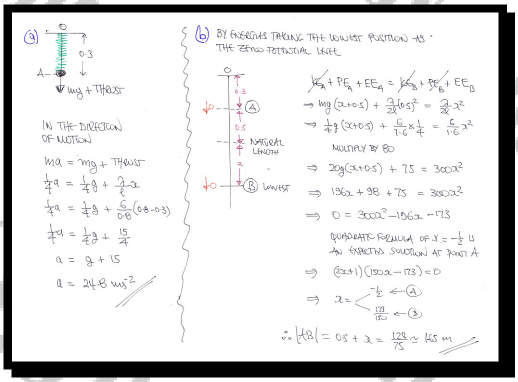
The particle is held level with A and released from rest. In the subsequent motion the particle reaches the lowest position at the point B, which lies vertically below A.

It is assumed that there is no air resistance during the motion.

a) Determine the initial acceleration of the particle as it is released from A.

b) Calculate the distance AB.

$\boxed{|a|_A = 24.8 \text{ ms}^{-2}}$, $\boxed{|AB| = \frac{124}{75} \approx 1.65 \text{ m}}$,

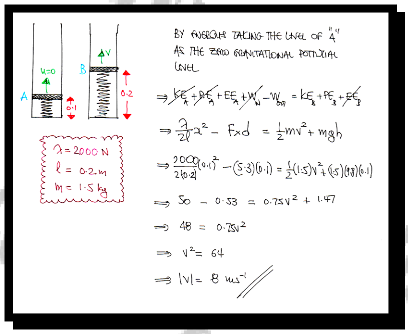


Question 9 (***)

A piston of mass 1.5 kg, enclosed in a fixed vertical tube, has one of its ends attached to a light spring of natural length 0.2 m and modulus of elasticity 2000 N. The other end of the spring is attached to the bottom of the vertical tube, so that the piston can oscillate inside the tube in a vertical direction. The motion of the piston inside the cylinder is subject to a constant resistance of 5.3 N.

The piston is pushed downwards so that the spring has length 0.1 m and released from rest. By modelling the piston as a particle determine the speed of the piston when the spring reaches its natural length.

$\boxed{|v| = 8 \text{ ms}^{-1}}$



Question 10 (***)

A particle P of mass 0.5 kg is attached to a light spring of natural length 0.6 m and modulus of elasticity 47 N. The other end of the spring is attached to a fixed point O on a ceiling, so that P is hanging at rest vertically below O. The particle is pulled vertically downwards so that $|OP| = 1.16$ m and released from rest.

Ignoring any external resistances, find the speed of P when $|OP| = 0.88$ m.

$\boxed{}$, $\boxed{|v| = 5.6 \text{ ms}^{-1}}$

BY ENERGIES TAKING THE LEVEL OF "A" AS THE POINT OF ZERO GRAVITATIONAL POTENTIAL LEVEL

$\Rightarrow \cancel{KE_A} + \cancel{PE_A} + EE_A = KE_B + PE_B + EE_B$

$\Rightarrow \frac{\lambda}{2l}x_1^2 = \frac{1}{2}mv^2 + mgh + \frac{\lambda}{2l}x_2^2$

$\Rightarrow \frac{\lambda x_1^2}{l} = mv^2 + 2mgh + \frac{\lambda}{l}x_2^2$

$\Rightarrow \frac{\lambda}{ml}x_1^2 = v^2 + 2gh + \frac{\lambda}{ml}x_2^2$

$\Rightarrow v^2 = \frac{\lambda}{ml}\left(x_1^2 - x_2^2\right) - 2gh$

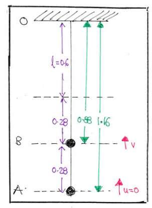
$\Rightarrow v^2 = \frac{47}{0.5 \times 0.6}\left[0.56^2 - 0.28^2\right] - 2(9.8)(0.28)$

$\Rightarrow v^2 = \frac{47}{0.3} \times \frac{147}{625} - \frac{686}{125}$

$\Rightarrow v^2 = \frac{4606}{125} - \frac{686}{125}$

$\Rightarrow v^2 = \frac{784}{25}$

$\Rightarrow |v| = \frac{28}{5} = 5.6 \text{ ms}^{-1}$



$\lambda = 47$ N
$l = 0.6$ m
$m = 0.5$

Question 11 (***)

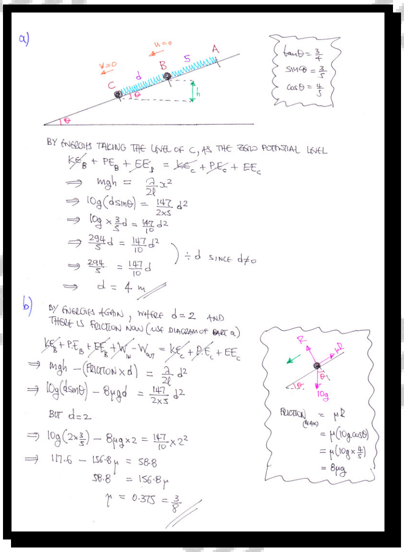
One end of a light elastic string, is attached to a point A on a fixed plane inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, and a particle of mass 10 kg is attached to the free end of the string.

The string has natural length 5 m and modulus of elasticity 147 N. The particle is first held at a point B on the plane, where B is below A and $|AB| = 5$ m. The string is parallel to a line of greatest slope of the plane. The particle is released from rest.

a) Given the plane is smooth, find the distance that the particle moves before first coming to instantaneous rest.

b) Given instead that the plane is rough, and the particle first comes after a distance of 2 m, determine the coefficient of friction between the particle and the plane.

$\boxed{d = 4 \text{ m}}$, $\boxed{\mu = \frac{3}{8} = 0.375}$



a)

$\tan\theta = \frac{3}{4}$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

BY ENERGIES TAKING THE LEVEL OF C, AS THE ZERO POTENTIAL LEVEL

$\cancel{KE_B} + PE_B + \cancel{EE_B} = \cancel{KE_C} + \cancel{PE_C} + EE_C$

$\Rightarrow mgh = \frac{\lambda}{2l}x^2$

$\Rightarrow 10g(d\sin\theta) = \frac{147}{2\times5}d^2$

$\Rightarrow 10g \times \frac{3}{5}d = \frac{147}{10}d^2$

$\Rightarrow \frac{294}{5}d = \frac{147}{10}d^2$) $\div d$ SINCE $d \neq 0$

$\Rightarrow \frac{294}{5} = \frac{147}{10}d$

$\Rightarrow d = 4$ m

b)

BY ENERGIES AGAIN, WHERE $d = 2$ AND THERE IS FRICTION NOW (USE DIAGRAM OF PART a)

$\cancel{KE_B} + PE_B + \cancel{EE_B} + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_C} + \cancel{PE_C} + EE_C$

$\Rightarrow mgh - (\text{FRICTION} \times d) = \frac{\lambda}{2l}d^2$

$\Rightarrow 10g(d\sin\theta) - 8\mu g d = \frac{147}{2\times5}d^2$

BUT $d = 2$

$\Rightarrow 10g(2\times\frac{3}{5}) - 8\mu g \times 2 = \frac{147}{10}\times 2^2$

$\Rightarrow 117.6 - 156.8\mu = 58.8$

$58.8 = 156.8\mu$

$\mu = 0.375 = \frac{3}{8}$

FRICTION (MAX) $= \mu R$

$= \mu(10g\cos\theta)$

$= \mu(10g \times \frac{4}{5})$

$= 8\mu g$

Question 12 (***)

A light elastic string AB has natural length 1.25 m and modulus of elasticity 24.5 N. Another light elastic string CD has natural length 1.25 m and modulus of elasticity 26.95 N.

The two strings AB and CD are joined together with B attached to C forming a longer string AD whose end A is fixed to a horizontal ceiling.

A particle of mass 5 kg is attached to the free end of the string at D and hangs in equilibrium, without touching the ground.

a) Determine the length of AD in this configuration.

The strings are next joined together at their ends with A joined to C and with B joined to D. The "A to C" end is fixed to the horizontal ceiling.

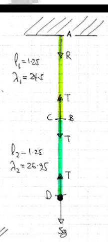
A particle of mass 5 kg is attached to the "B to D" end, and hangs in equilibrium, without touching the ground.

b) Calculate the tension in each string.

$\boxed{\quad}$, $\boxed{|AD| = \frac{80}{11} \approx 7.27 \text{ m}}$, $\boxed{T_{AB} = \frac{70}{3} \approx 23.3 \text{ N}}$, $\boxed{T_{CD} = \frac{77}{3} \approx 25.7 \text{ N}}$

a) LOOKING AT A DIAGRAM AND CONSIDERING THE TENSION

IN AB	IN CD
$T = \frac{\lambda_1 x_1}{l_1}$	$T = \frac{\lambda_2 x_2}{l_2}$
$5g = \frac{24.5 x_1}{1.25}$	$5g = \frac{26.95 x_2}{1.25}$
$x_1 = 2.5$ m	$x_2 = \frac{25}{11}$ m



$\therefore$ TOTAL LENGTH IS

$1.25 + 1.25 + 2.5 + \frac{25}{11} = \frac{80}{11}$

≈ 7.27 m

b) NEW DIAGRAM FOR THE NEW CONFIGURATION

IN THIS CONFIGURATION $x_1 = x_2 = x$

$\Rightarrow T_1 + T_2 = 5g$

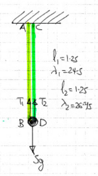
$\Rightarrow \frac{\lambda_1 x_1}{l_1} + \frac{\lambda_2 x_2}{l_2} = 5g$

$\Rightarrow \frac{24.5x}{1.25} + \frac{26.95x}{1.25} = 5g$

$\Rightarrow \frac{98}{5}x + \frac{539}{25}x = 5g$

$\Rightarrow \frac{1029}{25}x = 49$

$\Rightarrow x = \frac{25}{21}$



NOW THE TENSION IN EACH STRING CAN BE FOUND

$T_1 = \frac{\lambda_1 x_1}{l_1} = \frac{24.5 \times \frac{25}{21}}{1.25} = \frac{70}{3} = 23\frac{1}{3}$ N

$T_2 = 5g - \frac{70}{3} = \frac{77}{3} = 25\frac{2}{3}$ N

$\therefore$ TENSION IN AB IS $23\frac{1}{3}$ N

TENSION IN CD IS $25\frac{2}{3}$ N

Question 13 (***)

A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 1.2 m and modulus of elasticity 19.6 N.

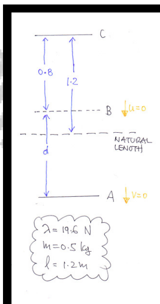
The other end of the spring is attached to a fixed point C on a horizontal ceiling.

The particle is held at the point B, where B is vertically below C and $|BC| = 0.8$ m. The spring remains straight in a vertical position.

The particle is released from rest and first comes to instantaneous rest at the point A.

Find the distance $|AC|$.

$\boxed{|AC| = 2.2 \text{ m}}$



SINCE WE HAVE A SPRING WE HAVE ELASTIC ENERGY AT ALL POSITIONS EXCEPT AT NATURAL LENGTH

$$\cancel{KE_B} + PE_B + EE_B = \cancel{KE_A} + PE_A + EE_A$$

TAKING THE LEVEL OF B AS THE ZERO GRAVITATIONAL POTENTIAL LEVEL.

$$EE_B = PE_A + EE_A$$

$$\Rightarrow \frac{\lambda}{2l}(1.2-0.8)^2 = -mgd + \frac{\lambda}{2l}(d-1.2)^2$$

$$\Rightarrow \frac{19.6}{2\times1.2}\times 0.4^2 = -0.5\times9.8d + \frac{19.6}{2\times1.2}(d-0.4)^2$$

$$\Rightarrow \frac{98}{75} = -\frac{49}{10}d + \frac{49}{6}(d^2 - 0.8d + 0.16)$$

$$\Rightarrow \cancel{\frac{98}{75}} = -\frac{49}{10}d + \frac{49}{6}d^2 - \frac{98}{15}d + \cancel{\frac{98}{75}}$$

$$\Rightarrow 0 = \frac{49}{6}d^2 - \frac{343}{30}d$$

$$\Rightarrow 5d^2 - 7d = 0$$

$$\Rightarrow d(5d-7) = 0$$

$$d = \begin{cases} 0 \\ \frac{7}{5} = 1.4 \end{cases}$$

$$\therefore |AC| = 0.8 + d = 0.8 + 1.4 = 2.2 \text{ m}$$

Question 14 (***+)

A particle P of mass 12 kg is attached to the midpoint of a light elastic string of natural length 0.5 m and modulus of elasticity λ N. The ends of the string are attached to two fixed points A and B, where $|AB| = 0.8$ m and AB is horizontal.

When P is held at the point M, where M is the midpoint of AB, the tension in the string is 216 N.

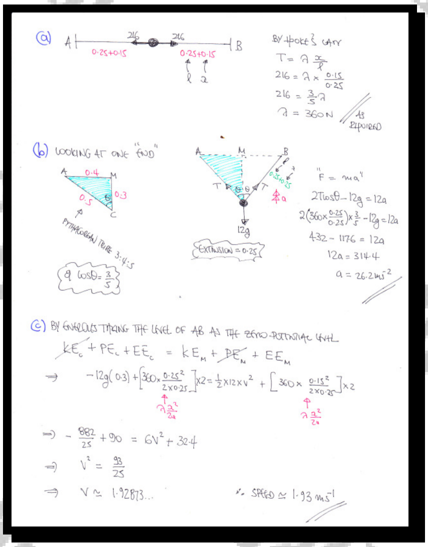
a) Show that $\lambda = 360$.

The particle is next held at the point C, where C is 0.3 m below M, and then it is released from rest.

b) Find the initial acceleration of P.

c) Calculate the speed of P as is passes through M.

$\boxed{|a|_{\text{initial}} = 26.2\,\text{ms}^{-2}}$, $\boxed{\text{speed} \approx 1.93 \text{ ms}^{-1}}$



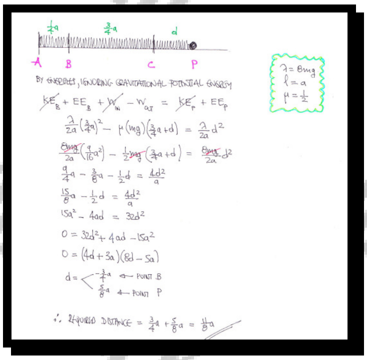
Question 15 (***+)

A particle P of mass m is attached to one end of a light elastic **spring**, of natural length a and modulus of elasticity $8mg$. The other end of the spring is attached to a fixed point A on a rough horizontal plane. The coefficient of friction between P and the plane is 0.5.

The particle is held at rest on the plane at a point B, where $AB = \frac{1}{4}a$ and released from rest.

Find the distance of P from B when P first comes to rest.

$$\boxed{d = \frac{11}{8}a}$$



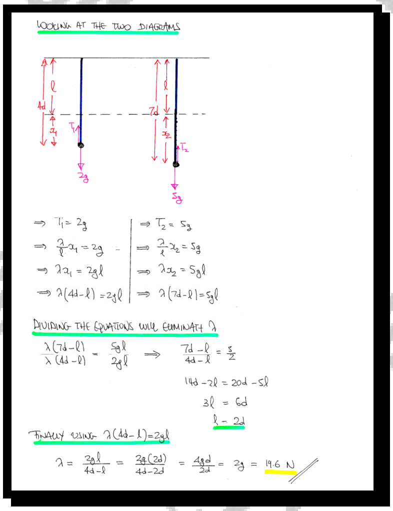
Question 16 (***+)

A particle of mass 2 kg is suspended from a fixed point P by a light elastic string, and rests in equilibrium at a vertical distance $4d$ below P.

When a different particle of mass 5 kg is suspended from the fixed point P by the same light elastic string, it rests in equilibrium at a vertical distance $7d$ below P.

Determine the modulus of elasticity of the string.

$\boxed{}$, $\boxed{\lambda = 19.6 \text{ N}}$



Question 17 (***+)

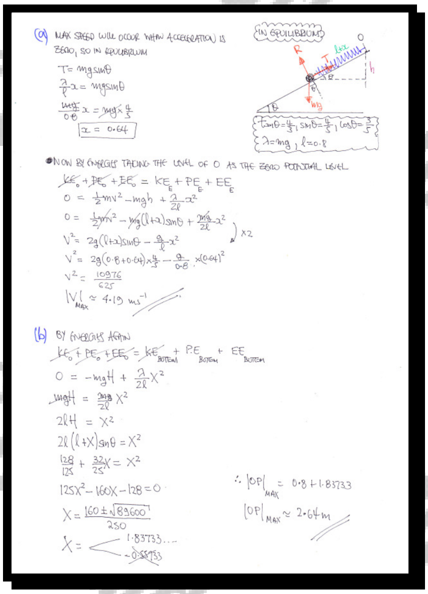
A particle P of mass m kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity mg N. The other end of the string is attached to a fixed point O on a smooth plane inclined at θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

The particle is released from rest from O and moves down the plane without any air resistance and without reaching the bottom of the plane.

a) Determine the greatest speed of P in the subsequent motion.

b) Find the distance of P from O, when it reaches the lowest point on the plane.

$\boxed{|v|_{\text{max}} = 4.19 \text{ ms}^{-1}}$, $\boxed{d \approx 2.64 \text{ m}}$



(a) MAX SPEED WILL OCCUR WHEN ACCELERATION IS ZERO, SO IN EQUILIBRIUM

IN EQUILIBRIUM

$T = mg\sin\theta$

$\frac{\lambda}{l}x = mg\sin\theta$

$\frac{\cancel{mg}}{0.8}x = \cancel{mg} \times \frac{4}{5}$

$\boxed{x = 0.64}$

$\tan\theta = \frac{4}{3}, \sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}$

$\lambda = mg, \ l = 0.8$

• NOW BY ENERGIES TAKING THE LEVEL OF O AS THE ZERO POTENTIAL LEVEL

$\cancel{KE_0} + \cancel{PE_0} + \cancel{EE_0} = KE_E + PE_E + EE_E$

$0 = \frac{1}{2}mv^2 - mgh + \frac{\lambda}{2l}x^2$

$0 = \frac{1}{2}\cancel{m}v^2 - \cancel{m}g(l+x)\sin\theta + \frac{\cancel{m}g}{2l}x^2$) ×2

$v^2 = 2g(l+x)\sin\theta - \frac{g}{l}x^2$

$v^2 = 2g(0.8+0.64)\times\frac{4}{5} - \frac{g}{0.8}\times(0.64)^2$

$v^2 = \frac{10976}{625}$

$|v|_{MAX} \approx 4.19 \text{ ms}^{-1}$

(b) BY ENERGIES AGAIN

$\cancel{KE_0} + \cancel{PE_0} + \cancel{EE_0} = \cancel{KE_{BOTTOM}} + PE_{BOTTOM} + EE_{BOTTOM}$

$0 = -mgH + \frac{\lambda}{2l}X^2$

$\cancel{mg}H = \frac{\cancel{mg}}{2l}X^2$

$2lH = X^2$

$2l(l+X)\sin\theta = X^2$

$\frac{128}{125} + \frac{32}{25}X = X^2$

$125X^2 - 160X - 128 = 0$

$X = \frac{160 \pm \sqrt{89600}}{250}$

$X = 1.83733\ldots$ or $\cancel{-0.55733}$

$\therefore |OP|_{MAX} = 0.8 + 1.83733$

$|OP|_{MAX} \approx 2.64 \text{ m}$

Question 18 (***+)

A light elastic string has natural length 1 m and modulus of elasticity 10 N.

The two ends of the string are attached to two points A and B, which are 1.2 m apart on a horizontal ceiling.

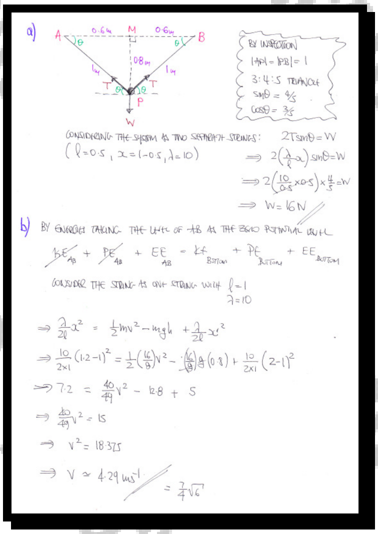
A particle P is attached to the midpoint of the string and hangs in equilibrium 0.8 m below the level of AB.

a) Calculate the weight of P.

P is then raised and released from rest from the midpoint of AB.

b) Calculate the speed of P when it has fallen vertically by 0.8 m.

$\boxed{W = 16 \text{ N}}$, $\boxed{V = \frac{7}{4}\sqrt{6} \approx 4.29... \text{ ms}^{-1}}$



Question 19 (***+)

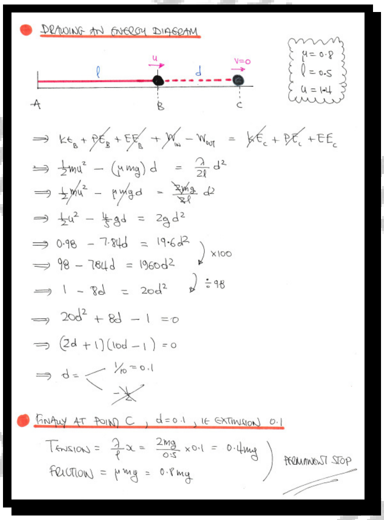
A particle P, of mass m, is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point A on a rough horizontal surface.

P is held at a point B, where $|AB| = 0.5$ m and given a speed of 1.4 ms^{-1} in the direction AB.

P comes at rest at the point C.

Determine whether this position of rest is instantaneous or permanent.

[illegible], $\boxed{\text{permanent}}$



Question 20 (***+)

A light elastic spring AB, of natural length 2 m, has its end A attached to a fixed point on a horizontal ceiling and a particle, of mass 3 kg, is attached to the other end of the spring, B, with the particle hanging in equilibrium.

The modulus of elasticity of the spring is $100g$ N

The particle is then pulled vertically downwards, so that $|AB| = 2.15$ m, and released from rest.

Determine the length of AB when the particle first comes to instantaneous rest.

[illegible], $\boxed{|AB| = 1.97 \text{ m}}$

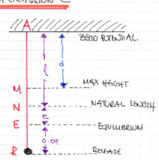
START BY FINDING THE EQUILIBRIUM EXTENSION e

$mg = \frac{\lambda}{l}e$

$e = \frac{mgl}{\lambda}$

$e = \frac{3g \times 2}{100g}$

$e = \frac{6}{100} = 0.06$



ADDITIONAL EXTENSION

$2.15 - 2 - 0.06 = 0.09$

$\lambda = 100g$ N
$l = 2$ m
$m = 3$ kg

BY ENERGIES TAKING THE LEVEL OF A, AS THE ZERO POTENTIAL LEVEL

$\Rightarrow \cancel{KE_R} + PE_R + EE_R = \cancel{KE_M} + PE_M + EE_M$

$\Rightarrow -mg(l+e+0.09) + \frac{\lambda}{2l}(e+0.09)^2 = -mgd + \frac{\lambda}{2l}(l-d)^2$

$\Rightarrow -m\cancel{g}(2.15) + \frac{100\cancel{g}}{4}(0.15)^2 = -m\cancel{g}d + \frac{100\cancel{g}}{4}(2-d)^2$

$\Rightarrow -\frac{129}{20} + \frac{9}{16} = -3d + 25(4 - 4d + d^2)$

$\Rightarrow -\frac{471}{80} = -3d + 100 - 100d + 25d^2$

$\Rightarrow -471 = -240d + 8000 - 8000d + 2000d^2$

$\Rightarrow 0 = 2000d^2 - 8240d + 8471$

BY THE QUADRATIC FORMULA

$\Rightarrow d = \frac{8240 \pm \sqrt{129600}}{2 \times 2000}$

$\Rightarrow d = \frac{8240 \pm 360}{4000}$

$\Rightarrow d = \begin{cases} \cancel{2.15} & \text{RELEASE POINT} \\ 1.97 \end{cases}$

ALTERNATIVE APPROACH

- PROVE THE PARTICLE IS MOVING IN S.H.M ABOUT EQUILIBRIUM POSITION
- THEN AS WE HAVE A SPRING THE MOTION IS "FULL S.H.M", SO THE ZERO SPEED POINTS DEFINES THE ENDPOINTS OF THE OSCILLATION
- SYMMETRY THEN YIELDS

EQUILIBRIUM AT $2 + 0.06 = 2.06$
$2.15 - 2.06 = 0.09$ ← AMPLITUDE
$2.06 - 0.09 = 1.97$

Question 21 (***+)

A light elastic string, of natural length 1.42 m, has each of its two ends attached to two fixed points, A and B, where AB is horizontal and $|AB| = 1.68$ m.

A particle, of mass 2 kg, is attached to the midpoint of the string, M.

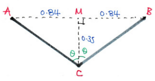
The particle is hanging in a equilibrium at the point C, where MC is vertical and $|MC| = 0.35$ m.

The particle is then held at M and released from rest.

Calculate, correct to 2 decimal places, the speed of particle as it passes through C.

$\boxed{}$, $\boxed{|v| \approx 1.98 \text{ ms}^{-1}}$

- START BY TRYING TO FIND THE MODULUS OF ELASTICITY



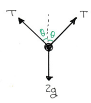
- BY PYTHAGORAS

$|BC| = \sqrt{0.35^2 + 0.84^2} = 0.91$

AND BY TRIGONOMETRY

$\sin\theta = \frac{0.84}{0.91} = \frac{12}{13}$

$\cos\theta = \frac{0.35}{0.91} = \frac{5}{13}$



- RESOLVING VERTICALLY, NOTING THAT THE EXTENSION IN EACH STRING IS $0.91 - 0.71 = 0.2$

$\Rightarrow \cancel{2}T\cos\theta = \cancel{2}g$

$\Rightarrow T \times \frac{5}{13} = g$

$\Rightarrow \frac{\lambda}{\ell}x \times \frac{5}{13} = g$

$\Rightarrow \frac{\lambda}{0.71} \times 0.2 \times \frac{5}{13} = g$

$\Rightarrow \frac{\lambda}{9.23} = g$

$\Rightarrow \lambda = 90.454$

- NOW BY ENERGIES TAKING THE LEVEL OF AB AS THE ZERO POTENTIAL LEVEL

$\Rightarrow \cancel{KE_M} + \cancel{PE_M} + EE_M + \cancel{W_{IN}} - \cancel{W_{OUT}} = KE_C + P.E_C + EE_C$

$\Rightarrow \frac{\lambda}{2\ell}x_M^2 = \frac{1}{2}mv_c^2 - mg|MC| + \frac{\lambda}{2\ell}x_c^2$

$\Rightarrow 2\left[\frac{90.454}{2 \times 0.71}(0.13)^2\right] = \frac{1}{2} \times 2 \times V_c^2 - 2g(0.35) + 2\left[\frac{90.454}{2 \times 0.71} \times (0.2)^2\right]$

"2 PIECES OF STRING" "2 PIECES OF STRING"

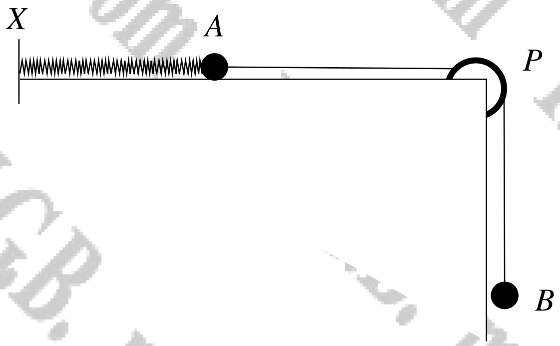
$\Rightarrow 2.15306 = V^2 - 6.86 + 5.096$

$\Rightarrow V^2 = 3.91706$

$\Rightarrow V \approx 1.979156386\ldots$

$\therefore V \approx 1.98 \text{ ms}^{-1}$

Question 22 (****)



A particle A, of mass m is at rest on a rough horizontal table.

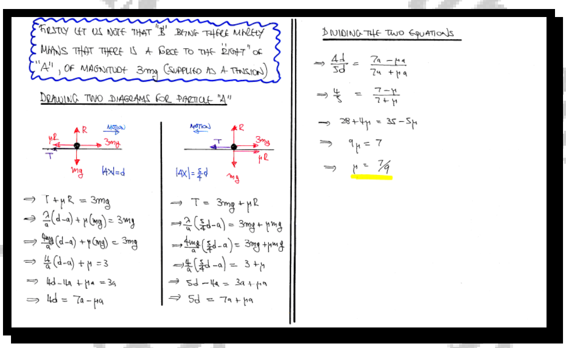
A is attached to a fixed point X on the table, by a light elastic string of natural length a and modulus of elasticity $4mg$.

A light inextensible string is attached to A and passes over a smooth pulley P with the other end of this string attached to another particle B, of mass $3m$, which hangs vertically below P.

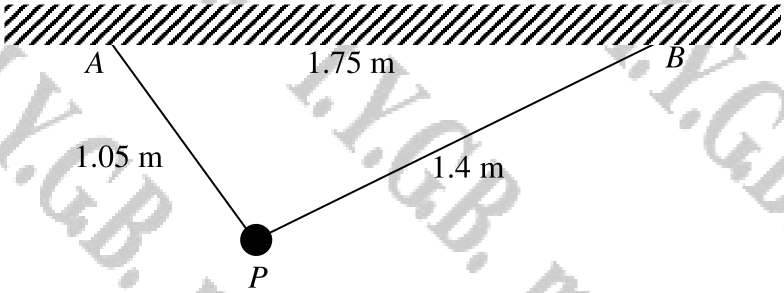
- When B is gently released from a position such that $|XA| = d$, A is about to slide towards P.
- When B is gently released from a position such that $|XA| = \frac{5}{4}d$, A is about to slide towards X.

Find the value of the coefficient of friction between A and the table.

FM-A, $\boxed{\mu = \frac{7}{9}}$



Question 23 (****)



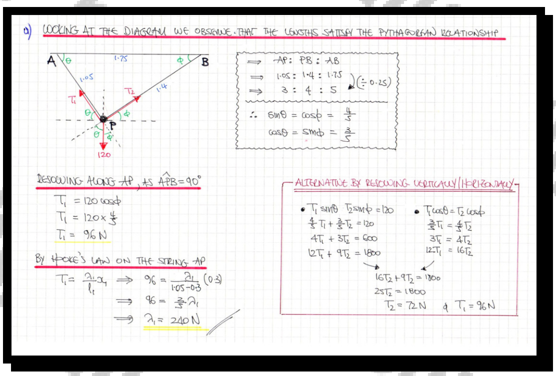
The figure above shows a particle P of weight 120 N is suspended by two light elastic strings AP and BP, where A and B are two fixed points on a horizontal ceiling, at a distance 1.75 m apart.

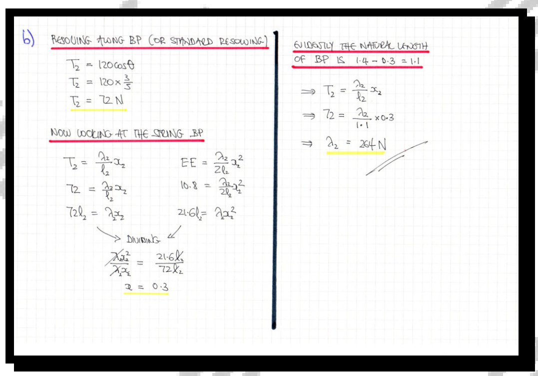
When the system is in equilibrium, AP is stretched by 0.3 m to a length of 1.05 m and BP is stretched to a length of 1.4 m.

a) Determine the modulus of elasticity of AP.

b) Given further that the energy stored in BP is 10.8 J find the modulus of elasticity of BP.

$\boxed{\lambda_{AP} = 240 \text{ N}}$, $\boxed{\lambda_{BP} = 264 \text{ N}}$





Question 24 (****)

A bungee jumper of mass 75 kg is attached to one end of a light elastic string, of natural length 25 m, and modulus of elasticity 3675 N.

The other end of the string is securely tied to a fixed point P on a horizontal platform, which is sufficiently high enough above the ground.

The bungee jumper steps off the platform at P and when his vertical distance from P is x m his speed is $v \text{ ms}^{-1}$.

The bungee jumper is modelled as a particle, falling without air resistance, with Hooke's law applying whilst the string is taut.

a) Show that for $x \geq 25$

$$25v^2 = -49x^2 + 2940x - 30625,$$

and hence calculate, correct to 2 decimal places, the greatest value of x.

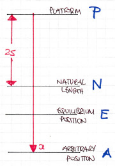
b) Determine the greatest value of v, during his jump.

$\boxed{}$, $\boxed{|x_{max}| = 30 + 5\sqrt{11} \approx 46.58 \text{ m}}$, $\boxed{|v_{max}| = 7\sqrt{11} \approx 23.22 \text{ ms}^{-1}}$

a) BY CONSIDERING ENERGIES TAKING THE LEVEL OF "P" AS THE ZERO GRAVITATIONAL POTENTIAL WE OBTAIN

$\Rightarrow \cancel{KE_P} + \cancel{PE_P} + \cancel{EE_P} = KE_A + PE_A + EE_A$

(IGNORED W_{IN} & W_{OUT} HERE)



$\Rightarrow 0 = \frac{1}{2}mv^2 - mgx + \frac{\lambda}{2l}(x-l)^2$

$\Rightarrow 2mgx - \frac{\lambda}{l}(x-l)^2 = mv^2$

$\Rightarrow 1470x - 147(x-25)^2 = 75v^2$

$\Rightarrow 1470x - 147(x^2 - 50x + 625) = 75v^2$

$\Rightarrow 1470x - 147x^2 + 7350x - 91875 = 75v^2$

$\Rightarrow 75v^2 = -147x^2 + 8820x - 91875$ $\div 3$

$\Rightarrow 25v^2 = -49x^2 + 2940x - 30625$ // AS REQUIRED

$m = 75$
$\lambda = 3675$
$l = 25$

NOW MAXIMUM VALUE OF x WILL OCCUR WHEN $v = 0$

$\Rightarrow 0 = -49x^2 + 2940x - 30625$

$\Rightarrow 49x^2 - 2940x + 30625 = 0$ $\div 49$

$\Rightarrow x^2 - 60x + 625 = 0$

$\Rightarrow (x-30)^2 - 900 + 625 = 0$

$\Rightarrow (x-30)^2 = 275$

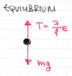
$\Rightarrow x - 30 = \begin{cases} 5\sqrt{11} \\ -5\sqrt{11} \end{cases}$

$\Rightarrow x = \begin{cases} 30 + 5\sqrt{11} \approx 46.58 \text{ m} \\ 30 - 5\sqrt{11} \approx \cancel{13.42} \text{ (STRING WILL SLACK)} \end{cases}$

b) NOW FOR MAX SPEED $\Rightarrow$ ZERO ACCELERATION

$\Rightarrow$ EQUILIBRIUM

$\Rightarrow$



$\Rightarrow \frac{\lambda}{l}e = mg$

$\Rightarrow e = \frac{lmg}{\lambda}$

$\Rightarrow e = \frac{25 \times 75 \times 9.8}{3675}$

$\Rightarrow e = 5$

$\Rightarrow x = 25 + 5 = 30$

USING THE ENERGY EQUATION WITH $x = 30$

$\Rightarrow 25v^2 = -49x^2 + 2940x - 30625$

$\Rightarrow 25v^2 = -49 \times 30^2 + 2940 \times 30 - 30625$

$\Rightarrow 25v^2 = 13475$

$\Rightarrow v^2 = 539$

$\Rightarrow |v| \approx 23.22 \text{ ms}^{-1}$ // $(7\sqrt{11})$

Question 25 (****)

A light elastic string AB, of modulus of elasticity mg and natural length l, is fixed to a point A on a rough plane inclined at an angle β to the horizontal.

The other end of the string B is attached to a particle of mass m which is held at rest on the plane so that $|AB| = l$. The string lies along a line of greatest slope of the plane, with B lower than A.

The coefficient of friction between the particle and the plane is μ, $\mu < \tan\beta$.

The particle is released from rest.

a) Show that when the particle first comes to rest it has moved down the plane by a distance

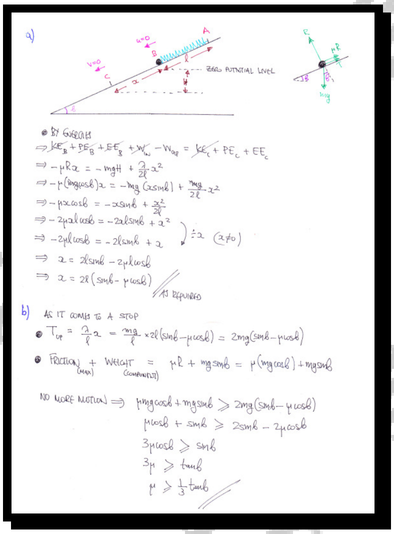
$$2l(\sin\theta - \mu\cos\beta).$$

Once the particle comes to rest there is no further motion.

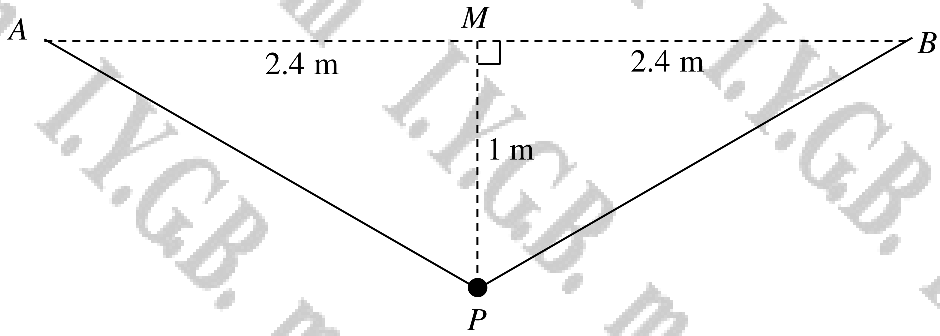
b) Show further that

$$\mu \ge \frac{1}{3}\tan\beta$$

proof



Question 26 (****)



One of the two ends of each of two identical light strings AP and BP, are attached to a particle P of mass m kg. The other ends of each of the strings, A and B, are fixed at the same horizontal level, 4.8 m apart. The particle rests in equilibrium 1 m below M, where M is the midpoint of AB. Each of the strings has natural length 2 m and modulus of elasticity of 637 N.

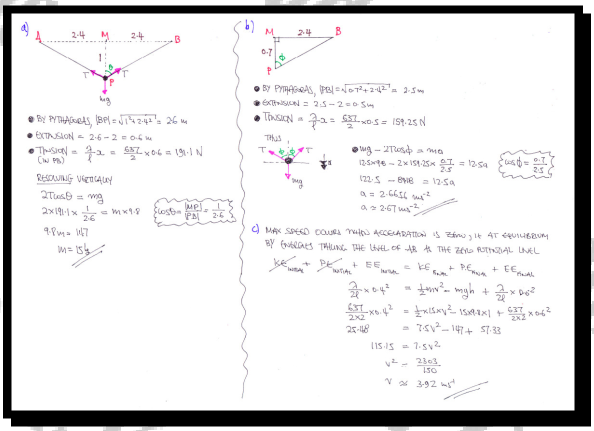
a) Show that $m = 15$.

The particle is then held at M and released from rest.

b) Find the acceleration of P, when it has fallen vertically down by 0.7 m.

c) Calculate the maximum speed of P in the subsequent motion.

$\boxed{a = 2.6656 \text{ ms}^{-2}}$, $\boxed{V_{max} \approx 3.91833... \text{ ms}^{-1}}$



Question 27 (****)

A long straight vertical wall stands on a **rough** horizontal plane. The fixed point O lies at the bottom of the wall, at some point along the edge between the wall and the plane. An elastic string has one end attached to O and the other end attached to a particle of mass 2 kg. The string has natural length 1.6 m and modulus of elasticity 200 N. The coefficient of friction between the particle and the plane is μ.

The particle is pulled at some point A on the plane, so that OA is perpendicular to the wall and $|OA| = 2\text{ m}$.

The particle is projected towards along AO, towards O with speed 10 ms^{-1} and travels in a straight line hitting the wall at O with speed $v\text{ ms}^{-1}$.

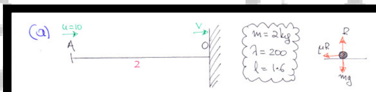
a) Assuming that air resistance can be ignored express v^2 in terms of μ.

The particle rebounds off the wall with half its speed and moves in a straight line towards A.

b) Given the particle comes to rest as it reaches A, show that $\mu = \frac{5}{14}$.

$$\boxed{v^2 = 110 - 39.2\mu}$$

(a)



IGNORING POTENTIAL ENERGY AS ALL MOTION TAKE PLACE AT THE SAME HORIZONTAL LEVEL

$\Rightarrow KE_A + EE_A + \cancel{W_{IN}} - W_{OUT} = KE_O + \cancel{EE_O}$

$\Rightarrow \frac{1}{2}mu^2 + \frac{\lambda}{2l}x^2 - \mu R \times d = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}\times 2\times 10^2 + \frac{200}{2\times 1.6}\times(2-1.6)^2 - \mu(mg)\times 2 = \frac{1}{2}\times 2\times v^2$

$\Rightarrow 100 + 10 - 39.2\mu = v^2$

$\Rightarrow v^2 = 110 - 39.2\mu$

(b) NOW SPEED REDUCES TO $w = \frac{1}{2}v$ — NOW MOTION BACK TO A BY ENERGY AGAIN

$\Rightarrow KE_O + \cancel{EE_O} + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_A} + EE_A$

$\Rightarrow \frac{1}{2}mw^2 - \mu R\times d = \frac{\lambda}{2l}x^2$

$\Rightarrow \frac{1}{2}\times 2\times\left(\frac{1}{2}v\right)^2 - \mu(mg)\times 2 = \frac{200}{2\times 1.6}\times 0.4^2$

$\Rightarrow \frac{1}{4}v^2 - 4\mu g = 10$

$\Rightarrow v^2 - 16\mu g = 40$

$\Rightarrow v^2 - 156.8\mu = 40$

$\Rightarrow (110 - 39.2\mu) - 156.8\mu = 40$

$\Rightarrow 70 = 196\mu$

$\Rightarrow \mu = \frac{5}{14}$ AS REQUIRED

Question 28 (****+)

A light elastic string is fixed to a point A on a level horizontal ceiling.

When a particle of mass m is attached to the other end of the string B and hangs in equilibrium, the length AB is x.

When a different particle of mass M, $M > m$, is attached to B and hangs in equilibrium, the length AB is y.

Find an expression for the natural length of the string, in terms of m, M, x and y and hence deduce that

$$Mx > my.$$

$$\boxed{}\ , \ \boxed{\dfrac{Mx - my}{M - m}}$$

BY HOOKE'S LAW – LET THE NATURAL LENGTH BE l

$$mg = \frac{\lambda}{l}(x - l) \qquad \& \qquad Mg = \frac{\lambda}{l}(y - l)$$

$$mgl = \lambda(x - l) \qquad Mgl = \lambda(y - l)$$

$$\frac{mgl}{\lambda} = x - l \qquad \frac{Mgl}{\lambda} = y - l$$

$$\frac{gl}{\lambda} = \frac{x - l}{m} \qquad \frac{gl}{\lambda} = \frac{y - l}{M}$$

EQUATING YIELDS

$$\Rightarrow \frac{x - l}{m} = \frac{y - l}{M}$$

$$\Rightarrow Mx - Ml = my - ml$$

$$\Rightarrow Mx - my = Ml - ml$$

$$\Rightarrow Mx - my = l(M - m)$$

$$\Rightarrow l = \frac{Mx - my}{M - m}$$

AS $l > 0$ & $M > m$, SO THAT $M - m > 0$, IT IMPLIES THAT

$$\rightarrow Mx - my > 0$$

$$\rightarrow Mx > my$$

AS REQUIRED

Question 29 (****+)

An elastic string has one end attached to a fixed point O on a **rough** horizontal plane.

The other end of the string is attached to a particle of mass 7 kg. The string has natural length 1.2 m and modulus of elasticity 40 N. The particle is pulled at some point A on the plane so that $|OA| = 4 \text{ m}$ and is released from rest. The particle travels in a straight line coming to rest at some point B so that $|AB| = 6 \text{ m}$.

a) Determine the frictional force acting on the particle, assumed **constant** throughout the motion.

b) Show that the particle does not remain at rest at B.

The particle next comes to rest at some point C.

c) Show further that the string is not slack at C.

d) Calculate the distance BC.

$\boxed{R = 20 \text{ N}}$, $\boxed{|BC| = 0.4 \text{ m}}$

(a)

• IGNORING GRAVITATIONAL POTENTIAL ENERGY AS ALL MOTION IS AT THE SAME LEVEL

$$\cancel{KE_A} + EE_A + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_B} + EE_B$$

$$\Rightarrow \frac{\lambda}{2l}x_A^2 - R \times d = \frac{\lambda}{2l}x_B^2$$

$$\Rightarrow \frac{40}{2 \times 1.2} \times (4-1.2)^2 - R \times 6 = \frac{40}{2 \times 1.2} \times (2-1.2)^2$$

$$\Rightarrow \frac{392}{3} - 6R = \frac{32}{3}$$

$$\Rightarrow 120 = 6R$$

$$\Rightarrow R = 20$$

(b) AT B $\quad$ TENSION $= \frac{\lambda}{l}x_B = \frac{40}{1.2} \times (2-1.2) = \frac{80}{3} = 26.666... > 20$

$\therefore$ PARTICLE MUST MOVE AS TENSION IS GREATER THAN R

(c) • BY ENERGIES AGAIN AND ASSUMING AT THE STRING IS SLACK AT C

$$\cancel{KE_B} + EE_B + \cancel{W_{IN}} - W_{OUT} = \cancel{KE_C} + \cancel{EE_C}$$

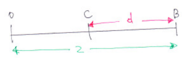
$$\frac{\lambda}{2l}x_B^2 - R \times d = 0$$

$$\frac{32}{3} = 20d$$

$$d = \frac{8}{15} \approx 0.533....$$

BUT THIS IMPLIES THAT $|OC| = 2 - 0.533... = 1.466... > 1.2$
WHICH IMPLIES THAT THE STRING IS NOT SLACK

(d)



$$EE_B - W_{OUT} = EE_C$$

EXTENSION AT POINT C

$$\Rightarrow \frac{32}{3} - 20d = \frac{40}{2 \times 1.2}(2-d-1.2)^2$$

$$\Rightarrow \frac{32}{3} - 20d = \frac{50}{3}(0.8-d)^2$$

$$\Rightarrow 32 - 60d = 50(0.8-d)^2$$

$$\Rightarrow 32 - 60d = 50(0.64 - 1.6d + d^2)$$

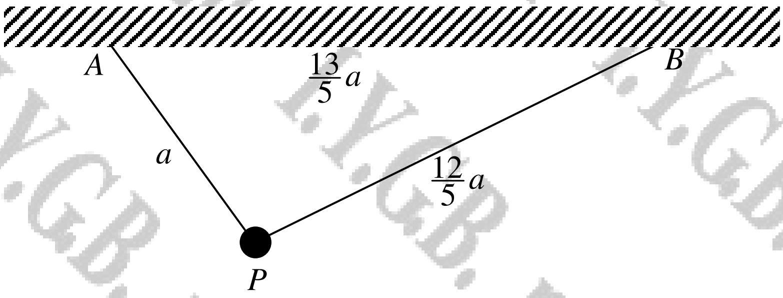
$$\Rightarrow \cancel{32} - 60d = \cancel{32} - 80d + 50d^2$$

$$\Rightarrow 0 = 50d^2 - 20d$$

$$\Rightarrow 0 = 10d(5d-2)$$

$$\Rightarrow d = \cancel{0},\ 0.4$$

Question 30 (****+)



The figure above shows a particle P of mass m is suspended by two light strings AP and BP, where A and B are two fixed points on a horizontal ceiling, at a distance $\frac{13}{5}a$ apart.

The string AP is inelastic and has length a.

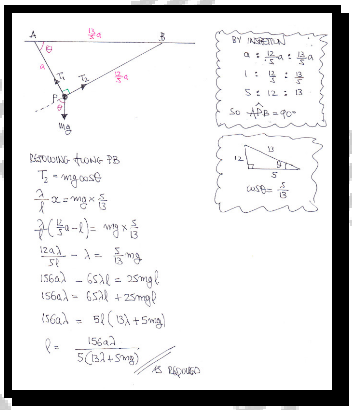
The string BP is elastic and has length $\frac{12}{5}a$.

The natural length of BP is l and its modulus of elasticity is λ.

Show clearly that

$$l = \frac{156\lambda a}{5(5mg + 13\lambda)}.$$

$\boxed{}$, $\boxed{\text{proof}}$



Question 31 (****+)

[In this question $g = 10 \text{ ms}^{-2}$]

Two particles A and B, or respective masses 8 kg and 2 kg, are attached to the ends of a light elastic string of natural length 2.5 m and modulus of elasticity 80 N.

The string passes through a small smooth hole on a rough horizontal table.

A is held at a distance of 2.5 m from the hole and B is held at a distance of 2 m vertically below the hole. The coefficient of friction between A and the table is 0.5.

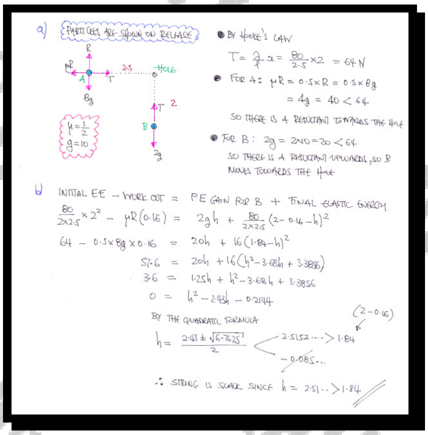
Both particles are released simultaneously from rest.

a) Show that both particles move towards the hole.

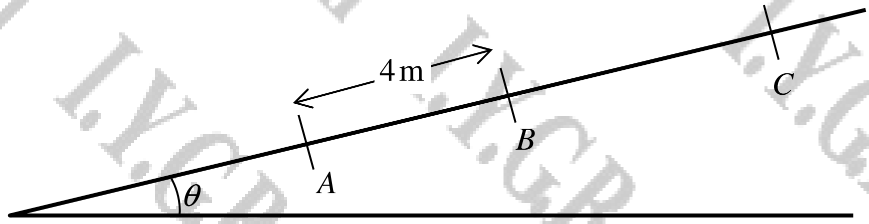
A comes to permanent rest after moving a distance of 0.16 m.

b) Show further that the string is slack when B comes to instantaneous rest for the first time.

$\boxed{\text{proof}}$



Question 32 (****+)



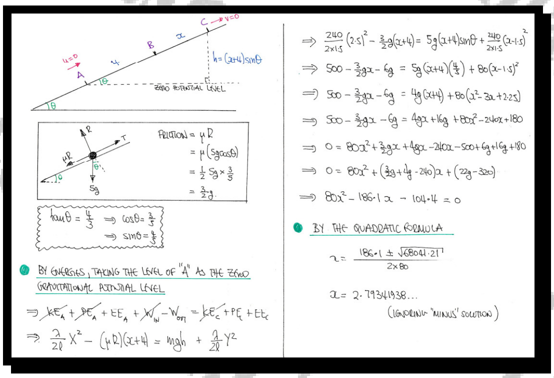
A light elastic string, with natural length $1\frac{1}{2}$ m and modulus of elasticity 240 N, has one end attached to a fixed point B on a rough plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{4}{3}$.

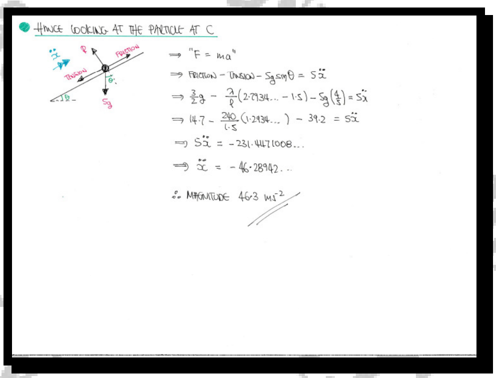
A particle of mass 5 kg is attached to the other end of the string. The coefficient of friction between the particle and the plane is 0.5. The particle is held at the point A on the plane, where $|AB| = 4$ m, and is released from rest.

The particle travels up the plane and comes to instantaneous rest at the point C, where the string is taut.

Given that A, B and C lie on a line of greatest slope of the plane, determine the magnitude of the acceleration of the particle at C.

$\boxed{}$, $\boxed{|\ddot{x}| = 46.28942...\text{ms}^{-2}}$





Question 33 (*****)

Two points A and B lie at the same horizontal level so that $|AB| = 4a$.

A light elastic string is just taut when its ends are fixed at A and B. A heavy particle is attached to the string at the point P where $|AP| = 3a$.

When the particle is allowed fall, eventually resting in equilibrium at some point below AB, $\measuredangle APB = 90°$.

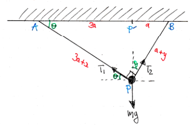
Show that

$$4\cos^2\theta - 12\sin^2\theta = 3(\cos\theta - \sin\theta),$$

where $\theta = \measuredangle BAP$.

$\boxed{}$, $\boxed{\text{proof}}$

LOOKING AT A DETAILED DIAGRAM FOR THE PROBLEM



LET x & y BE THE EXTENSIONS IN EACH SECTION

RESOLVING HORIZONTALLY & USING HOOKE'S LAW

$$T_1\cos\theta = T_2\sin\theta$$
$$\lambda\frac{x}{3a}\cos\theta = \frac{\lambda y}{a}\sin\theta$$
$$\frac{x\cos\theta}{3} = y\sin\theta$$
$$x\cos\theta = 3y\sin\theta$$

NOW LOOKING AT THE GEOMETRY OF THE RIGHT ANGLE TRIANGLE $A\hat{B}P$

- $\frac{3a+x}{4a} = \cos\theta$

$$3a + x = 4a\cos\theta$$
$$x = 4a\cos\theta - 3a$$
$$x = a(4\cos\theta - 3)$$

- $\frac{a+y}{4a} = \sin\theta$

$$a + y = 4a\sin\theta$$
$$y = 4a\sin\theta - a$$
$$y = a(4\sin\theta - 1)$$

COMBINING RESULTS

$$x\cos\theta = 3y\sin\theta$$
$$a(4\cos\theta - 3)\cos\theta = 3a(4\sin\theta - 1)\sin\theta$$
$$4\cos^2\theta - 3\cos\theta = 12\sin^2\theta - 3\sin\theta$$
$$4\cos^2\theta - 12\sin^2\theta = 3(\cos\theta - \sin\theta)$$

// AS REQUIRED