RELATIVE MOTION

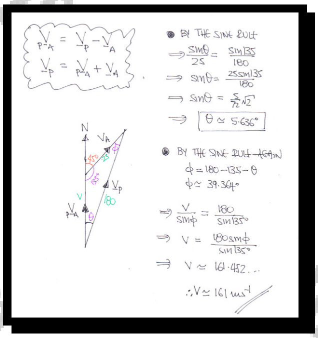
Question 1 (**)

An aeroplane is travelling at the same horizontal level. The speed of the aeroplane relative to the air is v ms^{-1} due north.

The air is blowing from south-west at 25 ms^{-1}.

Given the magnitude of the speed of the aeroplane relative to the ground is 180 ms^{-1}, determine the value of v.

$\boxed{v \approx 161 \text{ ms}^{-1}}$



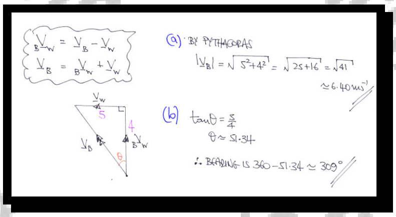
Question 2 (**)

As a boat moves, it travels at 4 ms^{-1} due north, relative to the water. The water is moving due west at 5 ms^{-1}.

a) Find the magnitude of the velocity of the boat relative to the ground.

b) Determine the bearing at which the boat is moving as viewed by a stationary observer outside the boat.

$\boxed{|v_B| = \sqrt{41} \approx 6.40 \text{ ms}^{-1}}$, $\boxed{\theta \approx 309°}$



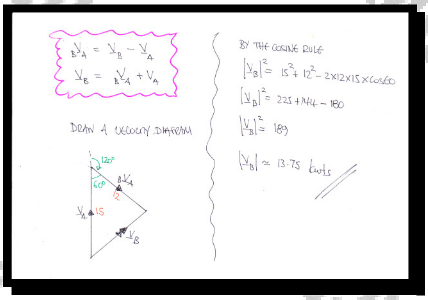
Question 3 (**)

A boat A is sailing at 15 knots due north.

To the captain of boat A another boat B is appearing to be sailing at 12 knots on a bearing of $120°$.

Determine the actual speed of B.

speed ≈ 13.75 knots



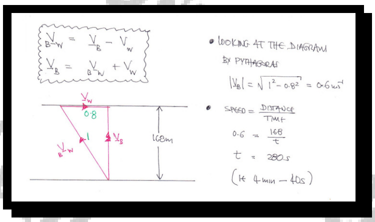
Question 4 (**)

A river which has parallel banks 168 m wide is flowing at constant speed 0.8 ms^{-1}.

A boy can capable of swimming at 1 ms^{-1} swims across at right angles to both banks.

Determine the time the boy takes to swim across the river.

280s



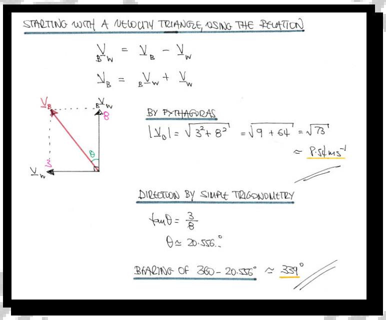
Question 5 (**)

William is standing on the observation platform of a tall lighthouse.

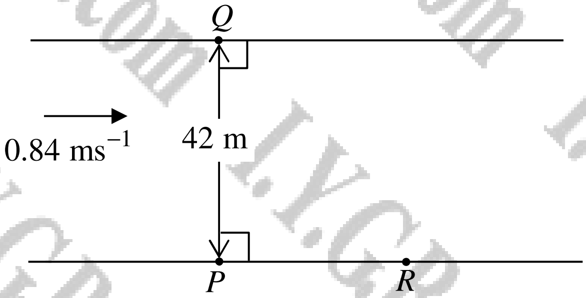
He is observing a boat is sailing through water, which flowing due west at 3 ms^{-1}. The velocity of the boat relative to the water is 8 ms^{-1}, due north.

Find the speed and direction, as a bearing, of the boat as it is observed by William.

$\boxed{}$, $\boxed{|v| \approx 8.54 \text{ ms}^{-1}}$, $\boxed{\theta \approx 339°}$



Question 6 (+)**



The banks of a river are modelled as parallel lines of constant width 42 m. The river flows with constant speed of 0.84 ms^{-1}, throughout its width. The points P and Q are on opposite river banks so that $PQ = 42 \text{ m}$, as shown in the figure above.

Alex and Bradley are swimmers, both capable of swimming relative to the water with a speed of 1.4 ms^{-1}.

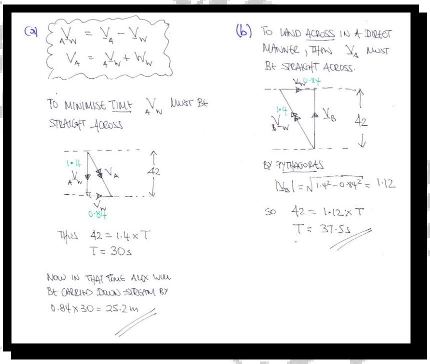
Alex sets from Q and decides to cross the river in the shortest possible time, and in doing so he reaches the opposite bank at the point R.

a) Calculate the distance PR.

Bradley sets from P and decides to cross the river directly across reaching the opposite bank at the point Q.

b) Calculate the time taken by Bradley to reach Q.

$\boxed{|PR| = 25.2 \text{ m}}$, $\boxed{t = 37.5 \text{ s}}$



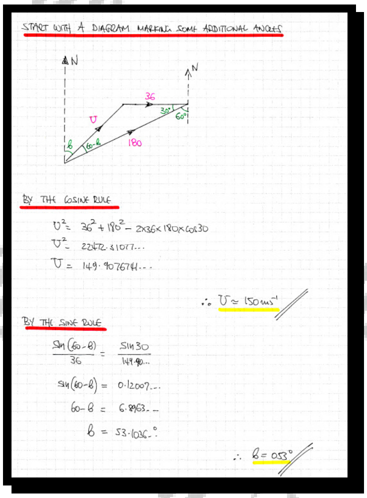
Question 7 (+)**

An aeroplane capable of speed U ms^{-1}, is flying with this speed on a bearing of β.

As observed from the ground, due to the cross wind this plane is flying with a speed of 180 ms^{-1}, on a bearing of $60°$.

If the cross wind is blowing from the west with a speed of 36 ms^{-1}, calculate the value of U and the value of β.

[illegible], $\boxed{U \approx 149.91}$, $\boxed{\beta \approx 053°}$

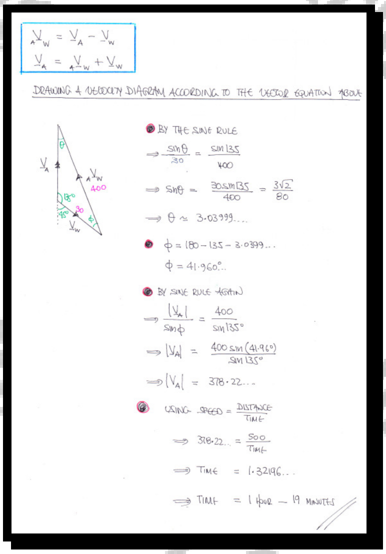


Question 8 (***)

An aeroplane is travelling at the same horizontal level and in a northerly direction relative to the ground. The speed of the aeroplane relative to the air is 400 km h^{-1}. The air is blowing from north-west at 30 km h^{-1}.

Determine the time, in hours and minutes, it takes the aeroplane to cover 500 km.

$t \approx 1 \text{ hour} - 19 \text{ minutes}$



Question 9 (***)

A bird is capable of flying at 70 km h^{-1}.

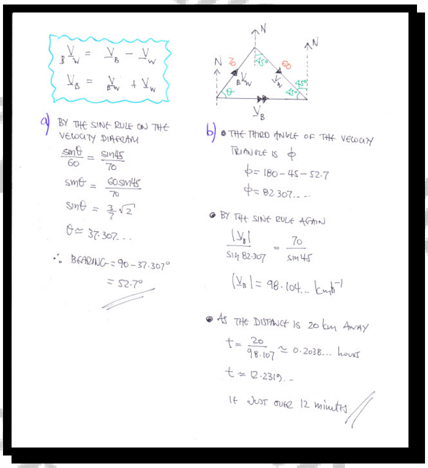
The bird wishes to fly to its next which 20 km due East from it current position.

There is a wind blowing from North-West at 60 km h^{-1}.

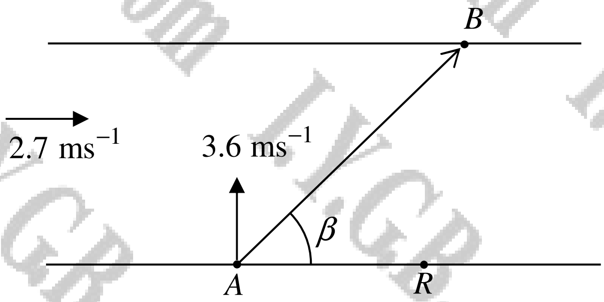
a) Find the direction, as a bearing, in which the bird must fly to reach its nest.

b) Calculate the time, in minutes, for its journey.

$\boxed{\approx 52.7°}$, $\boxed{\approx 12.23 \text{ min}}$



Question 10 (***)



The banks of a river are modelled as parallel lines of constant width. The river flows with constant speed of 2.7 ms^{-1}, throughout its width. A boat travels with constant velocity 3.6 ms^{-1} relative to the water, in a direction perpendicular to the river banks.

The boat starts at a point A on one river bank and ends up at a point B on the opposite river bank. The path AB forms an acute angle β with the river bank, as shown in the figure above.

a) Calculate …

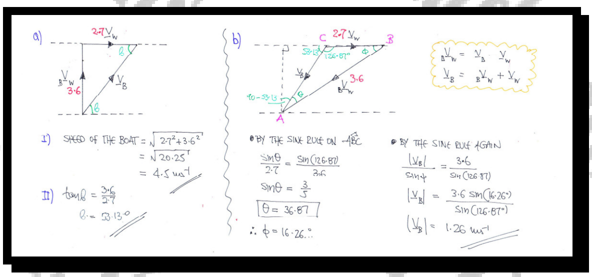
i. … the speed of the boat at it travels from A to B.

ii. … the value of β.

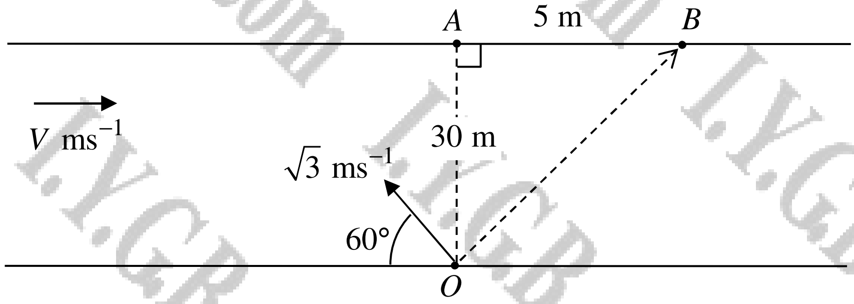
The boat returns from B to A, following exactly the same straight path.

b) Determine the velocity of the boat when rowed back from B to A, given that it is still rowed with a constant velocity 3.6 ms^{-1} relative to the water.

$\boxed{v = 4.5 \text{ ms}^{-1}}$, $\boxed{\beta = 53.13°}$, $\boxed{v = 1.26 \text{ ms}^{-1}}$



Question 11 (***)



A river flows with constant speed of $V\ \text{ms}^{-1}$, throughout its width. The banks of the river are modelled as parallel lines of constant width of $30\ \text{m}$.

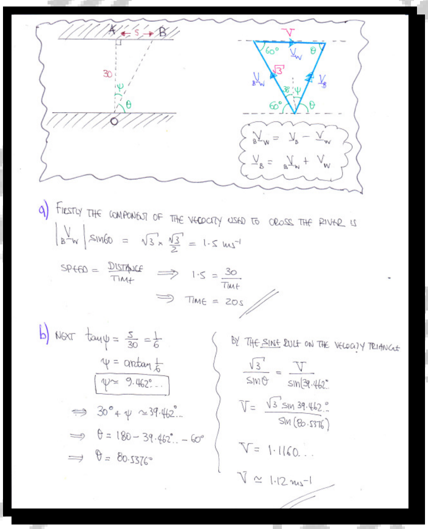
A boat starts at O and travels upstream with constant speed $\sqrt{3}\ \text{ms}^{-1}$ relative to the water, in a direction of $60°$ to the river bank, as shown in the figure above. The point A is on the opposite river bank so that OA is perpendicular to both river banks.

The point B is on the same bank as A, so that $|AB| = 5\ \text{m}$, downstream. To an observer on one of the banks of the river the boat sails on a straight line from O to B.

a) Calculate the time it takes the boat to travel from O to B.

b) Determine the value of V, correct to two decimal places.

$\boxed{t = 20\ \text{s}}$, $\boxed{V = 1.12\ \text{ms}^{-1}}$



Question 12 (***)

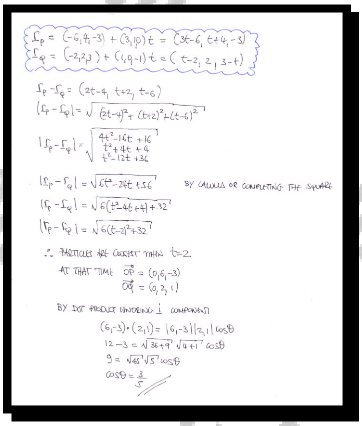
The vectors $\mathbf{i}$, $\mathbf{j}$ *and* $\mathbf{k}$ *are unit vectors mutually perpendicular to one another.*

At time $t = 0$ s, the respective position vectors of two particles P and Q, relative to a fixed origin O, are $(-6\mathbf{i}+4\mathbf{j}-3\mathbf{k})$ m and $(-2\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ m.

P has constant velocity $(3\mathbf{i}+\mathbf{j})$ ms^{-1} and Q has constant velocity $(\mathbf{i}-\mathbf{k})$ ms^{-1}.

Find the cosine of the angle POQ when the distance between P and Q is least.

$\boxed{\frac{3}{5}}$



$\mathbf{r}_P = (-6,4,-3) + (3,1,0)t = (3t-6, t+4, -3)$

$\mathbf{r}_Q = (-2,2,3) + (1,0,-1)t = (t-2, 2, 3-t)$

$\mathbf{r}_P - \mathbf{r}_Q = (2t-4, t+2, t-6)$

$|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{(2t-4)^2 + (t+2)^2 + (t-6)^2}$

$|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{4t^2-16t+16 + t^2+4t+4 + t^2-12t+36}$

$|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6t^2-24t+56}$ BY CALCULUS OR COMPLETING THE SQUARE

$|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6(t^2-4t+4)+32}$

$|\mathbf{r}_P - \mathbf{r}_Q| = \sqrt{6(t-2)^2+32}$

∴ PARTICLES ARE CLOSEST WHEN $t=2$

AT THAT TIME $\overrightarrow{OP} = (0,6,-3)$

$\overrightarrow{OQ} = (0,2,1)$

BY DOT PRODUCT IGNORING $\mathbf{i}$ COMPONENT

$(6,-3)\cdot(2,1) = |6,-3||2,1|\cos\theta$

$12-3 = \sqrt{36+9}\sqrt{4+1}\cos\theta$

$9 = \sqrt{45}\sqrt{5}\cos\theta$

$\cos\theta = \frac{3}{5}$

Question 13 (***)

Two straight horizontal roads meet at right angles at a junction O.

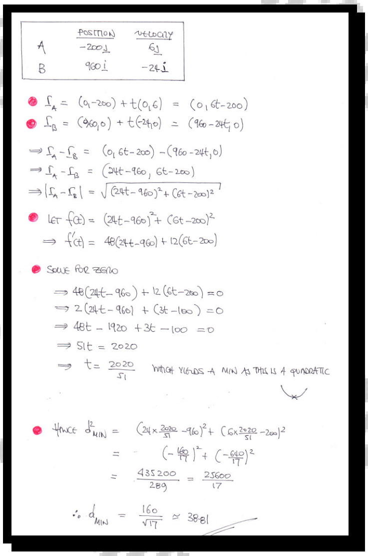
One of the roads is directed south to north and the other west to east.

A cyclist is travelling north on the first road at constant speed 6 ms^{-1} and at time $t = 0$ s is 200 m south of O.

A car is travelling west on the second road at constant speed 24 ms^{-1} and at time $t = 0$ s is 960 m east of O.

Determine the shortest distance between the cyclist and the car if they continue to move at the above described fashion.

$$d_{\text{min}} = \frac{160}{\sqrt{17}} \approx 38.81...$$



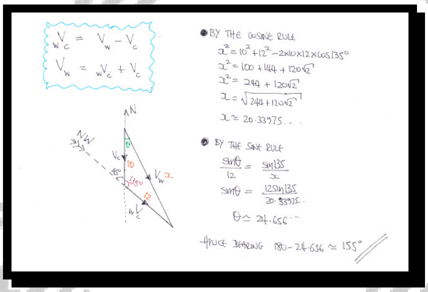
Question 14 (***)

A cyclist is travelling in a southerly direction at a constant speed 10 km h^{-1}, on a straight horizontal road.

To the cyclist the wind appear to be blowing from the north-west with a constant horizontal speed of 12 km h^{-1}.

Determine, as a three figure bearing, the direction from which the wind is blowing.

$\boxed{155°}$



Question 15 (***)

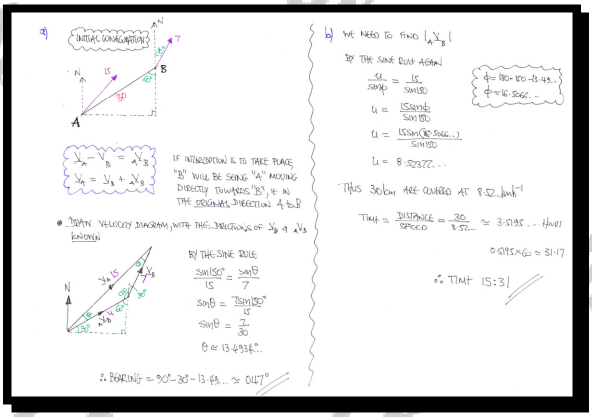
At noon, two ships A and B are 30 km apart, with A on a bearing of $240°$ from B.

Ship B is moving at 7 km h^{-1} on a bearing of $030°$. The maximum speed of A is 15 km h^{-1}. Ship A sets a course to intercept B as soon as possible.

a) Find the course set by A, giving the answer as a bearing to the nearest degree.

b) Determine the time at which A intercepts B.

$\boxed{\approx 047°}$, $\boxed{15:31}$



Question 16 (***)

An airplane flying at 600 km h^{-1} in still air travels directly from A to B.

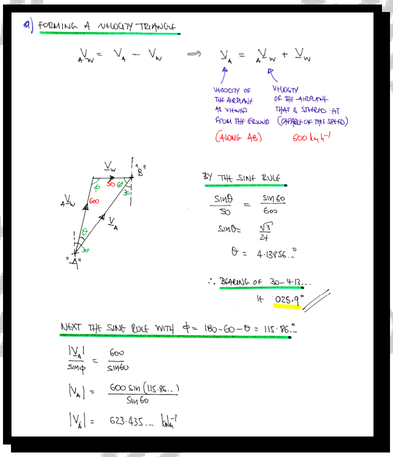
The point B is 700 km away from A, on a bearing $030°$ from A.

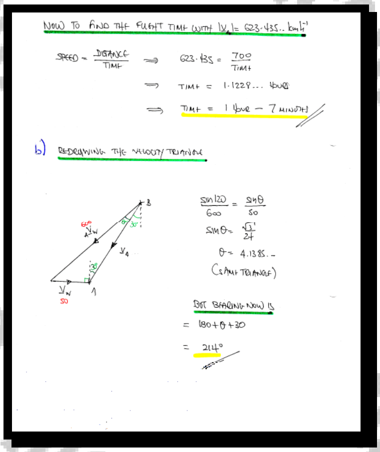
There is a steady wind, blowing from the west with a speed of 50 km h^{-1}, throughout the flight.

a) Determine, as a bearing, the course the pilot should steer the airplane in order to travel directly from A to B, and hence calculate the flight time.

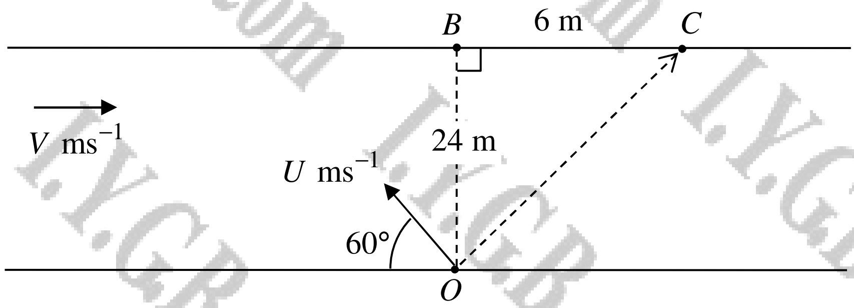
b) Find, as a bearing, the course the pilot should steer the airplane in order to travel directly from B to A, assuming the wind is blowing steadily from the west with the same speed.

[illegible], 025.9°, 1 hour, 7 minutes, 234°





Question 17 **(***)**



A river flows with constant speed of V ms^{-1}, throughout its width. The banks of the river are modelled as parallel lines of constant width of 24 m.

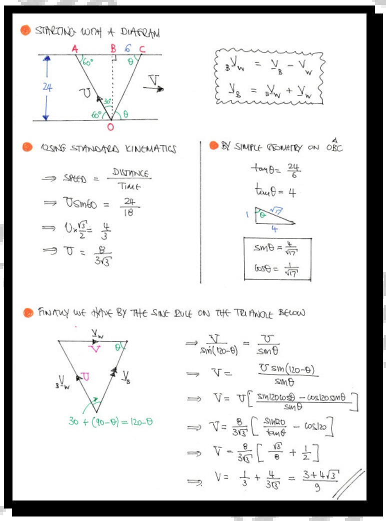
A boat starts at O and travels upstream with constant speed U ms^{-1} relative to the water, in a direction of 60° to the river bank, as shown in the figure above. The point B is on the opposite river bank so that OB is perpendicular to both river banks.

The point C is on the same bank as B, so that $|BC| = 6$ m, downstream.

To a stati0nary observer on one of the banks of the river the boat sails on a straight line from O to C.

Show that $V = \frac{1}{9}\left(3 + 4\sqrt{3}\right)$.

[ ], proof



Question 18 (***+)

At 14.00 hours a coastguard patrol ship first sights an unidentified boat, 12 km away on a bearing of $210°$.

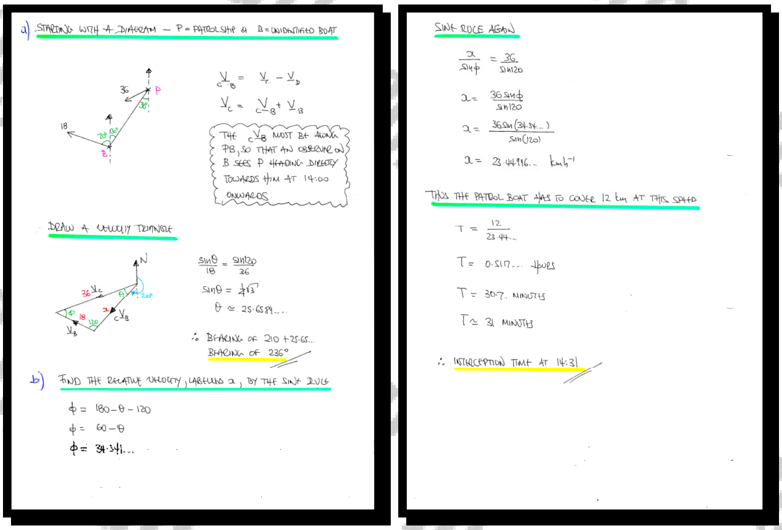
The unidentified boat is sailing at 18 km h^{-1} on a bearing of $290°$.

The coastguard patrol ship is sailing at 36 km h^{-1}.

a) Find, as a bearing, the course at which the coastguard patrol ship should steer in order to intercept the unidentified boat.

b) Calculate the time at which the interception will take place.

$\boxed{}$, $\boxed{\approx 236°}$, $\boxed{14:31}$


a) STARTING WITH A DIAGRAM — P = PATROL SHIP & B = UNIDENTIFIED BOAT

$_C\underline{V}_B = \underline{V}_C - \underline{V}_B$

$\underline{V}_C = {_C\underline{V}_B} + \underline{V}_B$

THE $_C\underline{V}_B$ MUST BE ALONG PB, SO THAT AN OBSERVER ON B SEES P HEADING DIRECTLY TOWARDS HIM AT 14:00 ONWARDS

DRAW A VELOCITY TRIANGLE

$\frac{\sin\theta}{18} = \frac{\sin 120}{36}$

$\sin\theta = \frac{1}{4}\sqrt{3}$

$\theta \simeq 25.6589...$

∴ BEARING OF 210 + 25.65...

BEARING OF 236°

b) FIND THE RELATIVE VELOCITY, LABELLED x, BY THE SINE RULE

$\phi = 180 - \theta - 120$

$\phi = 60 - \theta$

$\phi = 34.341...$

SINE RULE AGAIN

$\frac{x}{\sin\phi} = \frac{36}{\sin 120}$

$x = \frac{36\sin\phi}{\sin 120}$

$x = \frac{36\sin(34.34...)}{\sin(120)}$

$x = 23.44996... \text{ km h}^{-1}$

THUS THE PATROL BOAT HAS TO COVER 12 km AT THIS SPEED

$T = \frac{12}{23.44...}$

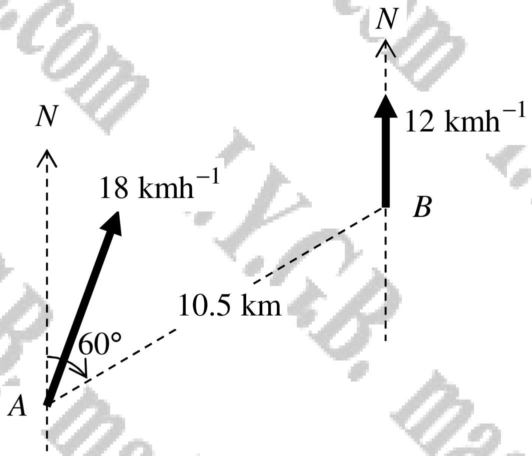
$T = 0.517...$ HOURS

$T = 30.7..$ MINUTES

$T \simeq 31$ MINUTES

∴ INTERCEPTION TIME AT 14:31

Question 19 (***+)



At noon two boats A and B are 10.5 km apart with B on a bearing $060°$ from A.

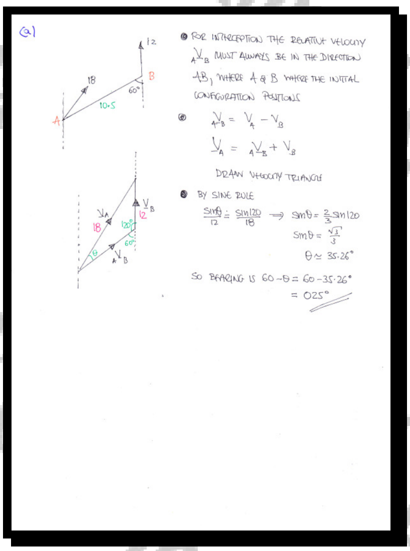
Boat B is travelling due North with a constant speed of 12 kmh^{-1}. Boat A is capable of a maximum speed of 18 kmh^{-1} and sets on a course to intercept A.

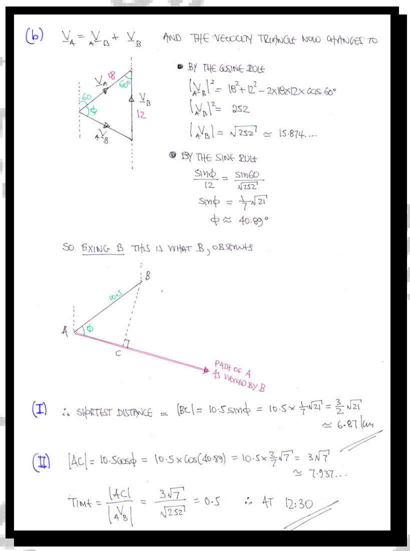
a) Calculate the bearing at which A must travel in order to intercept A in the least possible time.

b) Given instead that A travels on a bearing of $060°$ determine …

- **i.** … the closest distance between A and B.
- **ii.** … the time it takes for the two boats to get closest together.

$\boxed{\approx 025°}$, $\boxed{d_{\text{min}} = \frac{3}{2}\sqrt{21} \approx 6.87 \text{ km}}$, $\boxed{12:30}$





Question 20 (***+)

The radar of a battleship detects a destroyer, 50 km North of the battleship.

The destroyer is moving on a bearing of $120°$ with constant speed 40 km h^{-1}.

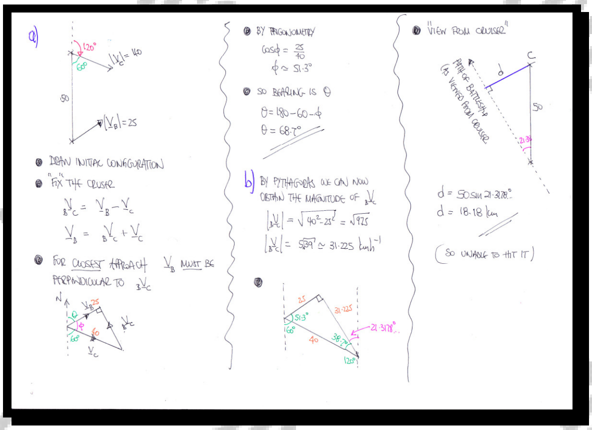
The maximum speed of the battleship is 25 km h^{-1} and on detecting the destroyer, it heads on a bearing $\theta°$ with maximum speed, in order to get as close as possible to the destroyer.

a) Find the value of θ.

The guns of the battleship have a range of 10 km.

b) Determine whether the destroyer gets within the range of the battleship's guns.

$\boxed{\theta \approx 68.7°}$, $\boxed{d_{\text{min}} \approx 18.2 \text{km} > 10 \text{km}}$



Question 21 (*+)**

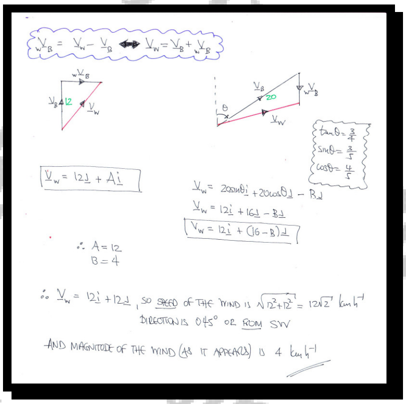
When a boat is sailing due North with constant speed 12 km h^{-1}, the wind appears to the crew on the boat to be blowing from the West.

The boat increases its speed to 20 km h^{-1} and changes its direction to a bearing $\theta°$, where $\tan\theta = \frac{3}{4}$. The wind now appears to the crew on the boat to be blowing from the North.

Assuming the true velocity of the wind is the same throughout the boat's journey determine in any order …

- … the **true** speed of the wind.
- … the **true** direction of the wind.
- … the **apparent** speed of the wind when the boat is sailing at 20 km h^{-1} on a $\theta°$ bearing.

$$\boxed{|v_w| = 12\sqrt{2} \approx 17.0 \text{ km h}^{-1}}, \quad \boxed{045° \text{ or from SW}}, \quad \boxed{|v_w| = 4 \text{ km h}^{-1}}$$



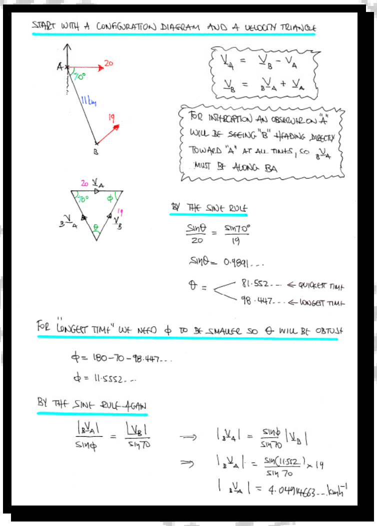
Question 22 (***+)

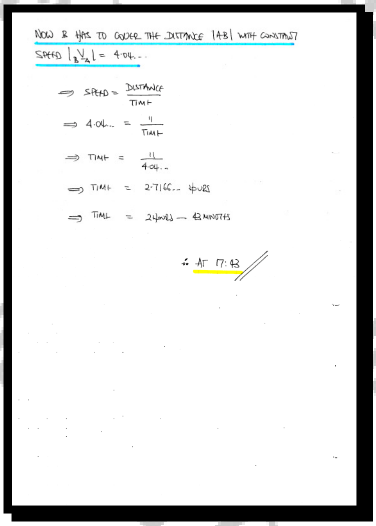
A ship A is sailing due east with constant speed 20 km h^{-1}.

At 15.00 hours, another ship B is 11 km away on a bearing of $160°$ from A.

Find the latest time by which B can intercept A, assuming that B will set on such course with constant speed 19 km h^{-1}.

$\boxed{}$, $\boxed{17:43}$





Question 23 (***+)

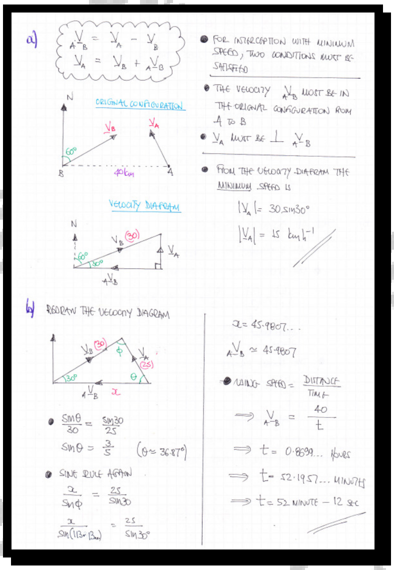
A ship B is moving on a bearing $060°$ at constant speed 30 km h^{-1}.

Another ship A moving with constant speed $V \text{ km h}^{-1}$ sets on a course to intercept B, when A gets to a position 40 km east of B.

a) Find the minimum value of V, required for interception.

b) Given further that $V = 25$, determine the time it takes A to intercept B.

$\boxed{52' - 12''}$



Question 24 (***+)

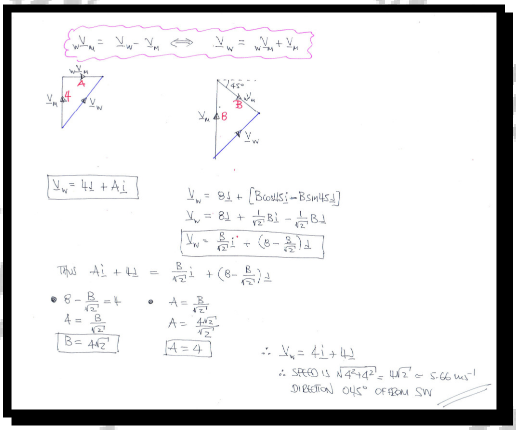
When a jogger is running due North with constant speed 4 ms^{-1}, the wind appears to him to be blowing from the West.

When the jogger is running due North with constant speed 8 ms^{-1}, the wind appears to him to be blowing from the North West.

Assuming the true velocity of the wind is the same throughout the joggers run, determine in any order …

- … the **true** speed of the wind.
- … the **true** direction of the wind.

$\boxed{|v_w| = 4\sqrt{2} \approx 5.66 \text{ ms}^{-1}}$, $\boxed{045° \text{ or from SW}}$



Question 25 (***+)

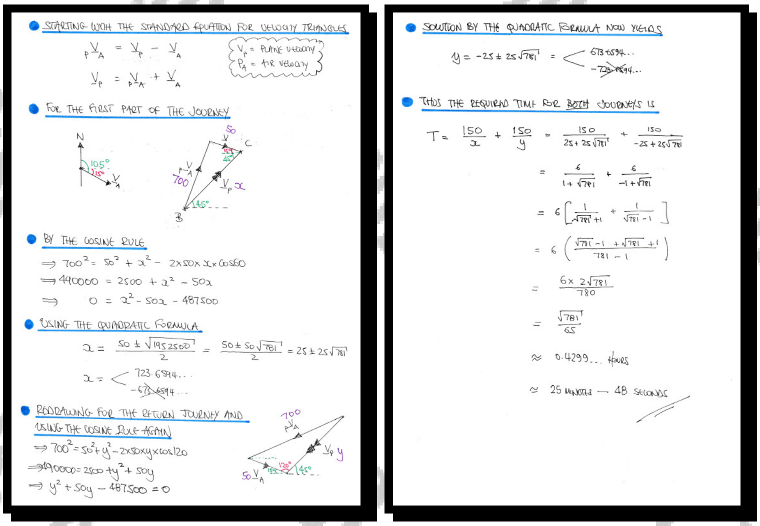
A coastal base C is on bearing $045°$ from an army airport base B, 150 km away.

As part of a training exercise, a plane leaves B on a direct path to C. On reaching C, the plane immediately returns directly back to B, along the same path.

The plane is flying with constant speed 700 km h^{-1} relative to the air. During the entire flight there is a wind blowing from a bearing of $105°$, with speed 50 km h^{-1}.

Determine the flight time in minutes and seconds.

$\boxed{}$, $\boxed{25 \text{ minutes } - 45 \text{ seconds}}$



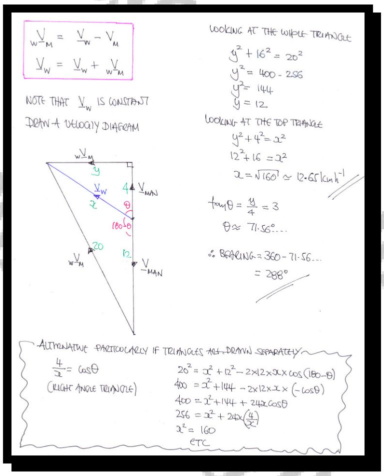
Question 26 (***+)

When a man is walking due North with constant speed 4 km h^{-1}, the wind appears to him to be blowing from the East.

When the man is jogging due South with constant speed 12 km h^{-1}, the wind appears to him to be blowing with constant speed 20 km h^{-1}.

Assuming the true velocity of the wind is the same throughout the man's walking and jogging, determine the true speed and direction of the wind.

$\boxed{|v_w| = 4\sqrt{10} \approx 12.65 \text{ km h}^{-1}}$, $\boxed{288°}$



Question 27 (***+)

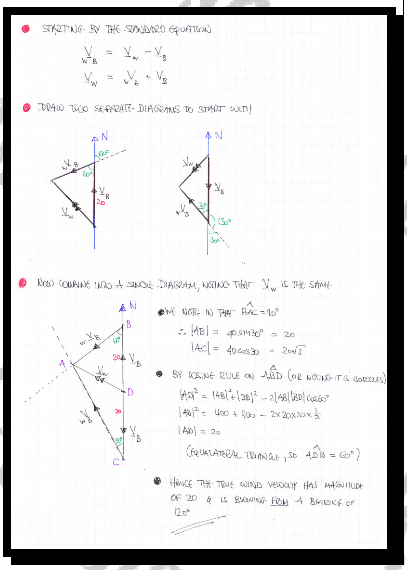
When a boat is sailing due North with constant speed 20 km h^{-1}, the wind appears to him to be blowing from a direction with bearing $060°$.

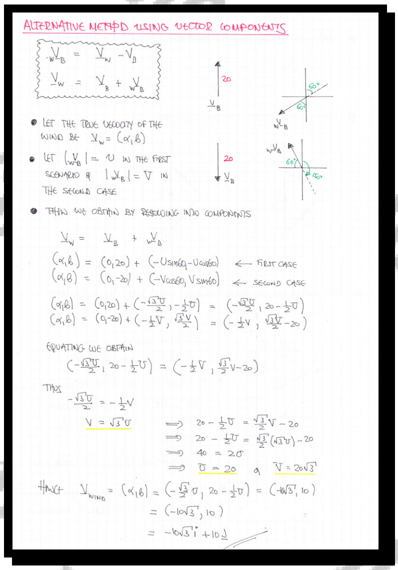
The boat turns around and starts sailing south with the same speed.

The wind now appears to be blowing from a direction with bearing $150°$.

Assuming the true velocity of the wind is the same for both parts of the journey, find the true velocity of the wind.

$\boxed{|v_w| = 20 \text{ ms}^{-1}}$, $\boxed{\text{from a bearing } 120°}$





Question 28 (***+)

The radar of a battleship detects a destroyer, 50 km west of the battleship.

The destroyer is moving on a bearing of $30°$ with constant speed 40 km h^{-1}.

The maximum speed of the battleship is 30 km h^{-1} and on detecting the destroyer, it heads on a bearing $\theta°$ with maximum speed, in order to get as close as possible to the destroyer.

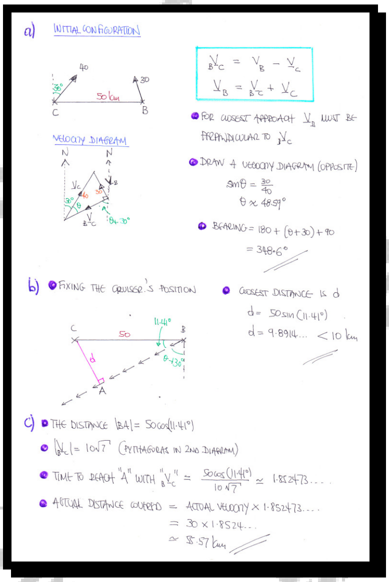
a) Find the value of θ.

The guns of the battleship have a range of 10 km.

b) Determine whether the destroyer gets within the range of the battleship's guns.

c) Calculate the actual distance the battleship covers from the instant it sets in pursuit of the destroyer until it gets as close to it.

$\boxed{\theta \approx 348.6°}$, $\boxed{d_{\text{min}} \approx 9.89 \text{ km} < 10 \text{ km}}$, $\boxed{\approx 55.57 \text{ km}}$



Question 29 (*+)**

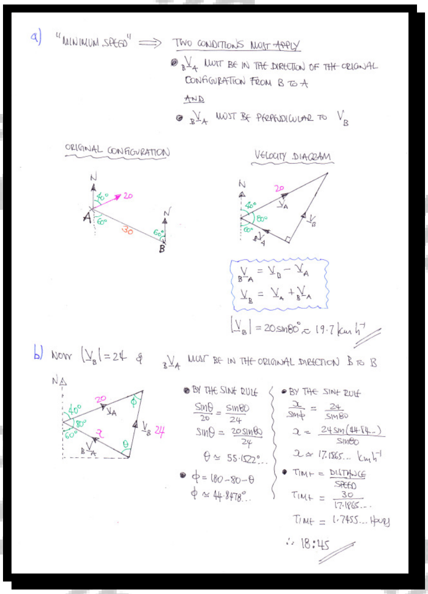
A ship A is travelling at a constant speed of 20 km h^{-1} on a bearing of $040°$.

At 17.00 hours, another ship B is 30 km away from A and the bearing of A from B is $300°$. Ship B is travelling at a constant speed of U km h^{-1} and sets on a course to intercept A.

a) Find the least possible value of U.

b) Given that $U = 24$, determine the earliest time at which B intercepts A.

$\boxed{U \approx 19.7}$, $\boxed{18:45}$



Question 30 (***+)

At time $t = 0$ two walkers A and B are 250 m apart with B due south of A. The park in which they are taking their walk is wide and its grounds are completely flat.

A is walking due east with constant speed 1.6 ms^{-1}.

B walks with constant speed 1.5 ms^{-1} in a straight line and in such a way so that he passes as close as possible to B.

a) Find, as a bearing, the direction of the path of B.

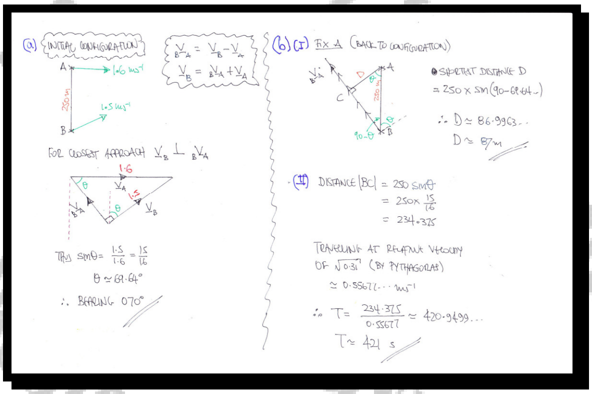
The two walkers are at their closest distance together, D m, at time T s.

b) Calculate, in any order, …

i. … the value of D.

ii. … the value of T.

$\boxed{\approx 070°}$, $\boxed{D \approx 87.0 \text{ m}}$, $\boxed{T \approx 421 \text{ s}}$



Question 31 **(***+)**

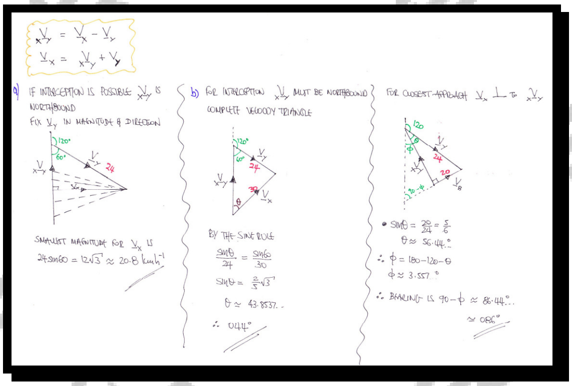
A yacht Y is moving with constant speed 24 km h^{-1} on a straight line course of bearing $120°$. At a given instant, a patrol boat X, is due south of Y and sets on a straight line course with constant speed $U \text{ km h}^{-1}$ to intercept Y.

a) Calculate the minimum value of U so that X can intercept Y.

b) Given that $U = 30$ determine the bearing that X must move on so that interception takes place.

c) Given instead that $U = 10$ find the bearing that X must move on so that it passes as close as possible to Y.

$\boxed{U_{\text{min}} = 12\sqrt{3} \approx 20.8 \text{ km h}^{-1}}$, $\boxed{\approx 044°}$, $\boxed{\approx 086°}$



Question 32 (***+)

A patrol boat is due south of a fishing trawler. The trawler is sailing at constant speed of 12 km h^{-1} on a bearing $120°$.

The patrol boat decides to intercept the trawler and travels in a straight line with constant speed $U \text{ km h}^{-1}$.

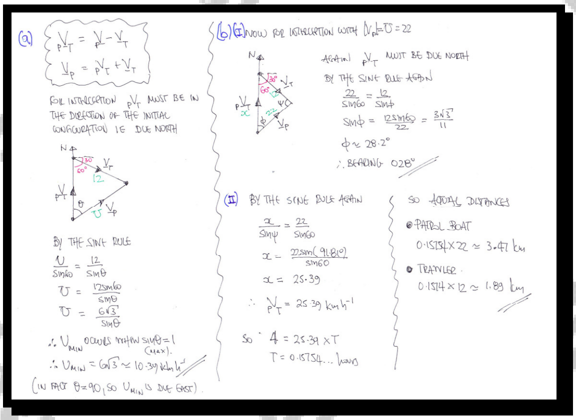
a) Find the minimum value of U.

b) Given instead that $U = 22$, determine …

  i. … the bearing of the course that the patrol must take to intercept the trawler.

  ii. … the distance the trawler and the distance the patrol boat cover until the interception takes place.

$\boxed{U_{\text{min}} = 6\sqrt{3} \approx 10.39 \text{ km h}^{-1}}$, $\boxed{\approx 028°}$, $\boxed{D_P \approx 3.47 \text{ km}}$, $\boxed{D_T \approx 1.89 \text{ km}}$



Question 33 (***+)

At noon a frigate is 18 km away from a ship and at that time the bearing of the frigate relative to the ship is $120°$.

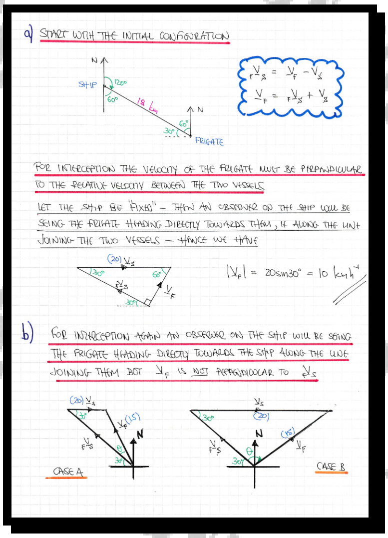
The ship is sailing east at a constant speed of 20 km h^{-1}.

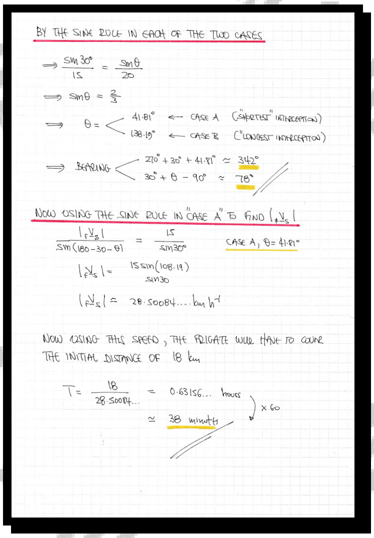
a) Determine the minimum speed with which the frigate can intercept the ship.

The frigate sets off to intercept the ship by sailing at a constant speed of 15 km h^{-1}.

b) Calculate, to the nearest degree, the two possible bearings which the frigate can follow, and hence find the shorter of the two possible interception times, correct to the nearest minute.

$V_{\text{min}} = 10 \text{ km h}^{-1}$, $\theta_1 \approx 78°$, $\theta_2 \approx 342°$, $t \approx 38$ minutes





Question 34 (***+)

A ship B is travelling due east at a constant speed of 15 km h^{-1}.

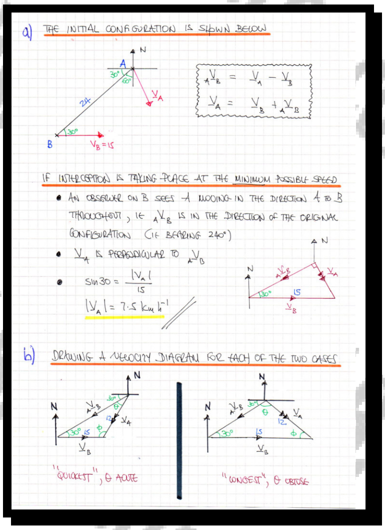
At midnight, another ship A is 24 km away from B so that the bearing of B from A is $240°$. Ship B is travelling at a constant speed of U km h^{-1} and sets on a course to intercept A.

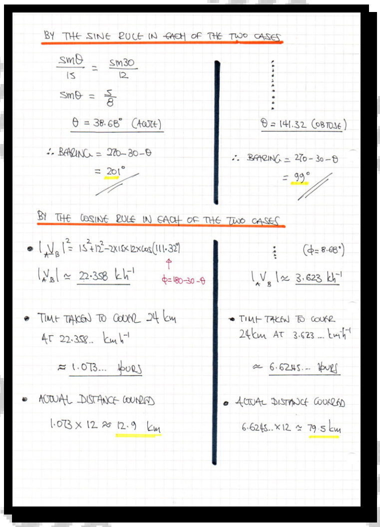
a) Find the least possible value of U.

b) Given that $U = 12$, determine the two possible bearings at which A can sail so it can intercept B.

Determine the actual distance covered by A in each of these two cases.

$\boxed{U = 7.5}$, $\boxed{\theta \approx 099°, \quad d \approx 79.5 \text{ km}}$, $\boxed{\theta \approx 201°, \quad d \approx 12.9 \text{ km}}$





Question 35 (****)

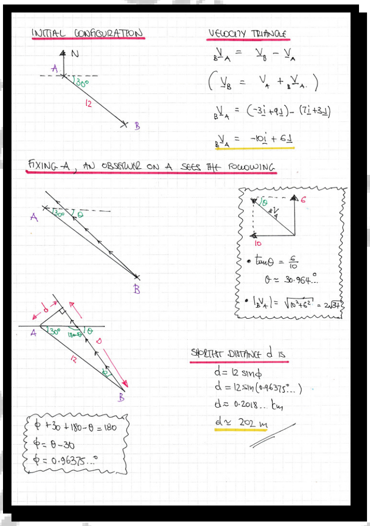
The unit vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are oriented due east and due north, respectively.*

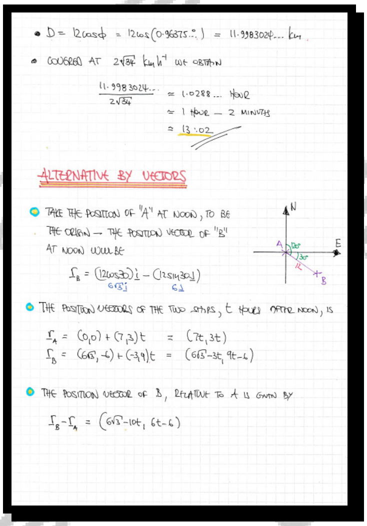
Two boats, A and B, are moving in the open sea with velocities $(7\mathbf{i}+3\mathbf{j})\ \text{km h}^{-1}$ and $(-3\mathbf{i}+9\mathbf{j})\ \text{km h}^{-1}$, respectively.

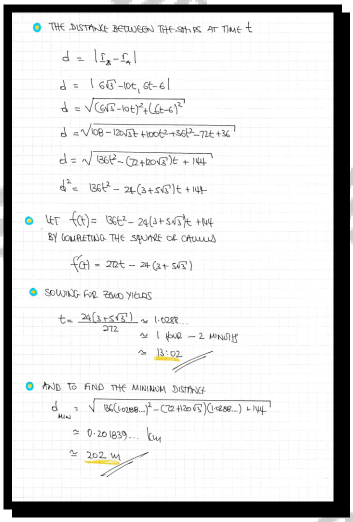
At noon, B is on a bearing of $120°$ from A, 12 km away.

Calculate, correct to the nearest m, the closest distance between the two boats and the time when they are at that closest distance.

, $d \approx 202$ m , 13:02







Question 36 (****)

At a certain instant, a ship A is sighted 35 km north of a fixed observation point O, sailing with constant speed 25 km h^{-1} on a bearing $045°$.

At the same instant another ship B is sighted 12 km east of O.

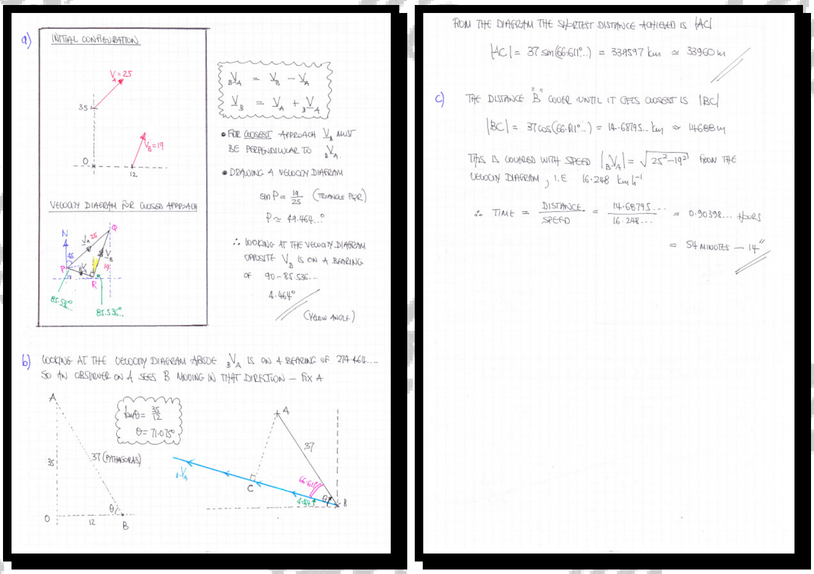
B sails with a maximum constant speed of 19 km h^{-1}, in a direction so that it passes as close as possible to A.

a) Determine, correct to three decimal places, the bearing at which B is sailing.

b) Find, correct to the nearest metre, the shortest distance between A and B.

c) Calculate, in minutes and seconds, the time B takes to pass closest to A.

$\boxed{\approx 4.464°}$, $\boxed{\approx 33960 \text{ m}}$, $\boxed{\approx 54'-14''}$



Question 37 (****)

On the radar screen of a plane A, an enemy aircraft B is observed on a bearing $120°$, 200 km way. The speed of the enemy craft is 1200 km h^{-1} on a bearing $330°$. The two aircrafts are at the same altitude.

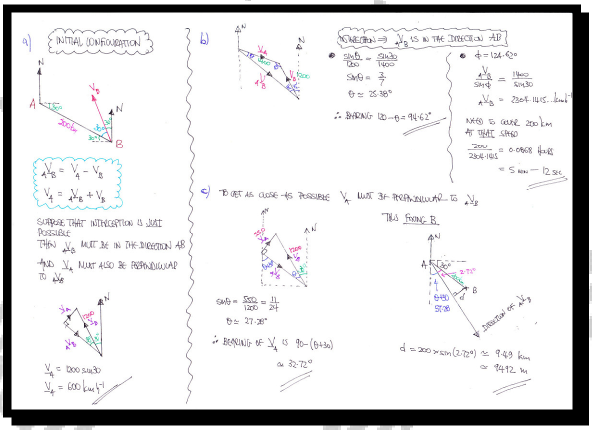
A immediately sets with constant speed V_A to intercept B.

a) Determine the minimum value of V_A which makes the interception possible.

b) Given that $V_A = 1400 \text{ km h}^{-1}$, determine the bearing A must follow in order to intercept B, in the shortest possible time T, and find the value of T, correct to the nearest second.

c) Given instead that $V_A = 550 \text{ km h}^{-1}$, determine the bearing A must follow in order to pass as close as possible to B, and find this closest distance correct to the nearest metre.

$\boxed{600 \text{ km h}^{-1}}$, $\boxed{94.62°}$, $\boxed{5'-12''}$, $\boxed{32.72°}$, $\boxed{9492 \text{ m}}$



Question 38 (****)

The vectors $\mathbf{i}$ *and* $\mathbf{j}$ *are unit vectors mutually perpendicular to one another.*

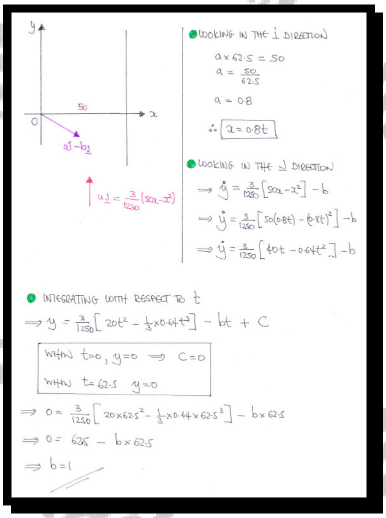
A man is about to swim across a river, starting at a fixed point O on one river bank to a point A in the opposite river bank. The position vector of A relative to O is $50\mathbf{i}$ m. The river flows parallel to $\mathbf{j}$ and the speed of the flow is given by

$$\frac{3}{125}\left(50x - x^2\right)\mathbf{j}\ \text{ms}^{-1}.$$

The man swims with constant velocity $(a\mathbf{i} - b\mathbf{j})\ \text{ms}^{-1}$, taking 62.5 s to reach A.

Determine the value of b.

$\boxed{b = 1}$



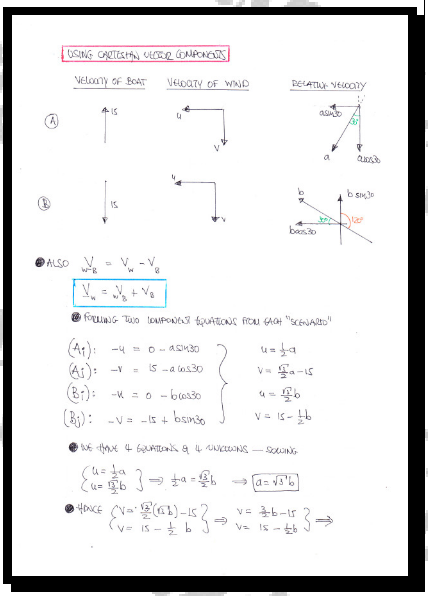
Question 39 (****)

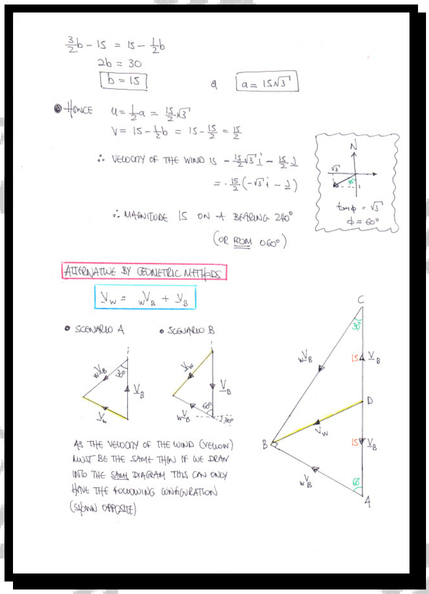
When a boat is sailing due North with constant speed 15 km h^{-1}, the wind appears to the crew on the boat to be blowing **from** the direction $030°$.

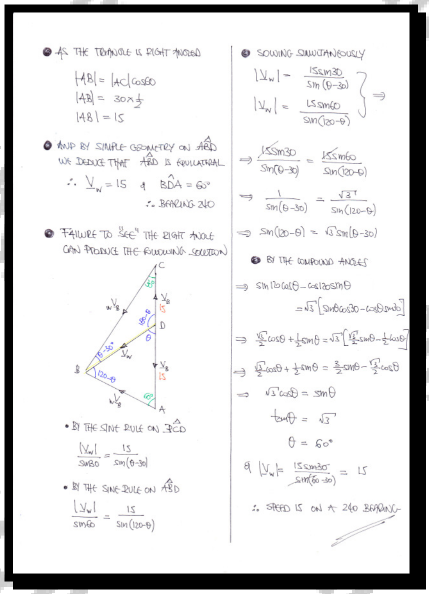
When another boat is sailing due South with constant speed 15 km h^{-1}, the wind appears to the crew on that boat to be blowing **from** the direction $120°$.

Assuming the true velocity of the wind is the same relative to the earth for both boat crews determine the velocity of the wind.

$$\mathbf{v}_w = -\frac{15}{2}\left(\sqrt{3}\mathbf{i}+\mathbf{j}\right) \quad \text{or} \quad |\mathbf{v}|_w = 15, \ \text{from bearing } 060°$$







Question 40 (****)

Three particles A, B and C are moving on a horizontal plane.

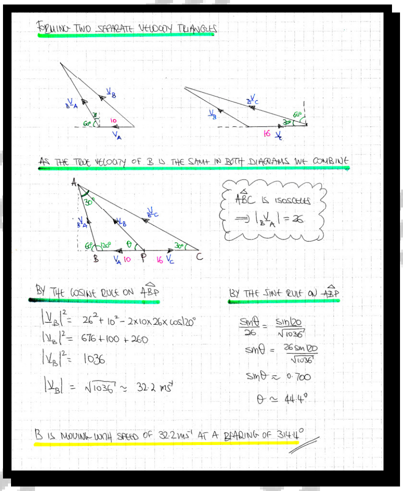
The speed of A is 10 ms^{-1} due west and the speed of C is 16 ms^{-1} due east.

Relative to A, B is moving on a bearing of $330°$.

Relative to C, B is moving on a bearing of $300°$.

Determine the speed and direction of motion of B.

$\boxed{}$, $\boxed{\approx 32.2 \text{ ms}^{-1} \text{ on a bearing } \approx 314.4°}$



Question 41 (****+)

Two particles A and B are moving on a horizontal plane with constant velocities.

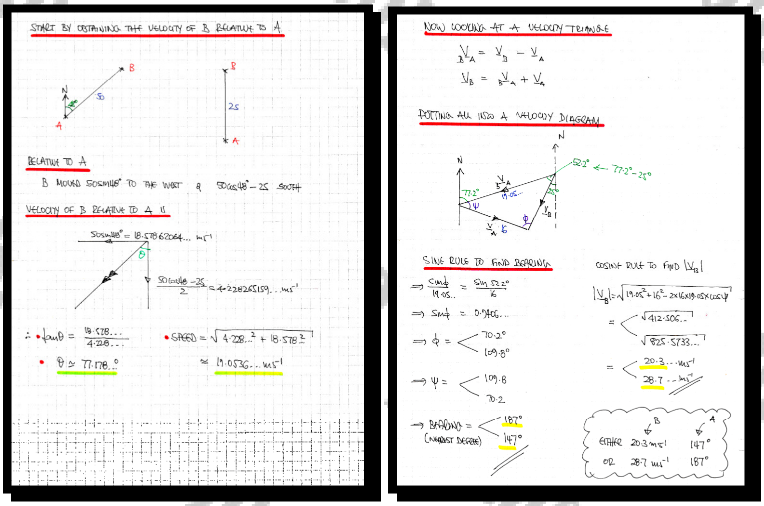
At a given instant B is on a bearing of $048°$ relative to A and the distance between A and B is 50 m.

The distance between A and B reduces to 25 m after 2 s with B due north from A.

It is further given that the actual speed of A is 16 ms^{-1} and B is moving on a bearing of $205°$.

Determine the two possible bearings in which A could be moving and the two possible speeds of B.

$$\left|\mathbf{v}_B\right| \approx 20.3 \text{ms}^{-1} \cap \mathbf{v}_A \text{ on a bearing} \approx 147° \cup \left|\mathbf{v}_B\right| \approx 28.7 \text{ms}^{-1} \cap \mathbf{v}_A \text{ on a bearing} \approx 187°$$



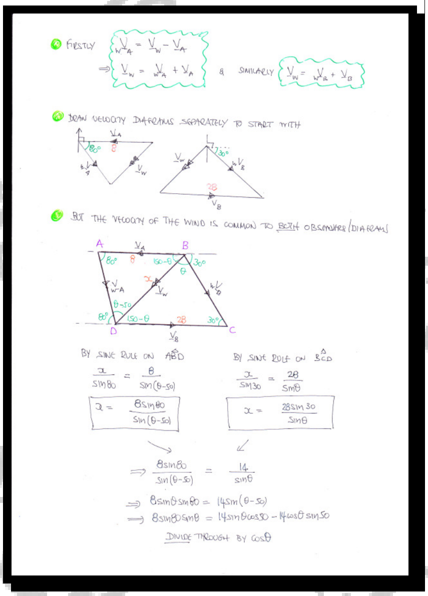
Question 42 (****+)

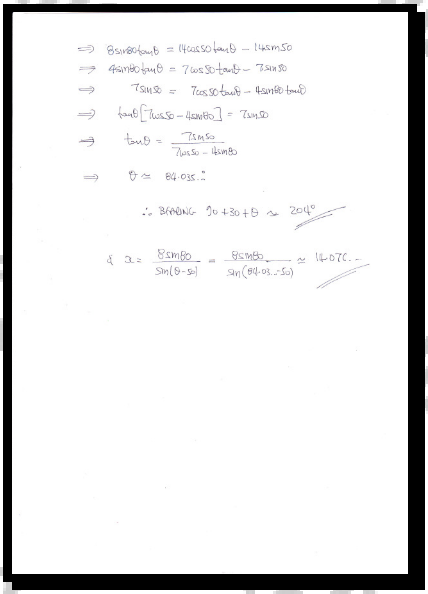
A ship A is sailing West at 8 mph and a ship B is sailing East at 28 mph.

To a man on A the wind appears to be blowing on a bearing of $170°$ and to a man on B the wind appears to be blowing on a bearing of $120°$.

Find the direction, as a bearing, and the speed of the wind.

$\boxed{\approx 204°}$, $\boxed{\approx 14.1 \text{ mph}}$





Question 43 (*****)

To a motorist P driving South on a level road with constant speed u, the wind appears to be blowing from a bearing $(90+\theta)°$.

To a motorist Q driving North on a level road with constant speed u, the wind appears to be blowing from a bearing $(90+\varphi)°$.

To a motorist R driving North on a level road with constant speed $2u$, the wind appears to be blowing from a bearing $(90+\psi)°$.

Assuming that the true speed and direction of the wind is the same for all three motorists, show that

$$2\tan\psi = 3\tan\varphi - \tan\theta .$$

proof