TRIGONOMETRIC EQUATIONS

Question 1

Solve each of the following trigonometric equations.

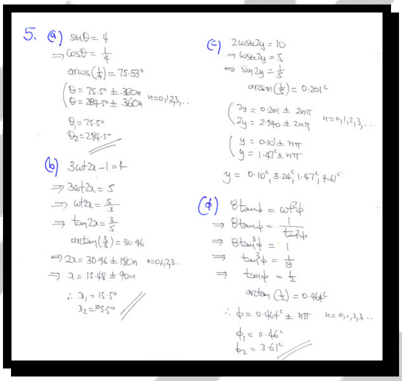
a) $\sec\theta = 4$, $\quad 0 \le \theta < 360°$

b) $3\cot 2x - 1 = 4$, $\quad 0 \le x < 180°$

c) $2\operatorname{cosec} 2y = 10$, $\quad 0 \le y < 2\pi$

d) $8\tan\varphi = \cot^2\varphi$, $\quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 75.5°,\ 284.5°}$, $\boxed{x \approx 15.5°,\ 105.5°}$, $\boxed{y \approx 0.10^{c},\ 1.47^{c},\ 3.24^{c},\ 4.61^{c}}$, $\boxed{\varphi \approx 0.46^{c},\ 3.61^{c}}$



Question 2

Solve each of the following trigonometric equations.

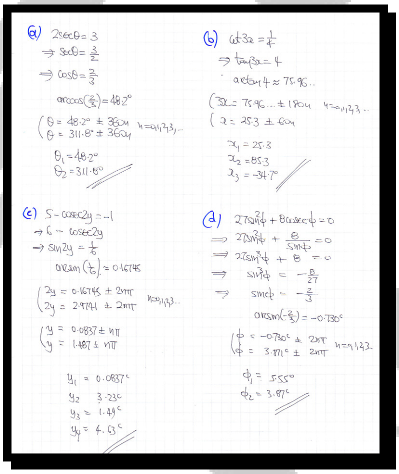
a) $2\sec\theta = 3, \quad 0 \le \theta < 360°$

b) $\cot 3x = \frac{1}{4}, \quad -90° \le x < 90°$

c) $5 - \operatorname{cosec} 2y = -1, \quad 0 \le y < 2\pi$

d) $27\sin^2\varphi + 8\operatorname{cosec}\varphi = 0, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 48.2°,\ 311.8°}$, $\boxed{x \approx -34.7°,\ 25.3°,\ 85.3°}$, $\boxed{y \approx 0.0837^c,\ 1.49^c,\ 3.23^c,\ 4.63^c}$, $\boxed{\varphi \approx 3.87^c,\ 5.55^c}$



Question 3

Solve each of the following trigonometric equations.

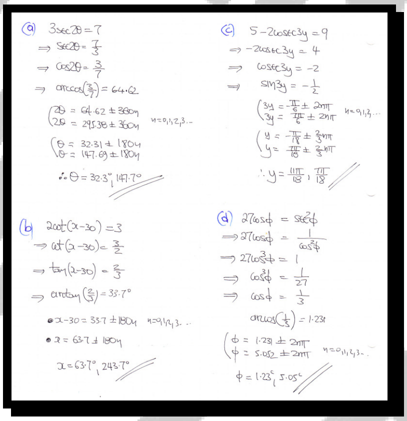
a) $3\sec 2\theta = 7$, $\quad 0 \le \theta < 180°$

b) $2\cot(x - 30°) = 3$, $\quad 0 \le x < 360°$

c) $5 - 2\operatorname{cosec} 3y = 9$, $\quad 0 \le y < \pi$

d) $27\cos\varphi = \sec^2\varphi$, $\quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 32.3°,\ 147.7°}$, $\boxed{x \approx 63.7°,\ 243.7°}$, $\boxed{y = \frac{7\pi}{18},\ \frac{11\pi}{18}}$, $\boxed{\varphi \approx 1.23^c,\ 5.05^c}$



(a) $3\sec 2\theta = 7$

$\Rightarrow \sec 2\theta = \frac{7}{3}$

$\Rightarrow \cos 2\theta = \frac{3}{7}$

$\Rightarrow \arccos\left(\frac{3}{7}\right) = 64.62$

$2\theta = 64.62 \pm 360n$, $2\theta = 295.38 \pm 360n$ $\quad n = 0,1,2,3\ldots$

$\theta = 32.31 \pm 180n$, $\theta = 147.69 \pm 180n$

$\therefore \theta = 32.3°, 147.7°$

(b) $2\cot(x-30) = 3$

$\Rightarrow \cot(x-30) = \frac{3}{2}$

$\Rightarrow \tan(x-30) = \frac{2}{3}$

$\Rightarrow \arctan\left(\frac{2}{3}\right) = 33.7°$

- $x - 30 = 33.7 \pm 180n$ $\quad n = 0,1,2,3\ldots$
- $x = 63.7 \pm 180n$

$x = 63.7°, 243.7°$

(c) $5 - 2\operatorname{cosec} 3y = 9$

$\Rightarrow -2\operatorname{cosec} 3y = 4$

$\Rightarrow \operatorname{cosec} 3y = -2$

$\Rightarrow \sin 3y = -\frac{1}{2}$

$3y = -\frac{\pi}{6} \pm 2n\pi$, $3y = \frac{7\pi}{6} \pm 2n\pi$ $\quad n = 0,1,2,3\ldots$

$y = -\frac{\pi}{18} \pm \frac{2}{3}n\pi$, $y = \frac{7\pi}{18} \pm \frac{2}{3}n\pi$

$\therefore y = \frac{11\pi}{18}, \frac{7\pi}{18}$

(d) $27\cos\phi = \sec^2\phi$

$\Rightarrow 27\cos\phi = \frac{1}{\cos^2\phi}$

$\Rightarrow 27\cos^3\phi = 1$

$\Rightarrow \cos^3\phi = \frac{1}{27}$

$\Rightarrow \cos\phi = \frac{1}{3}$

$\arccos\left(\frac{1}{3}\right) = 1.231$

$\phi = 1.231 \pm 2n\pi$, $\phi = 5.052 \pm 2n\pi$ $\quad n = 0,1,2,3\ldots$

$\phi = 1.23^c, 5.05^c$

Question 4

Solve each of the following trigonometric equations.

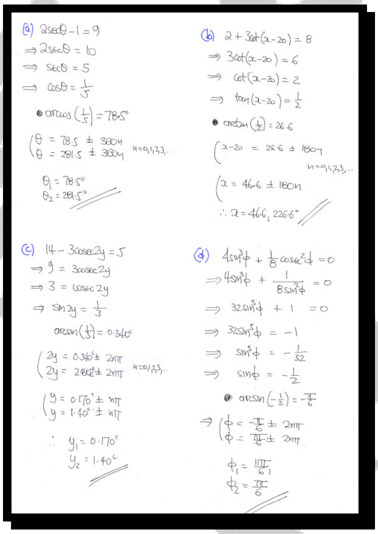
a) $2\sec\theta - 1 = 9\,, \qquad 0 \le \theta < 360°$

b) $2 + 3\cot\left(x - 20°\right) = 8\,, \qquad 0 \le x < 360°$

c) $14 - 3\operatorname{cosec} 2y = 5\,, \qquad 0 \le y < \pi$

d) $4\sin^3\varphi + \frac{1}{8}\operatorname{cosec}^2\varphi = 0\,, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 78.5°,\ 281.5°}$, $\boxed{x \approx 46.6°,\ 226.6°}$, $\boxed{y \approx 0.170^{\text{c}},\ 1.40^{\text{c}}}$, $\boxed{\varphi = \frac{7\pi}{6},\ \frac{11\pi}{6}}$



(a) $2\sec\theta - 1 = 9$

$\Rightarrow 2\sec\theta = 10$

$\Rightarrow \sec\theta = 5$

$\Rightarrow \cos\theta = \frac{1}{5}$

• $\arccos\left(\frac{1}{5}\right) = 78.5°$

$\theta = 78.5 \pm 360n$, $\theta = 281.5 \pm 360n$, $n = 0,1,2,3,\ldots$

$\theta_1 = 78.5°$, $\theta_2 = 281.5°$

(b) $2 + 3\cot(x - 20) = 8$

$\Rightarrow 3\cot(x - 20) = 6$

$\Rightarrow \cot(x - 20) = 2$

$\Rightarrow \tan(x - 20) = \frac{1}{2}$

• $\arctan\left(\frac{1}{2}\right) = 26.6$

$x - 20 = 26.6 \pm 180n$, $n = 0,1,2,3,\ldots$

$x = 46.6 \pm 180n$

$\therefore x = 46.6,\ 226.6°$

(c) $14 - 3\operatorname{cosec}2y = 5$

$\Rightarrow 9 = 3\operatorname{cosec}2y$

$\Rightarrow 3 = \operatorname{cosec}2y$

$\Rightarrow \sin 2y = \frac{1}{3}$

$\arcsin\left(\frac{1}{3}\right) = 0.340^c$

$2y = 0.340^c \pm 2n\pi$, $2y = 2.802^c \pm 2n\pi$, $n = 0,1,2,3,\ldots$

$y = 0.170^c \pm n\pi$, $y = 1.40^c \pm n\pi$

$\therefore y_1 = 0.170^c$, $y_2 = 1.40^c$

(d) $4\sin^3\phi + \frac{1}{8}\operatorname{cosec}^2\phi = 0$

$\Rightarrow 4\sin^3\phi + \frac{1}{8\sin^2\phi} = 0$

$\Rightarrow 32\sin^5\phi + 1 = 0$

$\Rightarrow 32\sin^5\phi = -1$

$\Rightarrow \sin^5\phi = -\frac{1}{32}$

$\Rightarrow \sin\phi = -\frac{1}{2}$

• $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\Rightarrow \phi = -\frac{\pi}{6} \pm 2n\pi$, $\phi = \frac{7\pi}{6} \pm 2n\pi$

$\phi_1 = \frac{11\pi}{6}$, $\phi_2 = \frac{7\pi}{6}$

Question 5

Solve each of the following trigonometric equations.

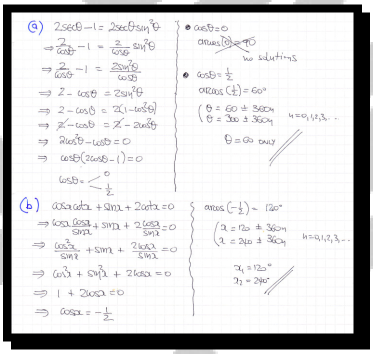
a) $2\sec\theta - 1 = 2\sec\theta\sin^2\theta$, $\quad 0 \le \theta < 180°$, $\theta \ne 90°$

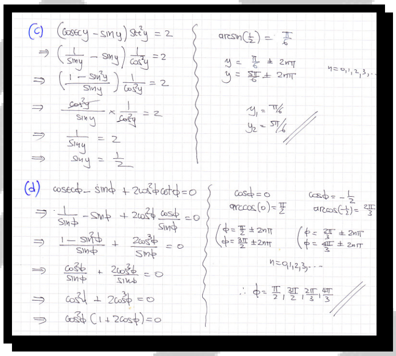
b) $\cos x\cot x + \sin x + 2\cot x = 0$, $\quad 0 < x < 360°$, $x \ne 180°$

c) $(\operatorname{cosec} y - \sin y)\sec^2 y = 2$, $\quad 0 \le y < \pi$, $y \ne \dfrac{\pi}{2}$

d) $\operatorname{cosec}\varphi - \sin\varphi + 2\cos^2\varphi\cot\varphi = 0$, $\quad 0 < \varphi < 2\pi$, $\varphi \ne \pi$

$\boxed{\theta = 60°}$, $\boxed{x = 120°, 240°}$, $\boxed{y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}}$, $\boxed{\varphi = \dfrac{\pi}{2}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{3\pi}{2}}$





Question 6

Solve each of the following trigonometric equations.

a) $\sec\theta + \cos\theta = \frac{5}{2}, \quad 0 \le \theta < 360°, \quad \theta \ne 90°, 270°$

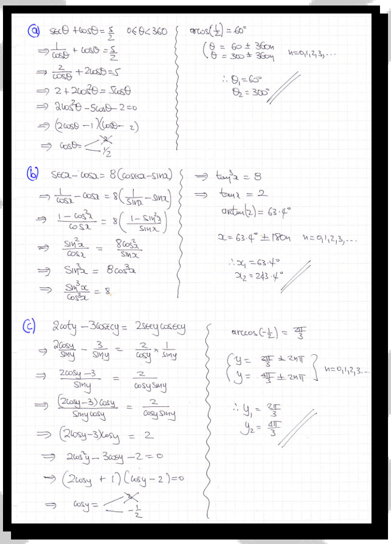
b) $\sec x - \cos x = 8(\operatorname{cosec} x - \sin x), \quad 0 \le x < 360°, \quad x \ne 90°$

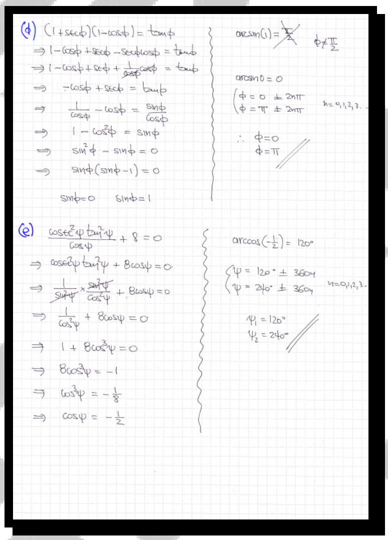
c) $2\cot y - 3\operatorname{cosec} y = 2\sec y \operatorname{cosec} y, \quad 0 < y < 2\pi, \quad y \ne \frac{k\pi}{2}, k \in \mathbb{Z}$

d) $(1+\sec\varphi)(1-\cos\varphi) = \tan\varphi, \quad 0 \le \varphi < 2\pi, \quad \varphi \ne \frac{\pi}{2}, \frac{3\pi}{2}$

e) $\frac{\operatorname{cosec}^2\psi \tan^2\psi}{\cos\psi} + 8 = 0, \quad 0 \le \psi < 360°, \quad \psi \ne 90°, 270°$

$\boxed{\theta = 60°, 300°}$, $\boxed{x = 63.4°, 243.6°}$, $\boxed{y = \frac{2\pi}{3}, \frac{4\pi}{3}}$, $\boxed{\varphi = 0, \pi}$, $\boxed{\psi = 120°, 240°}$





Question 7 **(hard questions)**

Solve each of the following trigonometric equations.

a) $2\sin\theta + 3\sec\theta = 6 + \tan\theta$, $\quad 0 \le \theta < 2\pi$, $\theta \ne \frac{\pi}{2}, \frac{3\pi}{2}$

b) $\sin^2 x \tan x + \cos^2 x \cot x + 2\sin x \cos x = 2$, $\quad 0 < x < 360°$, $x \ne 90°, 180°, 270°$

you may use in this part the fact that $\boxed{2\sin x \cos x \equiv \sin 2x}$

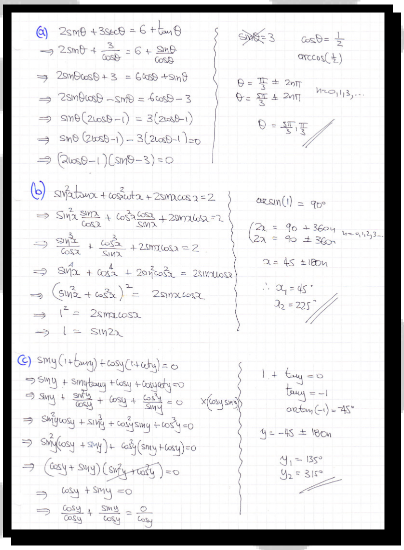
c) $\sin y(1 + \tan y) + \cos y(1 + \cot y) = 0$, $0 < y < 360°$, $y \ne 90°, 180°, 270°$

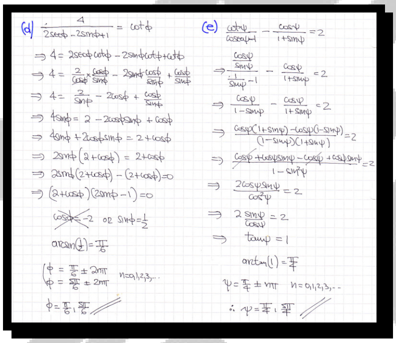
d) $\dfrac{4}{2\sec\varphi - 2\sin\varphi + 1} = \cot\varphi$, $\quad 0 < \varphi < 2\pi$, $\varphi \ne \pi$

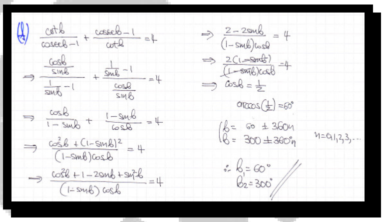
e) $\dfrac{\cot\psi}{\text{cosec}\,\psi - 1} - \dfrac{\cos\psi}{1 + \sin\psi} = 2$, $\quad 0 < \psi < 2\pi$, $\psi \ne \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

f) $\dfrac{\cot\beta}{\text{cosec}\,\beta - 1} + \dfrac{\text{cosec}\,\beta - 1}{\cot\beta} = 4$, $\qquad 0 \le x < 360°$, $x \ne 90°, 180°, 270°$

$\boxed{\varphi = \frac{\pi}{3}, \frac{5\pi}{3}}$, $\boxed{x = 45°, 225°}$, $\boxed{y = 135°, 315°}$, $\boxed{\varphi = \frac{\pi}{6}, \frac{5\pi}{6}}$, $\boxed{\psi = \frac{\pi}{4}, \frac{5\pi}{4}}$, $\boxed{\beta = 60°, 300°}$







Question 8

Solve each of the following equations.

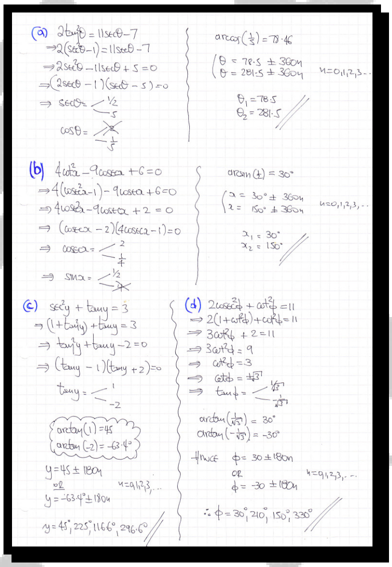
a) $2\tan^2\theta = 11\sec\theta - 7$, $\quad 0 \le \theta < 360°$

b) $4\cot^2 x - 9\operatorname{cosec} x + 6 = 0$, $\quad 0 \le x < 360°$

c) $\sec^2 y + \tan y = 3$, $\quad 0 \le y < 360°$

d) $2\operatorname{cosec}^2\varphi + \cot^2\varphi = 11$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 78.5°,\ 281.5°}$, $\boxed{x = 30°,\ 150°}$, $\boxed{y = 45°,\ 225° \quad y \approx 116.6°,\ 296.6°}$, $\boxed{\varphi = 30°,\ 150°,\ 210°,\ 330°}$



Question 9

Solve each of the following equations.

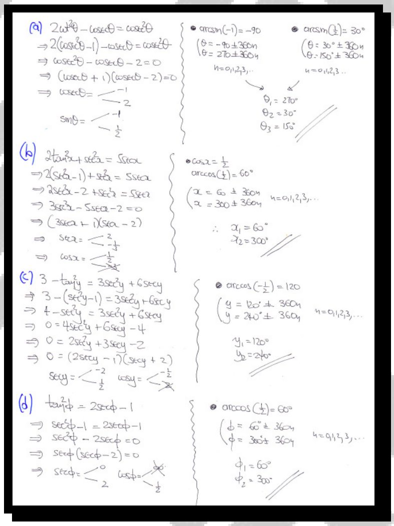
a) $2\cot^2\theta - \operatorname{cosec}\theta = \operatorname{cosec}^2\theta$, $\quad 0 \le \theta < 360°$

b) $2\tan^2 x + \sec^2 x = 5\sec x$, $\quad 0 \le x < 360°$

c) $3 - \tan^2 y = 3\sec^2 y + 6\sec y$, $\quad 0 \le y < 360°$

d) $\tan^2\varphi = 2\sec\varphi - 1$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 30°,\ 150°,\ 270°}$, $\boxed{x = 60°,\ 300°}$, $\boxed{y = 120°,\ 240°}$, $\boxed{\varphi = 60°,\ 300°}$



Question 10

Solve each of the following equations.

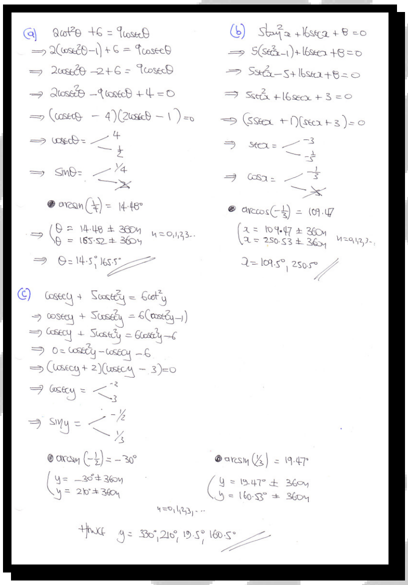
a) $2\cot^2\theta + 6 = 9\operatorname{cosec}\theta, \quad 0 \le \theta < 360°$

b) $5\tan^2 x + 16\sec x + 8 = 0, \quad 0 \le x < 360°$

c) $\operatorname{cosec} y + 5\operatorname{cosec}^2 y = 6\cot^2 y, \quad 0 \le y < 360°$

d) $2\tan^2\varphi = 15\sec\varphi - 9, \quad 0 \le \varphi < 360°$

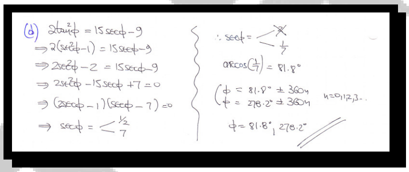
$\boxed{\theta \approx 14.5°,\ 165.5°}$, $\boxed{x \approx 109.5°,\ 250.5°}$, $\boxed{y \approx 19.5°,\ 160.5° \quad y = 210°,\ 330°}$, $\boxed{\varphi \approx 81.8°,\ 278.2°}$



(a) $2\cot^2\theta + 6 = 9\operatorname{cosec}\theta$
$\Rightarrow 2(\operatorname{cosec}^2\theta - 1) + 6 = 9\operatorname{cosec}\theta$
$\Rightarrow 2\operatorname{cosec}^2\theta - 2 + 6 = 9\operatorname{cosec}\theta$
$\Rightarrow 2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta + 4 = 0$
$\Rightarrow (\operatorname{cosec}\theta - 4)(2\operatorname{cosec}\theta - 1) = 0$
$\Rightarrow \operatorname{cosec}\theta = 4$ or $\frac{1}{2}$
$\Rightarrow \sin\theta = \frac{1}{4}$ or ~~2~~
$\arcsin\left(\frac{1}{4}\right) = 14.48°$
$\Rightarrow \theta = 14.48 \pm 360n$, $\theta = 165.52 \pm 360n$, $n = 0, 1, 2, 3 \ldots$
$\Rightarrow \theta = 14.5°, 165.5°$

(b) $5\tan^2 x + 16\sec x + 8 = 0$
$\Rightarrow 5(\sec^2 x - 1) + 16\sec x + 8 = 0$
$\Rightarrow 5\sec^2 x - 5 + 16\sec x + 8 = 0$
$\Rightarrow 5\sec^2 x + 16\sec x + 3 = 0$
$\Rightarrow (5\sec x + 1)(\sec x + 3) = 0$
$\Rightarrow \sec x = -3$ or $-\frac{1}{5}$
$\Rightarrow \cos x = -\frac{1}{3}$ or ~~-5~~
$\arccos\left(-\frac{1}{3}\right) = 109.47$
$x = 109.47 \pm 360n$, $x = 250.53 \pm 360n$, $n = 0, 1, 2, 3 \ldots$
$x = 109.5°, 250.5°$

(c) $\operatorname{cosec} y + 5\operatorname{cosec}^2 y = 6\cot^2 y$
$\Rightarrow \operatorname{cosec} y + 5\operatorname{cosec}^2 y = 6(\operatorname{cosec}^2 y - 1)$
$\Rightarrow \operatorname{cosec} y + 5\operatorname{cosec}^2 y = 6\operatorname{cosec}^2 y - 6$
$\Rightarrow 0 = \operatorname{cosec}^2 y - \operatorname{cosec} y - 6$
$\Rightarrow (\operatorname{cosec} y + 2)(\operatorname{cosec} y - 3) = 0$
$\Rightarrow \operatorname{cosec} y = -2$ or 3
$\Rightarrow \sin y = -\frac{1}{2}$ or $\frac{1}{3}$
$\arcsin\left(-\frac{1}{2}\right) = -30°$; $\arcsin\left(\frac{1}{3}\right) = 19.47°$
$y = -30° \pm 360n$, $y = 210° \pm 360n$; $y = 19.47° \pm 360n$, $y = 160.53° \pm 360n$
$n = 0, 1, 2, 3, \ldots$
thus $y = 330°, 210°, 19.5°, 160.5°$



(d) $2\tan^2\phi = 15\sec\phi - 9$
$\Rightarrow 2(\sec^2\phi - 1) = 15\sec\phi - 9$
$\Rightarrow 2\sec^2\phi - 2 = 15\sec\phi - 9$
$\Rightarrow 2\sec^2\phi - 15\sec\phi + 7 = 0$
$\Rightarrow (2\sec\phi - 1)(\sec\phi - 7) = 0$
$\Rightarrow \sec\phi = \frac{1}{2}$ or 7
$\therefore \sec\phi =$ ~~$\frac{1}{2}$~~ or 7, $\cos\phi = \frac{1}{7}$
$\arccos\left(\frac{1}{7}\right) = 81.8°$
$\phi = 81.8° \pm 360n$, $\phi = 278.2° \pm 360n$, $n = 0, 1, 2, 3 \ldots$
$\phi = 81.8°, 278.2°$

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Question 11

Solve each of the following equations.

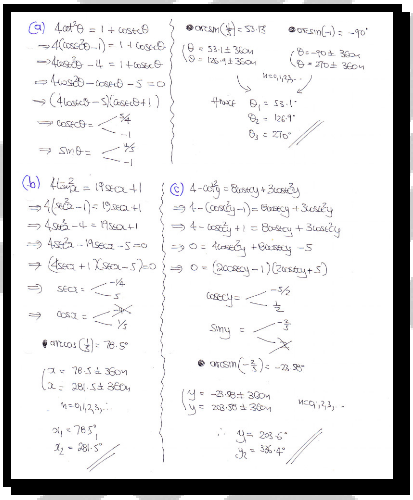
a) $4\cot^2\theta = 1 + \operatorname{cosec}\theta$, $\quad 0 \le \theta < 360°$

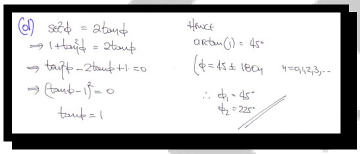
b) $4\tan^2 x = 19\sec x + 1$, $\quad 0 \le x < 360°$

c) $4 - \cot^2 y = 8\operatorname{cosec} y + 3\operatorname{cosec}^2 y$, $\quad 0 \le y < 360°$

d) $\sec^2\varphi = 2\tan\varphi$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta \approx 53.1°,\ 126.9° \quad \theta = 270°}$, $\boxed{x \approx 78.5°,\ 281.5°}$, $\boxed{y \approx 203.6°,\ 336.4°}$, $\boxed{\varphi = 45°,\ 225°}$





Question 12

Solve each of the following equations.

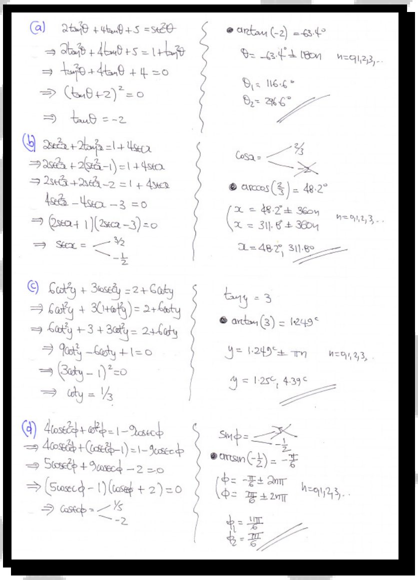
a) $2\tan^2\theta + 4\tan\theta + 5 = \sec^2\theta$, $\quad 0 \le \theta < 360°$

b) $2\sec^2 x + 2\tan^2 x = 1 + 4\sec x$, $\quad 0 \le x < 360°$

c) $6\cot^2 y + 3\text{cosec}^2 y = 2 + 6\cot y$, $\quad 0 \le y < 2\pi$

d) $4\text{cosec}^2\varphi + \cot^2\varphi = 1 - 9\,\text{cosec}\,\varphi$, $\quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 116.6°,\ 296.6°}$, $\boxed{x \approx 48.2°,\ 311.8°}$, $\boxed{y \approx 1.25^c,\ 4.39^c}$, $\boxed{\varphi = \frac{7\pi}{6},\ \frac{11\pi}{6}}$


(a) $2\tan^2\theta + 4\tan\theta + 5 = \sec^2\theta$
$\Rightarrow 2\tan^2\theta + 4\tan\theta + 5 = 1 + \tan^2\theta$
$\Rightarrow \tan^2\theta + 4\tan\theta + 4 = 0$
$\Rightarrow (\tan\theta + 2)^2 = 0$
$\Rightarrow \tan\theta = -2$

• $\arctan(-2) = -63.4°$
$\theta = -63.4° \pm 180n \quad n = 0,1,2,3,\dots$
$\theta_1 = 116.6°$
$\theta_2 = 296.6°$

(b) $2\sec^2 x + 2\tan^2 x = 1 + 4\sec x$
$\Rightarrow 2\sec^2 x + 2(\sec^2 x - 1) = 1 + 4\sec x$
$\Rightarrow 2\sec^2 x + 2\sec^2 x - 2 = 1 + 4\sec x$
$4\sec^2 x - 4\sec x - 3 = 0$
$\Rightarrow (2\sec x + 1)(2\sec x - 3) = 0$
$\Rightarrow \sec x = \frac{3}{2},\ -\frac{1}{2}$

$\cos x = \frac{2}{3},\ \cancel{-2}$
• $\arccos\left(\frac{2}{3}\right) = 48.2°$
$x = 48.2° \pm 360n$
$x = 311.8° \pm 360n \quad n = 0,1,2,3,\dots$
$x = 48.2°,\ 311.8°$

(c) $6\cot^2 y + 3\text{cosec}^2 y = 2 + 6\cot y$
$\Rightarrow 6\cot^2 y + 3(1 + \cot^2 y) = 2 + 6\cot y$
$\Rightarrow 6\cot^2 y + 3 + 3\cot^2 y = 2 + 6\cot y$
$\Rightarrow 9\cot^2 y - 6\cot y + 1 = 0$
$\Rightarrow (3\cot y - 1)^2 = 0$
$\Rightarrow \cot y = \frac{1}{3}$

$\tan y = 3$
• $\arctan(3) = 1.249^c$
$y = 1.249^c \pm \pi n \quad n = 0,1,2,3,\dots$
$y = 1.25^c,\ 4.39^c$

(d) $4\text{cosec}^2\phi + \cot^2\phi = 1 - 9\text{cosec}\,\phi$
$\Rightarrow 4\text{cosec}^2\phi + (\text{cosec}^2\phi - 1) = 1 - 9\text{cosec}\,\phi$
$\Rightarrow 5\text{cosec}^2\phi + 9\text{cosec}\,\phi - 2 = 0$
$\Rightarrow (5\text{cosec}\,\phi - 1)(\text{cosec}\,\phi + 2) = 0$
$\Rightarrow \text{cosec}\,\phi = \frac{1}{5},\ -2$

$\sin\phi = \cancel{5},\ -\frac{1}{2}$
• $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
$\phi = -\frac{\pi}{6} \pm 2n\pi$
$\phi = \frac{7\pi}{6} \pm 2n\pi \quad n = 0,1,2,3,\dots$
$\phi_1 = \frac{11\pi}{6}$
$\phi_2 = \frac{7\pi}{6}$

Question 13

Solve each of the following equations.

a) $10\sec^2\theta = 11\tan\theta + 16, \quad 0 \le \theta < 360°$

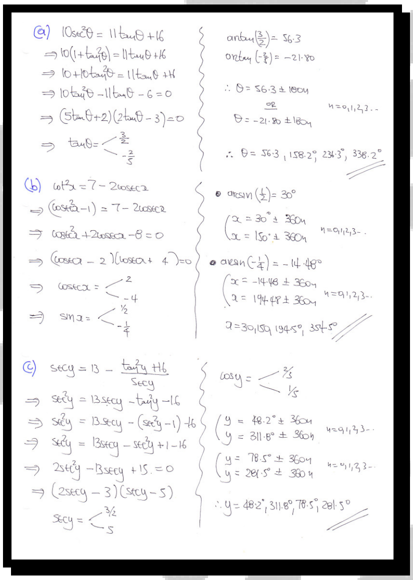
b) $\cot^2 x = 7 - 2\operatorname{cosec} x, \quad 0 \le x < 360°$

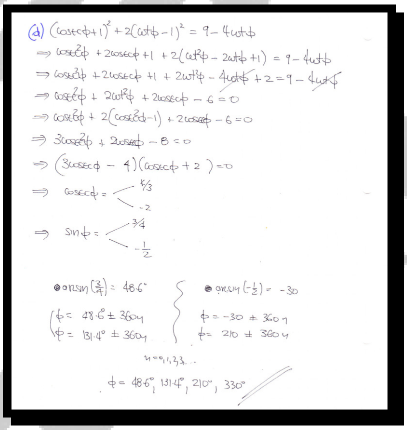
c) $\sec y = 13 - \dfrac{\tan^2 y + 16}{\sec y}, \quad 0 \le y < 360°$

d) $(\operatorname{cosec}\varphi + 1)^2 + 2(\cot\varphi - 1)^2 = 9 - 4\cot\varphi, \quad 0 \le \varphi < 360°$

$\boxed{\theta \approx 56.3°,\ 158.2°,\ 236.3°,\ 338.2°}$, $\boxed{x = 30°,\ 150° \quad x \approx 194.5°,\ 344.5°}$,

$\boxed{y \approx 48.2°,\ 78.5°,\ 281.5°,\ 311.8°}$, $\boxed{\varphi \approx 48.6°,\ 131.4° \quad \varphi = 210°,\ 330°}$





Question 14

Solve each of the following equations.

a) $3\tan^2\theta = 8\sec\theta\,, \qquad 0 \le \theta < 2\pi$

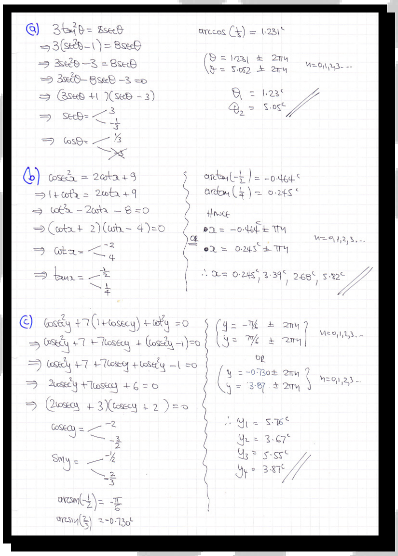
b) $\text{cosec}^2 x = 2\cot x + 9\,, \qquad 0 \le x < 2\pi$

c) $\text{cosec}^2 y + 7(1 + \text{cosec}\, y) + \cot^2 y = 0\,, \qquad 0 \le y < 2\pi$

d) $6\tan\varphi = \dfrac{2 - 3\sec^2\varphi}{\tan\varphi - 1}\,, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta \approx 1.23^{\text{c}},\ 5.05^{\text{c}}}$, $\boxed{x \approx 0.245^{\text{c}},\ 2.68^{\text{c}},\ 3.39^{\text{c}},\ 5.82^{\text{c}}}$, $\boxed{y \approx 3.67^{\text{c}},\ 3.87^{\text{c}},\ 5.55^{\text{c}},\ 5.76^{\text{c}}}$,

$\boxed{\varphi \approx 0.322^{\text{c}},\ 3.46^{\text{c}}}$



(a) $3\tan^2\theta = 8\sec\theta$

$\Rightarrow 3(\sec^2\theta - 1) = 8\sec\theta$

$\Rightarrow 3\sec^2\theta - 3 = 8\sec\theta$

$\Rightarrow 3\sec^2\theta - 8\sec\theta - 3 = 0$

$\Rightarrow (3\sec\theta + 1)(\sec\theta - 3)$

$\Rightarrow \sec\theta = 3,\ -\frac{1}{3}$

$\Rightarrow \cos\theta = \frac{1}{3},\ \cancel{-3}$

$\arccos\left(\frac{1}{3}\right) = 1.231^{c}$

$\theta = 1.231 \pm 2\pi n$, $\theta = 5.052 \pm 2\pi n$, $n = 0, 1, 2, 3, \dots$

$\theta_1 = 1.23^{c}$, $\theta_2 = 5.05^{c}$

(b) $\text{cosec}^2 x = 2\cot x + 9$

$\Rightarrow 1 + \cot^2 x = 2\cot x + 9$

$\Rightarrow \cot^2 x - 2\cot x - 8 = 0$

$\Rightarrow (\cot x + 2)(\cot x - 4) = 0$

$\Rightarrow \cot x = -2,\ 4$

$\Rightarrow \tan x = -\frac{1}{2},\ \frac{1}{4}$

$\arctan\left(-\frac{1}{2}\right) = -0.464^{c}$

$\arctan\left(\frac{1}{4}\right) = 0.245^{c}$

Hence

• $x = -0.464^{c} \pm \pi n$

or

• $x = 0.245^{c} \pm \pi n$, $n = 0, 1, 2, 3, \dots$

$\therefore x = 0.245^{c}, 3.39^{c}, 2.68^{c}, 5.82^{c}$

(c) $\text{cosec}^2 y + 7(1 + \text{cosec}\, y) + \cot^2 y = 0$

$\Rightarrow \text{cosec}^2 y + 7 + 7\text{cosec}\, y + (\text{cosec}^2 y - 1) = 0$

$\Rightarrow \text{cosec}^2 y + 7 + 7\text{cosec}\, y + \text{cosec}^2 y - 1 = 0$

$\Rightarrow 2\text{cosec}^2 y + 7\text{cosec}\, y + 6 = 0$

$\Rightarrow (2\text{cosec}\, y + 3)(\text{cosec}\, y + 2) = 0$

$\text{cosec}\, y = -2,\ -\frac{3}{2}$

$\sin y = -\frac{1}{2},\ -\frac{2}{3}$

$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

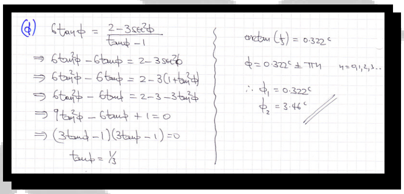
$\arcsin\left(\frac{2}{3}\right) = -0.730^{c}$

$y = -\pi/6 \pm 2\pi n$, $y = 7\pi/6 \pm 2\pi n$, $n = 0, 1, 2, 3, \dots$

or

$y = -0.730 \pm 2\pi n$, $y = 3.87 \pm 2\pi n$, $n = 0, 1, 2, 3, \dots$

$\therefore y_1 = 5.76^{c}$, $y_2 = 3.67^{c}$, $y_3 = 5.55^{c}$, $y_4 = 3.87^{c}$



(d) $6\tan\phi = \dfrac{2 - 3\sec^2\phi}{\tan\phi - 1}$

$\Rightarrow 6\tan^2\phi - 6\tan\phi = 2 - 3\sec^2\phi$

$\Rightarrow 6\tan^2\phi - 6\tan\phi = 2 - 3(1 + \tan^2\phi)$

$\Rightarrow 6\tan^2\phi - 6\tan\phi = 2 - 3 - 3\tan^2\phi$

$\Rightarrow 9\tan^2\phi - 6\tan\phi + 1 = 0$

$\Rightarrow (3\tan\phi - 1)(3\tan\phi - 1) = 0$

$\tan\phi = 1/3$

$\arctan\left(\frac{1}{3}\right) = 0.322^{c}$

$\phi = 0.322^{c} \pm \pi n$, $n = 0, 1, 2, 3, \dots$

$\therefore \phi_1 = 0.322^{c}$, $\phi_2 = 3.46^{c}$

Question 15

Solve each of the following equations.

a) $5\tan^2\theta - 12\sec\theta + 9 = 0$, $\quad 0 \le \theta < 360°$

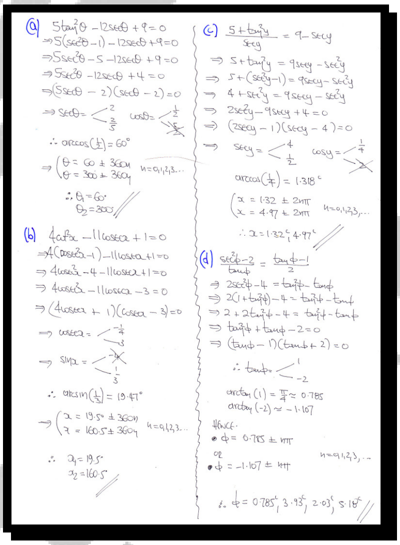
b) $4\cot^2 x - 11\operatorname{cosec} x + 1 = 0$, $\quad 0 \le x < 360°$

c) $\dfrac{5+\tan^2 y}{\sec y} = 9 - \sec y$, $\quad 0 \le y < 2\pi$

d) $\dfrac{\sec^2\varphi - 2}{\tan\varphi} = \dfrac{\tan\varphi - 1}{2}$, $\quad 0 \le \varphi < 2\pi$

$\boxed{\theta = 60°,\ 300°}$, $\boxed{x \approx 19.5°,\ 160.5°}$, $\boxed{y \approx 1.32^c,\ 4.97^c}$,

$\boxed{\varphi \approx 0.785^c,\ 2.03^c,\ 3.93^c,\ 5.18^c}$


(a) $5\tan^2\theta - 12\sec\theta + 9 = 0$

$\Rightarrow 5(\sec^2\theta - 1) - 12\sec\theta + 9 = 0$

$\Rightarrow 5\sec^2\theta - 5 - 12\sec\theta + 9 = 0$

$\Rightarrow 5\sec^2\theta - 12\sec\theta + 4 = 0$

$\Rightarrow (5\sec\theta - 2)(\sec\theta - 2) = 0$

$\Rightarrow \sec\theta = 2,\ \frac{2}{5}$ $\quad \cos\theta = \frac{1}{2},\ \cancel{\frac{5}{2}}$

$\therefore \arccos(\frac{1}{2}) = 60°$

$\Rightarrow \theta = 60 \pm 360n$, $\theta = 300 \pm 360n$, $n = 0,1,2,3\ldots$

$\therefore \theta_1 = 60°$, $\theta_2 = 300°$

(b) $4\cot^2 x - 11\operatorname{cosec} x + 1 = 0$

$\Rightarrow 4(\operatorname{cosec}^2 x - 1) - 11\operatorname{cosec} x + 1 = 0$

$\Rightarrow 4\operatorname{cosec}^2 x - 4 - 11\operatorname{cosec} x + 1 = 0$

$\Rightarrow 4\operatorname{cosec}^2 x - 11\operatorname{cosec} x - 3 = 0$

$\Rightarrow (4\operatorname{cosec} x + 1)(\operatorname{cosec} x - 3) = 0$

$\Rightarrow \operatorname{cosec} x = -\frac{1}{4},\ 3$

$\Rightarrow \sin x = \cancel{-4},\ \frac{1}{3}$

$\therefore \arcsin(\frac{1}{3}) = 19.47°$

$\Rightarrow x = 19.5° \pm 360n$, $x = 160.5° \pm 360n$, $n = 0,1,2,3\ldots$

$\therefore x_1 = 19.5°$, $x_2 = 160.5°$

(c) $\frac{5+\tan^2 y}{\sec y} = 9 - \sec y$

$\Rightarrow 5 + \tan^2 y = 9\sec y - \sec^2 y$

$\Rightarrow 5 + (\sec^2 y - 1) = 9\sec y - \sec^2 y$

$\Rightarrow 4 + \sec^2 y = 9\sec y - \sec^2 y$

$\Rightarrow 2\sec^2 y - 9\sec y + 4 = 0$

$\Rightarrow (2\sec y - 1)(\sec y - 4) = 0$

$\Rightarrow \sec y = 4,\ \frac{1}{2}$ $\quad \cos y = \frac{1}{4},\ \cancel{2}$

$\arccos(\frac{1}{4}) = 1.318^c$

$x = 1.32 \pm 2n\pi$, $x = 4.97 \pm 2n\pi$, $n = 0,1,2,3,\ldots$

$\therefore x = 1.32^c,\ 4.97^c$

(d) $\frac{\sec^2\phi - 2}{\tan\phi} = \frac{\tan\phi - 1}{2}$

$\Rightarrow 2\sec^2\phi - 4 = \tan^2\phi - \tan\phi$

$\Rightarrow 2(1 + \tan^2\phi) - 4 = \tan^2\phi - \tan\phi$

$\Rightarrow 2 + 2\tan^2\phi - 4 = \tan^2\phi - \tan\phi$

$\Rightarrow \tan^2\phi + \tan\phi - 2 = 0$

$\Rightarrow (\tan\phi - 1)(\tan\phi + 2) = 0$

$\therefore \tan\phi = 1,\ -2$

$\arctan(1) = \frac{\pi}{4} \approx 0.785$

$\arctan(-2) \approx -1.107$

Hence

- $\phi = 0.785 \pm n\pi$

or

- $\phi = -1.107 \pm n\pi$, $n = 0,1,2,3,\ldots$

$\therefore \phi = 0.785^c,\ 3.93^c,\ 2.03^c,\ 5.18^c$

Question 16

Solve each of the following equations.

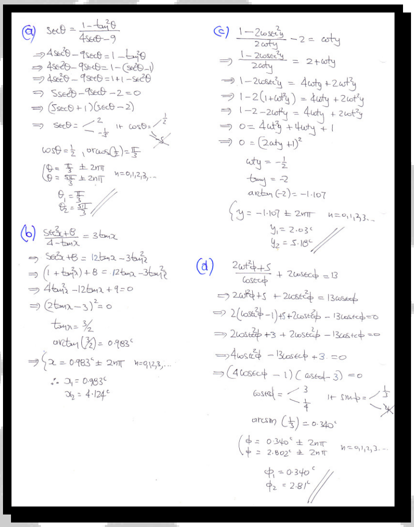
a) $\sec\theta = \dfrac{1-\tan^2\theta}{4\sec\theta - 9}, \qquad 0 \le \theta < 2\pi$

b) $\dfrac{\sec^2 x + 8}{4 - \tan x} = 3\tan x, \qquad 0 \le x < 2\pi$

c) $\dfrac{1 - 2\text{cosec}^2 y}{2\cot y} - 2 = \cot y, \qquad 0 \le y < 2\pi$

d) $\dfrac{2\cot^2\varphi + 5}{\text{cosec}\,\varphi} + 2\,\text{cosec}\,\varphi = 13, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}}$, $\boxed{x \approx 0.983^{\text{c}},\ 4.12^{\text{c}}}$, $\boxed{y \approx 2.03^{\text{c}},\ 5.18^{\text{c}}}$, $\boxed{\varphi \approx 0.340^{\text{c}},\ 2.80^{\text{c}}}$



Question 17

Solve each of the following trigonometric equations.

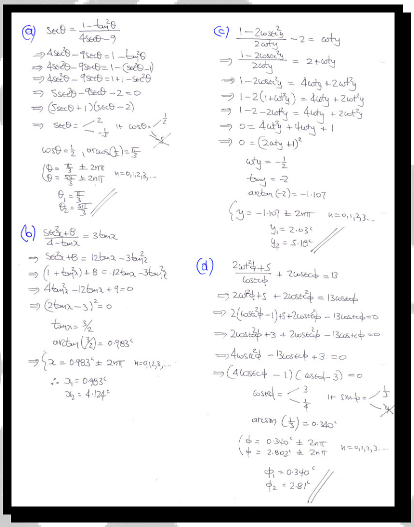
a) $\sec\theta = \dfrac{1-\tan^2\theta}{4\sec\theta - 9}, \qquad 0 \le \theta < 2\pi$

b) $\dfrac{\sec^2 x + 8}{4 - \tan x} = 3\tan x, \qquad 0 \le x < 2\pi$

c) $\dfrac{1-2\text{cosec}^2 y}{2\cot y} - 2 = \cot y, \qquad 0 \le y < 2\pi$

d) $\dfrac{2\cot^2\varphi + 5}{\text{cosec}\,\varphi} + 2\,\text{cosec}\,\varphi = 13, \quad 0 \le \varphi < 2\pi$

$\boxed{\theta = \dfrac{\pi}{3}, \ \dfrac{5\pi}{3}}$, $\boxed{x = 0.983^c, \ 4.12^c}$, $\boxed{y = 2.03^c, \ 5.18^c}$, $\boxed{\varphi = 0.340^c, \ 2.80^c}$



Question 17

Solve each of the following trigonometric equations.

a) $\cos(\theta + 30°) = \sin\theta$, $\quad 0 \le \theta < 360°$

b) $3\cos(x + 30°) = \sin(x - 60°)$, $\quad 0 \le x < 360°$

c) $\sin(y - 30°) = \sin(y + 45°)$, $\quad 0 \le y < 360°$

d) $\sin(\varphi + 30°) = \cos(\varphi - 45°)$, $\quad 0 \le \varphi < 360°$

e) $\cos(\alpha - 60°) = \cos(\alpha - 45°)$, $\quad 0 \le \alpha < 360°$

$\boxed{\theta = 30°,\ 210°}$, $\boxed{x = 60°,\ 240°}$, $\boxed{y = 82.5°,\ 262.5°}$, $\boxed{\varphi = 52.5°,\ 232.5°}$, $\boxed{\alpha = 52.5°,\ 232.5°}$

Question 18

Solve each of the following trigonometric equations.

a) $\sin(\theta - 45°) = \sin\theta$, $\quad 0 \le \theta < 360°$

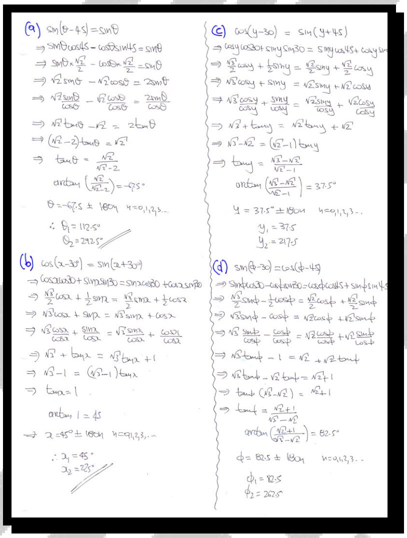
b) $\cos(x - 30°) = \sin(x + 30°)$, $\quad 0 \le x < 360°$

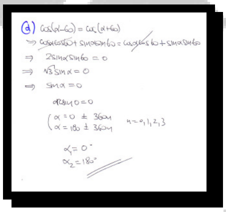
c) $\cos(y - 30°) = \sin(y + 45°)$, $\quad 0 \le y < 360°$

d) $\sin(\varphi - 30°) = \cos(\varphi - 45°)$, $\quad 0 \le \varphi < 360°$

e) $\cos(\alpha - 60°) = \cos(\alpha + 60°)$, $\quad 0 \le \alpha < 360°$

$\boxed{\theta = 112.5°,\ 292.5°}$, $\boxed{x = 45°,\ 225°}$, $\boxed{y = 37.5°,\ 217.5°}$, $\boxed{\varphi = 82.5°,\ 262.5°}$, $\boxed{\alpha = 0°,\ 180°}$





Question 19

Solve each of the following trigonometric equations.

a) $\sin\left(\theta+\dfrac{\pi}{4}\right)=\sin\theta, \qquad 0 \le \theta < 2\pi$

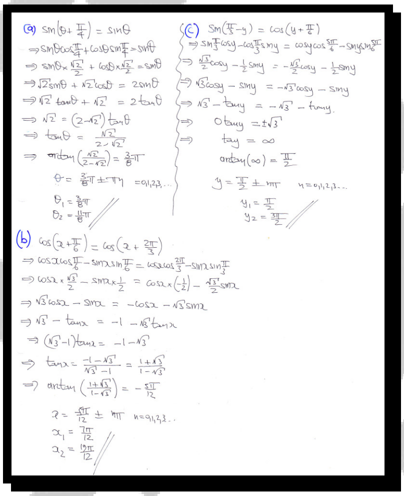
b) $\cos\left(x+\dfrac{\pi}{6}\right)=\cos\left(x+\dfrac{2\pi}{3}\right), \qquad 0 \le \theta < 2\pi$

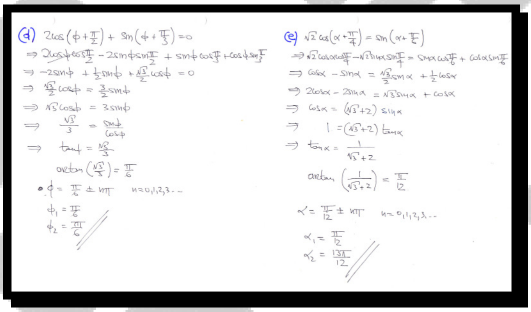
c) $\sin\left(\dfrac{\pi}{3}-y\right)=\cos\left(y+\dfrac{5\pi}{6}\right), \qquad 0 \le y < 2\pi$ (very hard)

d) $2\cos\left(\varphi+\dfrac{\pi}{2}\right)+\sin\left(\varphi+\dfrac{\pi}{3}\right)=0, \qquad 0 \le \varphi < 2\pi$

e) $\sqrt{2}\cos\left(\alpha+\dfrac{\pi}{4}\right)=\sin\left(\alpha+\dfrac{\pi}{6}\right), \qquad 0 \le \alpha < 2\pi$

$\boxed{\theta=\dfrac{3\pi}{8},\ \dfrac{11\pi}{8}}, \ \boxed{x=\dfrac{7\pi}{12},\ \dfrac{19\pi}{12}}, \ \boxed{y=\dfrac{\pi}{2},\ \dfrac{3\pi}{2}}, \ \boxed{\varphi=\dfrac{\pi}{6},\ \dfrac{7\pi}{6}}, \ \boxed{\alpha=\dfrac{\pi}{12},\ \dfrac{13\pi}{12}}$





Question 20

Solve each of the following trigonometric equations.

a) $\sin(\theta - 20°) = \sin(\theta + 60°)$, $\quad 0 \le \theta < 360°$

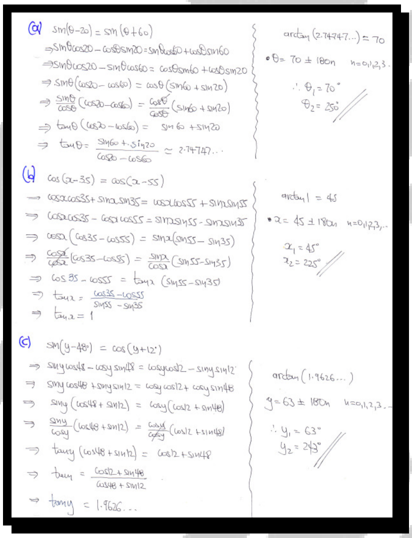
b) $\cos(x - 35°) = \cos(x - 55°)$, $\quad 0 \le x < 360°$

c) $\sin(y - 48°) = \cos(y + 12°)$, $\quad 0 \le y < 360°$

d) $\sin(\varphi + 72°) = \cos(\varphi - 38°)$, $\quad 0 \le \varphi < 360°$

e) $\cos(\alpha - 36°) = \cos(\alpha - 72°)$, $\quad 0 \le \alpha < 360°$

$\boxed{\theta = 70°,\ 250°}$, $\boxed{x = 45°,\ 225°}$, $\boxed{y = 63°,\ 243°}$, $\boxed{\varphi = 28°,\ 208°}$, $\boxed{\alpha = 54°,\ 234°}$



(a) $\sin(\theta-20) = \sin(\theta+60)$

$\Rightarrow \sin\theta\cos20 - \cos\theta\sin20 = \sin\theta\cos60 + \cos\theta\sin60$

$\Rightarrow \sin\theta\cos20 - \sin\theta\cos60 = \cos\theta\sin60 + \cos\theta\sin20$

$\Rightarrow \sin\theta(\cos20 - \cos60) = \cos\theta(\sin60 + \sin20)$

$\Rightarrow \frac{\sin\theta}{\cos\theta}(\cos20 - \cos60) = \frac{\cos\theta}{\cos\theta}(\sin60 + \sin20)$

$\Rightarrow \tan\theta(\cos20 - \cos60) = \sin60 + \sin20$

$\Rightarrow \tan\theta = \frac{\sin60 + \sin20}{\cos20 - \cos60} \simeq 2.74747...$

$\arctan(2.74747...) \simeq 70$

$\theta = 70 \pm 180n \quad n = 0,1,2,3...$

$\therefore \theta_1 = 70°$, $\theta_2 = 250°$

(b) $\cos(x-35) = \cos(x-55)$

$\Rightarrow \cos x\cos35 + \sin x\sin35 = \cos x\cos55 + \sin x\sin55$

$\Rightarrow \cos x\cos35 - \cos x\cos55 = \sin x\sin55 - \sin x\sin35$

$\Rightarrow \cos x(\cos35 - \cos55) = \sin x(\sin55 - \sin35)$

$\Rightarrow \frac{\cos x}{\cos x}(\cos35 - \cos55) = \frac{\sin x}{\cos x}(\sin55 - \sin35)$

$\Rightarrow \cos35 - \cos55 = \tan x(\sin55 - \sin35)$

$\Rightarrow \tan x = \frac{\cos35 - \cos55}{\sin55 - \sin35}$

$\Rightarrow \tan x = 1$

$\arctan 1 = 45$

$x = 45 \pm 180n \quad n = 0,1,2,3,...$

$x_1 = 45°$, $x_2 = 225°$

(c) $\sin(y-48°) = \cos(y+12°)$

$\Rightarrow \sin y\cos48 - \cos y\sin48 = \cos y\cos12 - \sin y\sin12$

$\Rightarrow \sin y\cos48 + \sin y\sin12 = \cos y\cos12 + \cos y\sin48$

$\Rightarrow \sin y(\cos48 + \sin12) = \cos y(\cos12 + \sin48)$

$\Rightarrow \frac{\sin y}{\cos y}(\cos48 + \sin12) = \frac{\cos y}{\cos y}(\cos12 + \sin48)$

$\Rightarrow \tan y(\cos48 + \sin12) = \cos12 + \sin48$

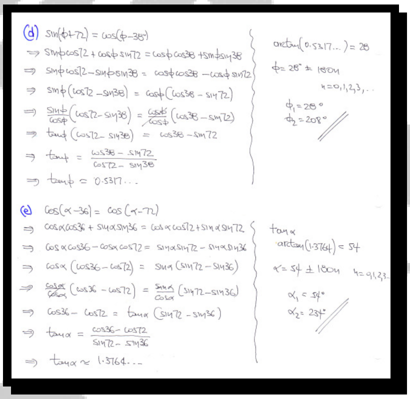
$\Rightarrow \tan y = \frac{\cos12 + \sin48}{\cos48 + \sin12}$

$\Rightarrow \tan y = 1.9626...$

$\arctan(1.9626...)$

$y = 63 \pm 180n \quad n = 0,1,2,3...$

$\therefore y_1 = 63°$, $y_2 = 243°$



(d) $\sin(\phi+72) = \cos(\phi-38°)$

$\Rightarrow \sin\phi\cos72 + \cos\phi\sin72 = \cos\phi\cos38 + \sin\phi\sin38$

$\Rightarrow \sin\phi\cos72 - \sin\phi\sin38 = \cos\phi\cos38 - \cos\phi\sin72$

$\Rightarrow \sin\phi(\cos72 - \sin38) = \cos\phi(\cos38 - \sin72)$

$\Rightarrow \frac{\sin\phi}{\cos\phi}(\cos72 - \sin38) = \frac{\cos\phi}{\cos\phi}(\cos38 - \sin72)$

$\Rightarrow \tan\phi(\cos72 - \sin38) = \cos38 - \sin72$

$\Rightarrow \tan\phi = \frac{\cos38 - \sin72}{\cos72 - \sin38}$

$\Rightarrow \tan\phi = 0.5317...$

$\arctan(0.5317...) = 28$

$\phi = 28° \pm 180n \quad n = 0,1,2,3,...$

$\phi_1 = 28°$, $\phi_2 = 208°$

(e) $\cos(\alpha-36) = \cos(\alpha-72)$

$\Rightarrow \cos\alpha\cos36 + \sin\alpha\sin36 = \cos\alpha\cos72 + \sin\alpha\sin72$

$\Rightarrow \cos\alpha\cos36 - \cos\alpha\cos72 = \sin\alpha\sin72 - \sin\alpha\sin36$

$\Rightarrow \cos\alpha(\cos36 - \cos72) = \sin\alpha(\sin72 - \sin36)$

$\Rightarrow \frac{\cos\alpha}{\cos\alpha}(\cos36 - \cos72) = \frac{\sin\alpha}{\cos\alpha}(\sin72 - \sin36)$

$\Rightarrow \cos36 - \cos72 = \tan\alpha(\sin72 - \sin36)$

$\Rightarrow \tan\alpha = \frac{\cos36 - \cos72}{\sin72 - \sin36}$

$\Rightarrow \tan\alpha \approx 1.3764...$

$\tan\alpha$ $\arctan(1.3764) = 54$

$\alpha = 54 \pm 180n \quad n = 0,1,2,3...$

$\alpha_1 = 54°$, $\alpha_2 = 234°$

Question 21

Solve each of the following trigonometric equations.

a) $\sin 2\theta = \tan\theta$, $\quad 0 \le \theta \le 180°$

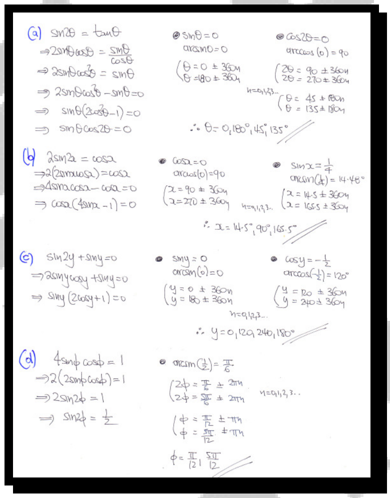
b) $2\sin 2x = \cos x$, $\quad 0 \le x < 180°$

c) $\sin 2y + \sin y = 0$, $\quad 0 \le y < 360°$

d) $4\sin\varphi\cos\varphi = 1$, $\quad 0 \le \varphi < \pi$

$\boxed{\theta = 0°,\ 45°,\ 135°,\ 180°}$, $\boxed{x = 90°,\ x \approx 14.5°,\ 165.5°}$, $\boxed{y = 0°,\ 120°,\ 180°,\ 240°}$,

$\boxed{\varphi = \frac{\pi}{12},\ \frac{5\pi}{12}}$



Question 22

Solve each of the following trigonometric equations.

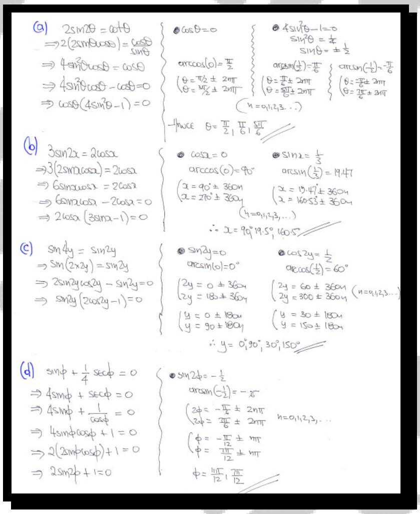
a) $2\sin 2\theta = \cot\theta$, $\quad 0 \le \theta \le \pi$

b) $3\sin 2x = 2\cos x$, $\quad 0 \le x < 180°$

c) $\sin 4y = \sin 2y$, $\quad 0 \le y < 180°$

d) $\sin\varphi + \frac{1}{4}\sec\varphi = 0$, $\quad 0 \le \varphi < \pi$

$\boxed{\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}}$, $\boxed{x = 90°,\ x \approx 19.5°,\ 160.5°}$, $\boxed{y = 0°,\ 30°,\ 90°,\ 150°}$, $\boxed{\varphi = \frac{7\pi}{12}, \frac{11\pi}{12}}$



Question 23

Solve each of the following trigonometric equations.

a) $\cos\theta - \sin 2\theta = 0$, $\quad 0 \le \theta \le 360°$

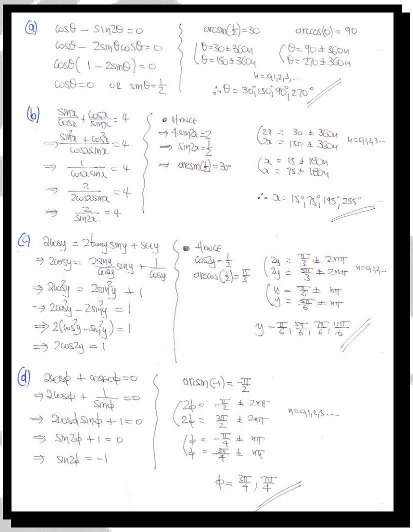
b) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = 4$, $\quad 0 \le x < 360°$

c) $2\cos y = 2\tan y \sin y + \sec y$, $\quad 0 \le y < 2\pi$

d) $2\cos\varphi + \operatorname{cosec}\varphi = 0$, $\quad 0 \le \varphi < 2\pi$

$\boxed{\theta = 30°,\ 90°,\ 150°,\ 270°}$, $\boxed{x = 15°,\ 75°,\ 195°,\ 255°}$, $\boxed{y = \dfrac{\pi}{6},\ \dfrac{5\pi}{6},\ \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}}$,

$\boxed{\varphi = \dfrac{3\pi}{4},\ \dfrac{7\pi}{4}}$



Question 24

Solve each of the following trigonometric equations.

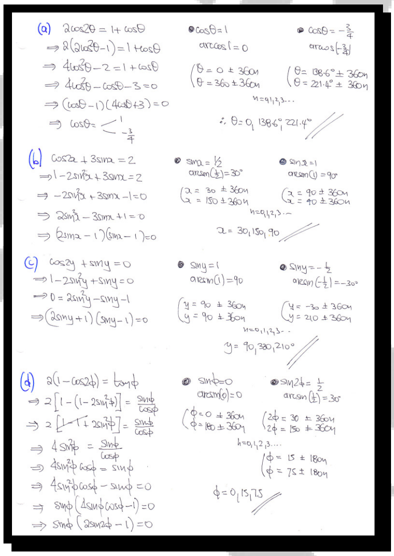
a) $2\cos 2\theta = 1 + \cos\theta$, $\quad 0 \le \theta < 360°$

b) $\cos 2x + 3\sin x = 2$, $\quad 0 \le x < 360°$

c) $\cos 2y + \sin y = 0$, $\quad 0 \le y < 360°$

d) $2(1 - \cos 2\varphi) = \tan\varphi$, $\quad 0 \le \varphi < 180°$

$\boxed{\theta = 0°,\ \theta \approx 138.6°,\ 221.4°}$, $\boxed{x = 30°,\ 90°,\ 150°}$, $\boxed{y = 90°,\ 210°,\ 330°}$, $\boxed{\varphi = 0°,\ 15°,\ 75°}$



Question 25

Solve each of the following trigonometric equations.

a) $\cos 2\theta = 1 + \sin\theta$, $\quad 0 \le \theta < 360°$

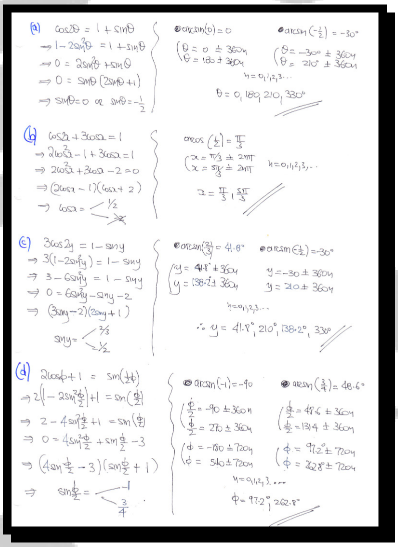
b) $\cos 2x + 3\cos x = 1$, $\quad 0 \le x < 2\pi$

c) $3\cos 2y = 1 - \sin y$, $\quad 0 \le y < 360°$

d) $2\cos\varphi + 1 = \sin\left(\frac{1}{2}\varphi\right)$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 0°,\ 180°,\ 210°,\ 330°}$, $\boxed{x = \frac{\pi}{3},\ \frac{5\pi}{3}}$, $\boxed{y \approx 41.8°,\ 138.2° \quad y = 210°,\ 330°}$,

$\boxed{\varphi = 97.2°,\ 262.8°}$



Question 26

a) $\cos 2\theta - 7\sin\theta - 4 = 0$, $\quad 0 \le \theta < 360°$

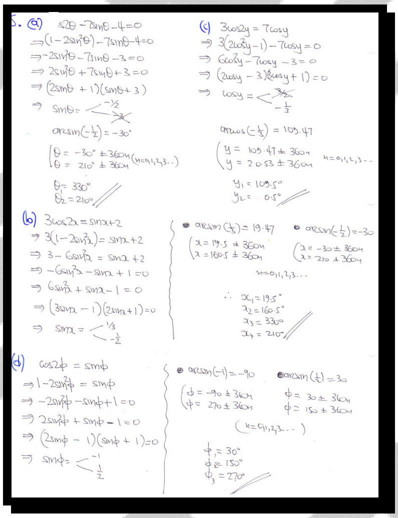
b) $3\cos 2x = \sin x + 2$, $\quad 0 \le x < 360°$

c) $3\cos 2y = 7\cos y$, $\quad 0 \le y < 360°$

d) $\cos 2\varphi = \sin\varphi$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 19.5°,\ 160.5° \quad x = 210°,\ 330°}$, $\boxed{y \approx 109.5°,\ 250.5°}$,

$\boxed{\varphi = 30°,\ 150°,\ 270°}$



Question 27

Solve each of the following trigonometric equations.

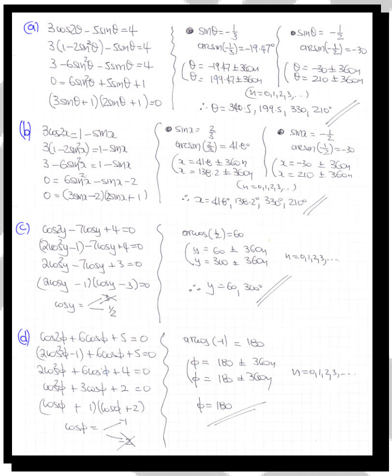
a) $3\cos 2\theta - 5\sin\theta = 4$, $\quad 0 \le \theta < 360°$

b) $3\cos 2x = 1 - \sin x$, $\quad 0 \le x < 360°$

c) $\cos 2y - 7\cos y + 4 = 0$, $\quad 0 \le y < 360°$

d) $\cos 2\varphi + 6\cos\varphi + 5 = 0$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330° \quad \theta \approx 199.5°,\ 340.5°}$, $\boxed{x \approx 41.8°,\ 138.2° \quad x = 210°,\ 330°}$, $\boxed{y = 60°,\ 300°}$, $\boxed{\varphi = 180°}$



Question 28

Solve each of the following trigonometric equations.

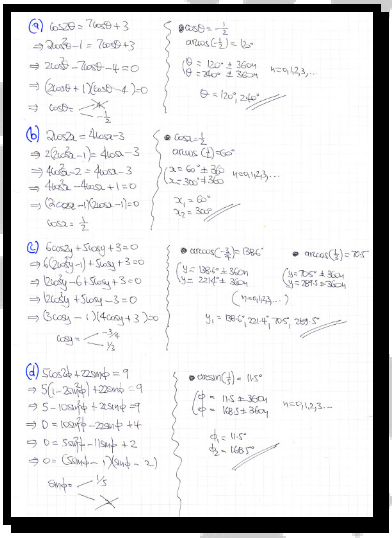
a) $\cos 2\theta = 7\cos\theta + 3$, $\quad 0 \le \theta < 360°$

b) $2\cos 2x = 4\cos x - 3$, $\quad 0 \le x < 360°$

c) $6\cos 2y + 5\cos y + 3 = 0$, $\quad 0 \le y < 360°$

d) $5\cos 2\varphi + 22\sin\varphi = 9$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 120°,\ 240°}$, $\boxed{x = 60°,\ 300°}$, $\boxed{y \approx 70.5°,\ 138.6°,\ 221.4°,\ 289.5°}$, $\boxed{\varphi \approx 11.5°,\ 168.5°}$



Question 29

Solve each of the following trigonometric equations.

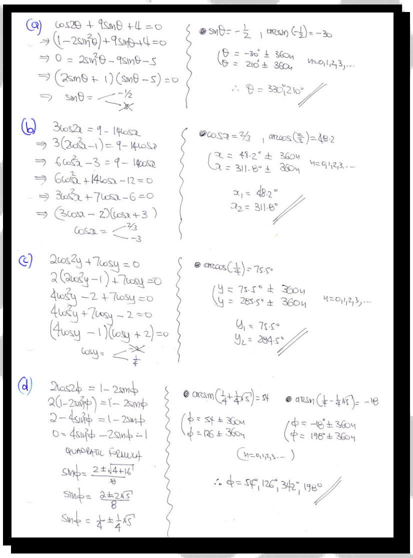
a) $\cos 2\theta + 9\sin\theta + 4 = 0$, $\quad 0 \le \theta < 360°$

b) $3\cos 2x = 9 - 14\cos x$, $\quad 0 \le x < 360°$

c) $2\cos 2y + 7\cos y = 0$, $\quad 0 \le y < 360°$

d) $2\cos 2\varphi = 1 - 2\sin\varphi$, $\quad 0 \le \varphi < 360°$

$\boxed{\theta = 210°,\ 330°}$, $\boxed{x \approx 48.2°,\ 311.8°}$, $\boxed{y \approx 75.5°,\ 284.5°}$, $\boxed{\varphi = 54°,\ 126°,\ 198°,\ 342°}$



Question 30

Solve each of the following trigonometric equations.

a) $\tan\theta(1+\cos 2\theta) = 2\sin^2 2\theta$, $\quad 0 \le \theta \le 90°$

b) $4\tan 2\varphi + 3\cot\varphi\sec^2\varphi = 0$, $\quad 0 \le \varphi < 2\pi$ (hard)

$\boxed{\theta = 0°, 15°, 75°, 90°}$, $\boxed{\varphi = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$