

TRIGONOMETRIC EQUATIONS & IDENTITIES INTRODUCTION

Question 1

Solve each of the following trigonometric equations in the range given.

a) $\sin x = \frac{1}{2}$

$0 \leq x < 360^\circ$

$x = 30^\circ, 150^\circ$

b) $\cos 2x = \frac{1}{2}$

$0 \leq x < 360^\circ$

$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

c) $\tan(\theta - 20^\circ) = \sqrt{3}$

$-180^\circ \leq \theta < 180^\circ$

$\theta = -100^\circ, 80^\circ$

d) $\sin(2\theta + 30^\circ) = \frac{\sqrt{3}}{2}$

$-180^\circ \leq \theta < 180^\circ$

$\theta = -165^\circ, -135^\circ, 15^\circ, 45^\circ$

e) $\cos(3x - 45^\circ) = -\frac{1}{2}$

$0 \leq x < 180^\circ$

$x = 55^\circ, 95^\circ, 175^\circ$

Handwritten solutions for Question 1:

(a) $\sin x = \frac{1}{2}$, $0 \leq x < 360^\circ$
 $\sin^{-1}(\frac{1}{2}) = 30^\circ$
 $x = 30^\circ \pm 360^\circ$ and $x = 150^\circ \pm 360^\circ$
 $x = 30^\circ, 150^\circ$

(b) $\cos 2x = \frac{1}{2}$, $0 \leq x < 360^\circ$
 $\cos^{-1}(\frac{1}{2}) = 60^\circ$
 $2x = 60^\circ \pm 360^\circ$ and $2x = 300^\circ \pm 360^\circ$
 $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

(c) $\tan(\theta - 20^\circ) = \sqrt{3}$, $-180^\circ \leq \theta < 180^\circ$
 $\tan^{-1}(\sqrt{3}) = 60^\circ$
 $\theta - 20^\circ = 60^\circ$
 $\theta = 80^\circ$
 $\theta - 20^\circ = 240^\circ$
 $\theta = 260^\circ$ (out of range)
 $\theta = 80^\circ$

(d) $\sin(2\theta + 30^\circ) = \frac{\sqrt{3}}{2}$, $-180^\circ \leq \theta < 180^\circ$
 $\sin^{-1}(\frac{\sqrt{3}}{2}) = 60^\circ$
 $2\theta + 30^\circ = 60^\circ$
 $2\theta = 30^\circ$
 $\theta = 15^\circ$
 $2\theta + 30^\circ = 120^\circ$
 $2\theta = 90^\circ$
 $\theta = 45^\circ$
 $2\theta + 30^\circ = 240^\circ$
 $2\theta = 210^\circ$
 $\theta = 105^\circ$ (out of range)
 $\theta = 15^\circ, 45^\circ$

(e) $\cos(3x - 45^\circ) = -\frac{1}{2}$, $0 \leq x < 180^\circ$
 $\cos^{-1}(-\frac{1}{2}) = 120^\circ$
 $3x - 45^\circ = 120^\circ$
 $3x = 165^\circ$
 $x = 55^\circ$
 $3x - 45^\circ = 240^\circ$
 $3x = 285^\circ$
 $x = 95^\circ$
 $3x - 45^\circ = 360^\circ$
 $3x = 405^\circ$
 $x = 135^\circ$ (out of range)
 $x = 55^\circ, 95^\circ$

Question 2

Solve each of the following trigonometric equations in the range given.

a) $\sin x = -\frac{1}{2}$

$0 \leq x < 360^\circ$

$x = 210^\circ, 330^\circ$

b) $\cos 2x = -0.1736$

$0 \leq x < 360^\circ$

$x = 50^\circ, 130^\circ, 230^\circ, 310^\circ$

c) $\tan(x - 40^\circ) = 0.7$

$-180^\circ \leq x < 180^\circ$

$x = -105^\circ, 75^\circ$

d) $\sin(2\theta + 40^\circ) = 0.9848$

$-180^\circ \leq \theta < 180^\circ$

$\theta = -160^\circ, -150^\circ, 20^\circ, 30^\circ$

e) $\cos(4y - 60^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$

$0 \leq y < 90^\circ$

$y = 33.8^\circ, 86.3^\circ$

Handwritten solutions for Question 2:

(a) $\sin x = -\frac{1}{2}$, $0 \leq x < 360^\circ$
 $\arcsin(-\frac{1}{2}) = -30^\circ$
 $x = -30^\circ \pm 360^\circ$
 $x = 210^\circ, 330^\circ$

(b) $\cos 2x = -0.1736$, $0 \leq x < 360^\circ$
 $\arccos(-0.1736) = 100.0^\circ$
 $2x = 100^\circ \pm 360^\circ$
 $2x = 220^\circ \pm 360^\circ$
 $x = 50^\circ, 130^\circ, 230^\circ, 310^\circ$

(c) $\tan(x - 40^\circ) = 0.7$, $-180^\circ \leq x < 180^\circ$
 $\arctan(0.7) = 35.0^\circ$
 $x - 40^\circ = 35.0^\circ$
 $x = 75.0^\circ$
 $x - 40^\circ = 35.0^\circ + 180^\circ$
 $x = 215.0^\circ$
 $x = -105.0^\circ$

(d) $\sin(2\theta + 40^\circ) = 0.9848$, $-180^\circ \leq \theta < 180^\circ$
 $\arcsin(0.9848) = 82.0^\circ$
 $2\theta + 40^\circ = 82.0^\circ \pm 360^\circ$
 $2\theta = 42.0^\circ \pm 360^\circ$
 $\theta = 21.0^\circ \pm 180^\circ$
 $\theta = 21.0^\circ, -159.0^\circ$
 $2\theta + 40^\circ = 82.0^\circ + 180^\circ$
 $2\theta = 118.0^\circ \pm 360^\circ$
 $\theta = 59.0^\circ \pm 180^\circ$
 $\theta = 59.0^\circ, -121.0^\circ$

(e) $\cos(4y - 60^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$, $0 \leq y < 90^\circ$
 $\arccos(\frac{\sqrt{6} - \sqrt{2}}{4}) = 75.0^\circ$
 $4y - 60^\circ = 75.0^\circ \pm 360^\circ$
 $4y = 135.0^\circ \pm 360^\circ$
 $y = 33.8^\circ \pm 90.0^\circ$
 $y = 33.8^\circ, 123.8^\circ$
 $4y - 60^\circ = 75.0^\circ + 180^\circ$
 $4y = 255.0^\circ \pm 360^\circ$
 $y = 63.8^\circ \pm 90.0^\circ$
 $y = 63.8^\circ, 153.8^\circ$

Question 3

Solve each of the following trigonometric equations in the range given.

a) $\tan\left(\frac{3}{2}x\right) = -\frac{\sqrt{3}}{3}$

$0 \leq x < 360^\circ$

$x = 100^\circ, 220^\circ, 340^\circ$

b) $\sin(4y - 45^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$

$-90^\circ \leq y < 90^\circ$

$y = -75^\circ, -37.5^\circ, 15^\circ, 52.5^\circ$

c) $5 \cos 2x = 2$

$0 \leq x < 360^\circ$

$x = 33.2^\circ, 146.8^\circ, 213.2^\circ, 326.8^\circ$

d) $3 - \tan(\theta - 10^\circ) = 1$

$0 \leq \theta < 360^\circ$

$\theta = 73.4^\circ, 253.4^\circ$

e) $3 \sin(3\theta - 30^\circ) + 4 = 5$

$-90^\circ \leq \theta < 90^\circ$

$\theta = -56.5^\circ, 16.5^\circ, 63.5^\circ$

Handwritten solutions for Question 3:

(a) $\tan\left(\frac{3}{2}x\right) = -\frac{\sqrt{3}}{3}$
 $\Rightarrow \frac{3}{2}x = -30^\circ \pm 180^\circ$
 $x = -20^\circ \pm 120^\circ$
 $x_1 = 100^\circ$
 $x_2 = 220^\circ$
 $x_3 = 340^\circ$

(b) $\sin(4y - 45^\circ) = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\Rightarrow 4y - 45^\circ = 15^\circ \pm 360^\circ$
 $4y = 60^\circ \pm 360^\circ$
 $y = 15^\circ \pm 90^\circ$
 $y_1 = 15^\circ$
 $y_2 = 105^\circ$
 $y_3 = -75^\circ$
 $y_4 = -37.5^\circ$

(c) $5 \cos 2x = 2$
 $\Rightarrow \cos 2x = \frac{2}{5}$
 $\Rightarrow 2x = \cos^{-1}\left(\frac{2}{5}\right) = 66.42^\circ$
 $x = 33.21^\circ \pm 180^\circ$
 $x_1 = 33.2^\circ$
 $x_2 = 213.2^\circ$
 $x_3 = 146.8^\circ$
 $x_4 = 326.8^\circ$

(d) $3 - \tan(\theta - 10^\circ) = 1$
 $\Rightarrow \tan(\theta - 10^\circ) = 2$
 $\Rightarrow \theta - 10^\circ = \tan^{-1}(2) = 63.4^\circ$
 $\theta = 73.4^\circ \pm 180^\circ$
 $\theta_1 = 73.4^\circ$
 $\theta_2 = 253.4^\circ$

(e) $3 \sin(3\theta - 30^\circ) + 4 = 5$
 $\Rightarrow \sin(3\theta - 30^\circ) = \frac{1}{3}$
 $\Rightarrow 3\theta - 30^\circ = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$
 $3\theta = 49.47^\circ \pm 360^\circ$
 $\theta = 16.49^\circ \pm 120^\circ$
 $\theta_1 = 16.5^\circ$
 $\theta_2 = 136.5^\circ$
 $\theta_3 = -56.5^\circ$

Question 4

Solve each of the following trigonometric equations in the range given.

a) $4 \cos 2\theta = 3$

$0 \leq \theta < 360^\circ$

$\theta \approx 20.7^\circ, 159.3^\circ, 200.7^\circ, 339.3^\circ$

b) $3 \tan 4x - 4 = 5$

$0 \leq x < 90^\circ$

$x \approx 17.9^\circ, 62.9^\circ$

c) $5 \sin(3\psi - 70^\circ) = 4$

$-90^\circ \leq \psi < 90^\circ$

$\psi \approx -79.0^\circ, -54.4^\circ, 41.0^\circ, 65.6^\circ$

d) $5 - 3 \tan 2\phi = 1$

$0 \leq \phi < 180^\circ$

$\phi \approx 26.6^\circ, 116.6^\circ$

e) $2 \sin(y + 50^\circ) + 3 = 4$

$0 \leq y < 360^\circ$

$y = 100^\circ, 340^\circ$

Handwritten solutions for Question 4:

a) $4 \cos 2\theta = 3$ $0 \leq \theta < 360^\circ$
 $\cos 2\theta = \frac{3}{4}$
 $\arccos(\frac{3}{4}) = 41.41^\circ$
 $2\theta = 41.41^\circ \pm 360^\circ$ $\cos(41.41^\circ)$
 $2\theta = 318.59^\circ \pm 360^\circ$
 $\theta = 20.7^\circ$
 $\theta = 159.3^\circ$
 $\theta = 200.7^\circ$
 $\theta = 339.3^\circ$

b) $3 \tan 4x - 4 = 5$ $0 \leq x < 90^\circ$
 $3 \tan 4x = 9$
 $\tan 4x = 3$
 $\arctan(3) = 71.57^\circ$
 $4x = 71.57^\circ \pm 180^\circ$
 $x = 17.9^\circ \pm 45^\circ$
 $x = 17.9^\circ$
 $x = 62.9^\circ$

c) $5 \sin(3\psi - 70^\circ) = 4$ $-90^\circ \leq \psi < 90^\circ$
 $\sin(3\psi - 70^\circ) = \frac{4}{5}$
 $\arcsin(\frac{4}{5}) = 53.13^\circ$
 $3\psi - 70^\circ = 53.13^\circ \pm 360^\circ$
 $3\psi = 123.13^\circ \pm 360^\circ$
 $\psi = 41.0^\circ \pm 120^\circ$
 $\psi = 41.0^\circ$
 $\psi = -79.0^\circ$

d) $5 - 3 \tan 2\phi = 1$ $0 \leq \phi < 180^\circ$
 $3 \tan 2\phi = 4$
 $\tan 2\phi = \frac{4}{3}$
 $\arctan(\frac{4}{3}) = 53.13^\circ$
 $2\phi = 53.13^\circ \pm 180^\circ$
 $\phi = 26.6^\circ \pm 90^\circ$
 $\phi = 26.6^\circ$
 $\phi = 116.6^\circ$

e) $2 \sin(y + 50^\circ) + 3 = 4$ $0 \leq y < 360^\circ$
 $2 \sin(y + 50^\circ) = 1$
 $\sin(y + 50^\circ) = \frac{1}{2}$
 $\arcsin(\frac{1}{2}) = 30^\circ$
 $y + 50^\circ = 30^\circ \pm 360^\circ$
 $y = -20^\circ \pm 360^\circ$
 $y = 100^\circ$
 $y = 340^\circ$

Question 5

Solve each of the following trigonometric equations in the range given.

a) $4 \cos(2\theta - 20^\circ) = 1$

$0 \leq \theta < 360^\circ$

$\theta \approx 47.8^\circ, 152.2^\circ, 227.8^\circ, 332.2^\circ$

b) $2 + \tan 4x = 5$

$-90^\circ \leq x < 90^\circ$

$x \approx -72.1^\circ, -27.1^\circ, 17.9^\circ, 62.9^\circ$

c) $7 \sin(4t + 10^\circ) = 4$

$-90^\circ \leq t < 90^\circ$

$t \approx -83.8^\circ, -56.2^\circ, 6.2^\circ, 33.8^\circ$

d) $5 \cos(4\phi) + 1 = 0$

$0 \leq \phi < 90^\circ$

$\phi \approx 25.4^\circ, 64.6^\circ$

e) $\tan(y + 50^\circ) = 3$

$0 \leq y < 360^\circ$

$y \approx 21.6^\circ, 201.6^\circ$

Handwritten solutions for Question 5:

a) $4 \cos(2\theta - 20^\circ) = 1$, $0 \leq \theta < 360^\circ$
 $\cos(2\theta - 20^\circ) = \frac{1}{4}$
 $\cos^{-1}(\frac{1}{4}) = 75.52^\circ$
 $2\theta - 20^\circ = 75.52^\circ \pm 360^\circ$
 $2\theta = 95.52^\circ \pm 360^\circ$
 $2\theta = 354.48^\circ \pm 360^\circ$
 $\theta = 47.76^\circ \pm 180^\circ$
 $\theta = 152.24^\circ \pm 180^\circ$
 $\therefore \theta = 47.8^\circ, 152.2^\circ, 227.8^\circ, 332.2^\circ$

b) $2 + \tan 4x = 5$, $-90^\circ \leq x < 90^\circ$
 $\tan 4x = 3$
 $\tan^{-1}(3) = 71.57^\circ$
 $4x = 71.57^\circ \pm 180^\circ$
 $x = 17.89^\circ \pm 45^\circ$
 $\therefore x = 17.9^\circ, 62.9^\circ, -27.1^\circ, -72.1^\circ$

c) $7 \sin(4t + 10^\circ) = 4$, $-90^\circ \leq t < 90^\circ$
 $\sin(4t + 10^\circ) = \frac{4}{7}$
 $\sin^{-1}(\frac{4}{7}) = 33.8^\circ$
 $4t + 10^\circ = 33.8^\circ \pm 360^\circ$
 $4t = 23.8^\circ \pm 360^\circ$
 $t = 5.95^\circ \pm 90^\circ$
 $t = 33.8^\circ \pm 90^\circ$
 $\therefore t = 6.2^\circ, 33.8^\circ, -83.8^\circ, -56.2^\circ$

d) $5 \cos(4\phi) + 1 = 0$, $0 \leq \phi < 90^\circ$
 $\cos(4\phi) = -\frac{1}{5}$
 $\cos^{-1}(-\frac{1}{5}) = 101.54^\circ$
 $4\phi = 101.54^\circ \pm 360^\circ$
 $\phi = 25.39^\circ \pm 90^\circ$
 $\phi = 64.61^\circ \pm 90^\circ$
 $\therefore \phi = 25.4^\circ, 64.6^\circ$

e) $\tan(y + 50^\circ) = 3$, $0 \leq y < 360^\circ$
 $\tan^{-1}(3) = 71.57^\circ$
 $y + 50^\circ = 71.57^\circ \pm 180^\circ$
 $y = 21.57^\circ \pm 180^\circ$
 $\therefore y = 21.6^\circ, 201.6^\circ$

Question 6

Solve each of the following trigonometric equations in the range given.

- a) $\cos(2\theta + 25)^\circ = -0.454$ $0 \leq \theta < 360^\circ$ $\theta \approx 46^\circ, 109^\circ, 226^\circ, 289^\circ$
- b) $\cos(2y - 35)^\circ = 0.891$ $0 \leq y < 360^\circ$ $\theta \approx 4^\circ, 31^\circ, 184^\circ, 211^\circ$
- c) $3\cos 3x - 1 = 0.22$ $-90^\circ \leq x < 90^\circ$ $x \approx -22^\circ, 22^\circ$
- d) $\tan(5y - 35)^\circ = -2 - \sqrt{3}$ $0 \leq y < 90^\circ$ $\theta \approx 28^\circ, 64^\circ$
- e) $1 + 2\sin(\theta + 25)^\circ = 2.532$ $0 \leq \theta < 360^\circ$ $\theta \approx 25^\circ, 105^\circ$

The image shows handwritten solutions for the five parts of Question 6. The solutions are written in a clear, legible hand and include all the steps from the original equations to the final answers. Key steps include using the arccos function for parts a and b, isolating the cosine term for part c, using the arctan function for part d, and isolating the sine term for part e. The final answers are boxed in the original image, matching the printed solutions above.

a) $\cos(2\theta + 25)^\circ = -0.454$
 $\arccos(-0.454) = 117.0^\circ$
 $2\theta + 25 = 117.0 \pm 360n$
 $2\theta = 92.0 \pm 360n$
 $\theta = 46.0 \pm 180n$
 $\theta = 46.0^\circ, 109.0^\circ, 226.0^\circ, 289.0^\circ$

b) $\cos(2y - 35)^\circ = 0.891$
 $\arccos(0.891) = 27.0^\circ$
 $2y - 35 = 27.0 \pm 360n$
 $2y = 62.0 \pm 360n$
 $y = 31.0 \pm 180n$
 $y = 31.0^\circ, 211.0^\circ$

c) $3\cos 3x - 1 = 0.22$
 $3\cos 3x = 1.22$
 $\cos 3x = 0.4066$
 $\arccos(0.4066) = 66.0^\circ$
 $3x = 66.0 \pm 360n$
 $x = 22.0 \pm 120n$
 $x = 22.0^\circ, -22.0^\circ$

d) $\tan(5y - 35)^\circ = -2 - \sqrt{3}$
 $\arctan(-2 - \sqrt{3}) = -75^\circ$
 $5y - 35 = -75 \pm 180n$
 $5y = -40 \pm 180n$
 $y = -8 \pm 36n$
 $y = 28^\circ, 64^\circ$

e) $1 + 2\sin(\theta + 25)^\circ = 2.532$
 $2\sin(\theta + 25)^\circ = 1.532$
 $\sin(\theta + 25)^\circ = 0.766$
 $\arcsin(0.766) = 50.0^\circ$
 $\theta + 25 = 50.0 \pm 360n$
 $\theta = 25 \pm 360n$
 $\theta = 25^\circ, 105^\circ$

Question 7

Solve each of the following trigonometric equations in the range given.

a) $\cos 2\theta = -0.7374$

$0 \leq \theta < 2\pi$

$\theta \approx 1.20^\circ, 1.94^\circ, 4.34^\circ, 5.08^\circ$

b) $\cos\left(x - \frac{\pi}{4}\right) = 0.61$

$0 \leq x < 2\pi$

$x \approx 1.70^\circ, 6.15^\circ$

c) $\sqrt{2} + 4\sin x = \sqrt{6}$

$0 \leq x < 2\pi$

$x = \frac{\pi}{12}, \frac{11\pi}{12}$

d) $2 + \tan\left(y + \frac{\pi}{3}\right) = \sqrt{3}$

$0 \leq y < 2\pi$

$y = \frac{7\pi}{12}, \frac{19\pi}{12}$

e) $4 - 2\sin 3x = 2.5$

$0 \leq x < \pi$

$x \approx 0.28^\circ, 0.76^\circ, 2.38^\circ, 2.86^\circ$

Handwritten solutions for Question 7:

a) $\cos 2\theta = -0.7374$
 $\arccos(-0.7374) = 2.40^\circ$
 $2\theta = 2.400 \pm 2\pi n$
 $2\theta = 3.883 \pm 2\pi n$
 $\theta = 1.200 \pm \pi n$
 $\theta = 1.942 \pm \pi n$
 $\theta_1 = 1.20^\circ$
 $\theta_2 = 4.34^\circ$
 $\theta_3 = 1.94^\circ$
 $\theta_4 = 5.08^\circ$

b) $\cos\left(x - \frac{\pi}{4}\right) = 0.61$
 $\arccos(0.61) = 0.9143$
 $x - \frac{\pi}{4} = 0.9146 \pm 2\pi n$
 $x - \frac{\pi}{4} = 5.365 \pm 2\pi n$
 $x = 1.700 \pm 2\pi n$
 $x = 6.154 \pm 2\pi n$
 $x = 1.70^\circ, 6.15^\circ$

c) $\sqrt{2} + 4\sin x = \sqrt{6}$
 $4\sin x = \sqrt{6} - \sqrt{2}$
 $\sin x = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\arcsin\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) = \frac{\pi}{12}$
 $x = \frac{\pi}{12} \pm 2\pi n$
 $x = \frac{11\pi}{12} \pm 2\pi n$
 $x = \frac{\pi}{12}, \frac{11\pi}{12}$

d) $2 + \tan\left(y + \frac{\pi}{3}\right) = \sqrt{3}$
 $\tan\left(y + \frac{\pi}{3}\right) = \sqrt{3} - 2$
 $\arctan(\sqrt{3} - 2) = -0.3465$
 $y + \frac{\pi}{3} = -0.3465 \pm \pi n$
 $y = -\frac{\pi}{3} - 0.3465 \pm \pi n$
 $y = -\frac{\pi}{3} \pm \pi n$
 $y_1 = \frac{7\pi}{12}$
 $y_2 = \frac{19\pi}{12}$

e) $4 - 2\sin 3x = 2.5$
 $-2\sin 3x = -1.5$
 $\sin 3x = 0.75$
 $\arcsin(0.75) = 0.8481$
 $3x = 0.8481 \pm 2\pi n$
 $3x = 2.294 \pm 2\pi n$
 $x = 0.2827 \pm \frac{2\pi n}{3}$
 $x = 0.765 \pm \frac{2\pi n}{3}$
 $x_1 = 0.28^\circ$
 $x_2 = 2.38^\circ$
 $x_3 = 0.76^\circ$
 $x_4 = 2.86^\circ$

Question 8

Solve each of the following trigonometric equations in the range given.

a) $\sin 2x = \sin 48^\circ$

$0 \leq x < 360^\circ$

$x = 24^\circ, 66^\circ, 204^\circ, 246^\circ$

b) $\cos 3x = \cos 96^\circ$

$0 \leq x < 180^\circ$

$x = 32^\circ, 88^\circ, 152^\circ$

c) $\tan(\theta + 50^\circ) = \tan 12^\circ$

$0 \leq \theta < 360^\circ$

$\theta = 142^\circ, 322^\circ$

d) $\sin 2x = \sin 324^\circ$

$0 \leq x < 360^\circ$

$x = 108^\circ, 162^\circ, 288^\circ, 342^\circ$

e) $\cos 4x = \cos 240^\circ$

$0 \leq x < 180^\circ$

$x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$

Handwritten solutions for Question 8:

(a) $\sin 2x = \sin 48^\circ$
 $2x = 48^\circ \pm 360^\circ n$
 $2x = 152^\circ \pm 360^\circ n$
 $x = 24^\circ \pm 180^\circ n$
 $x = 66^\circ \pm 180^\circ n$
 $x = 24^\circ, 66^\circ, 204^\circ, 246^\circ$

(b) $\cos 3x = \cos 96^\circ$
 $3x = 96^\circ \pm 360^\circ n$
 $3x = 264^\circ \pm 360^\circ n$
 $x = 32^\circ \pm 120^\circ n$
 $x = 88^\circ \pm 120^\circ n$
 $x = 32^\circ, 88^\circ, 152^\circ, 216^\circ$

(c) $\tan(\theta + 50^\circ) = \tan 12^\circ$
 $\theta + 50^\circ = 12^\circ \pm 180^\circ n$
 $\theta = -38^\circ \pm 180^\circ n$
 $\theta = 142^\circ, 322^\circ$

(d) $\sin 2x = \sin 324^\circ$
 $2x = 324^\circ \pm 360^\circ n$
 $2x = -36^\circ \pm 360^\circ n$
 $x = 162^\circ \pm 180^\circ n$
 $x = 108^\circ \pm 180^\circ n$
 $x = 108^\circ, 162^\circ, 288^\circ, 342^\circ$

(e) $\cos 4x = \cos 240^\circ$
 $4x = 240^\circ \pm 360^\circ n$
 $4x = 120^\circ \pm 360^\circ n$
 $x = 60^\circ \pm 90^\circ n$
 $x = 30^\circ \pm 90^\circ n$
 $x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$

Question 9

Solve each of the following trigonometric equations in the range given.

a) $\sin x = \frac{1}{2}$

$0 \leq x < 2\pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

b) $\cos 2x = \frac{1}{2}$

$0 \leq x < 2\pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

c) $\tan\left(\theta - \frac{\pi}{9}\right) = \sqrt{3}$

$-\pi \leq \theta < \pi$

$\theta = -\frac{5\pi}{9}, \frac{4\pi}{9}$

d) $\sin\left(2\theta + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$-\pi \leq \theta < \pi$

$\theta = -\frac{11\pi}{12}, -\frac{3\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}$

e) $\cos\left(3x - \frac{\pi}{4}\right) = -\frac{1}{2}$

$0 \leq x < \pi$

$x = \frac{11\pi}{36}, \frac{19\pi}{36}, \frac{35\pi}{36}$

Handwritten solutions for Question 9:

(a) $\sin x = \frac{1}{2}$
 $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$
 $x = \frac{\pi}{6} \pm 2\pi n$
 $x = \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\cos 2x = \frac{1}{2}$
 $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$
 $2x = \frac{\pi}{3} \pm 2\pi n$
 $x = \frac{\pi}{6} \pm \pi n$
 $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(c) $\tan\left(\theta - \frac{\pi}{9}\right) = \sqrt{3}$
 $\arctan(\sqrt{3}) = \frac{\pi}{3}$
 $\theta - \frac{\pi}{9} = \frac{\pi}{3} \pm \pi n$
 $\theta = \frac{4\pi}{9} \pm \pi n$
 $\theta = \frac{4\pi}{9}, -\frac{5\pi}{9}$

(d) $\sin\left(2\theta + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$
 $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
 $2\theta + \frac{\pi}{6} = \frac{\pi}{3} \pm 2\pi n$
 $2\theta = \frac{\pi}{6} \pm 2\pi n$
 $\theta = \frac{\pi}{12} \pm \pi n$
 $\theta = \frac{\pi}{12}, -\frac{11\pi}{12}, \frac{\pi}{4}, -\frac{3\pi}{4}$

(e) $\cos\left(3x - \frac{\pi}{4}\right) = -\frac{1}{2}$
 $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$
 $3x - \frac{\pi}{4} = \frac{2\pi}{3} \pm 2\pi n$
 $3x = \frac{11\pi}{12} \pm 2\pi n$
 $x = \frac{11\pi}{36} \pm \frac{2\pi n}{3}$
 $x = \frac{11\pi}{36}, \frac{19\pi}{36}, \frac{35\pi}{36}$

Question 10

Solve each of the following trigonometric equations in the range given.

a) $\tan\left(\frac{3x}{2}\right) = -\frac{\sqrt{3}}{3}$

$0 \leq x < 2\pi$

$x = \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{17\pi}{9}$

b) $\sin\left(4y - \frac{\pi}{4}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$

$-\frac{\pi}{2} \leq y < \frac{\pi}{2}$

$y = -\frac{5\pi}{12}, -\frac{5\pi}{24}, \frac{\pi}{12}, \frac{7\pi}{24}$

c) $5\cos 2x = 2$

$0 \leq x < 2\pi$

$x = 0.58^\circ, 2.56^\circ, 3.72^\circ, 5.70^\circ$

d) $3 - \tan\left(\theta - \frac{\pi}{18}\right) = 1$

$0 \leq \theta < 2\pi$

$\theta = 1.28^\circ, 4.42^\circ$

e) $3\sin\left(3\theta - \frac{\pi}{6}\right) + 4 = 5$

$-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

$\theta = -0.99^\circ, 0.29^\circ, 1.11^\circ$

Handwritten solutions for Question 10:

(a) $\tan\left(\frac{3x}{2}\right) = -\frac{\sqrt{3}}{3}$
 $\theta = \arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$
 $\left(\frac{3x}{2}\right) = -\frac{\pi}{6} + n\pi$
 $3x = -\frac{\pi}{3} + 2n\pi$
 $x = -\frac{\pi}{9} + \frac{2n\pi}{3}$
 $x_1 = -\frac{\pi}{9}$
 $x_2 = \frac{5\pi}{9}$
 $x_3 = \frac{11\pi}{9}$

(b) $\sin\left(4y - \frac{\pi}{4}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$
 $\arcsin\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) = \frac{\pi}{12}$
 $4y - \frac{\pi}{4} = \frac{\pi}{12} + 2n\pi$
 $4y = \frac{\pi}{3} + 2n\pi$
 $y = \frac{\pi}{12} + \frac{n\pi}{2}$
 $y_1 = \frac{\pi}{12}$
 $y_2 = \frac{5\pi}{12}$
 $y_3 = \frac{13\pi}{12}$
 $y_4 = \frac{17\pi}{12}$

(c) $5\cos 2x = 2$
 $\cos 2x = \frac{2}{5}$
 $\arccos\left(\frac{2}{5}\right) = 1.107^\circ$
 $2x = 1.107^\circ + 2n\pi$
 $x = 0.553^\circ + n\pi$
 $x_1 = 0.553^\circ$
 $x_2 = 3.73^\circ$
 $x_3 = 4.88^\circ$
 $x_4 = 6.04^\circ$

(d) $3 - \tan\left(\theta - \frac{\pi}{18}\right) = 1$
 $\tan\left(\theta - \frac{\pi}{18}\right) = 1$
 $\arctan(1) = \frac{\pi}{4}$
 $\theta - \frac{\pi}{18} = \frac{\pi}{4} + n\pi$
 $\theta = \frac{5\pi}{18} + n\pi$
 $\theta_1 = 1.28^\circ$
 $\theta_2 = 4.42^\circ$

(e) $3\sin\left(3\theta - \frac{\pi}{6}\right) + 4 = 5$
 $3\sin\left(3\theta - \frac{\pi}{6}\right) = 1$
 $\sin\left(3\theta - \frac{\pi}{6}\right) = \frac{1}{3}$
 $\arcsin\left(\frac{1}{3}\right) = 0.339^\circ$
 $3\theta - \frac{\pi}{6} = 0.339^\circ + 2n\pi$
 $3\theta = 0.339^\circ + \frac{\pi}{6} + 2n\pi$
 $\theta = 0.113^\circ + \frac{\pi}{18} + \frac{2n\pi}{3}$
 $\theta_1 = 0.113^\circ$
 $\theta_2 = 1.04^\circ$
 $\theta_3 = 2.19^\circ$

Question 11

Solve each of the following trigonometric equations in the range given.

a) $4 \cos\left(2\theta - \frac{\pi}{9}\right) = 1$

$0 \leq \theta < 2\pi$

$\theta \approx 0.83^\circ, 2.66^\circ, 3.98^\circ, 5.80^\circ$

b) $2 + \tan 4x = 5$

$-\frac{\pi}{2} \leq x < \frac{\pi}{2}$

$x \approx -1.26^\circ, -0.47^\circ, 0.31^\circ, 1.10^\circ$

c) $7 \sin\left(4t + \frac{\pi}{18}\right) = 4$

$-\frac{\pi}{2} \leq t < \frac{\pi}{2}$

$t \approx -1.46^\circ, -0.98^\circ, 0.11^\circ, 0.59^\circ$

d) $5 \cos 4\phi + 1 = 0$

$0 \leq \phi < \frac{\pi}{2}$

$\phi \approx 0.44^\circ, 1.13^\circ$

e) $\tan\left(y + \frac{5\pi}{18}\right) = 3$

$0 \leq y < 2\pi$

$y \approx 0.38^\circ, 3.52^\circ$

Handwritten solutions for Question 11:

(a) $4 \cos\left(2\theta - \frac{\pi}{9}\right) = 1$
 $\cos\left(2\theta - \frac{\pi}{9}\right) = \frac{1}{4}$
 $\arccos\left(\frac{1}{4}\right) = 1.107^\circ$
 $2\theta - \frac{\pi}{9} = 1.107^\circ \pm 2\pi n$
 $2\theta = 1.107^\circ \pm 2\pi n + \frac{\pi}{9}$
 $2\theta = 5.314 \pm 2\pi n$
 $\theta = 2.657 \pm \pi n$
 $\therefore \theta = 0.83^\circ, 2.66^\circ, 3.98^\circ, 5.80^\circ$

(b) $2 + \tan 4x = 5$
 $\tan 4x = 3$
 $\arctan(3) = 1.107^\circ$
 $4x = 1.107^\circ \pm \pi n$
 $x = 0.277 \pm \frac{\pi n}{4}$
 $\therefore x = -1.26^\circ, -0.47^\circ, 0.31^\circ, 1.10^\circ$

(c) $7 \sin\left(4t + \frac{\pi}{18}\right) = 4$
 $\sin\left(4t + \frac{\pi}{18}\right) = \frac{4}{7}$
 $\arcsin\left(\frac{4}{7}\right) = 0.648^\circ$
 $4t + \frac{\pi}{18} = 0.648^\circ \pm 2\pi n$
 $4t = 0.648^\circ \pm 2\pi n - \frac{\pi}{18}$
 $t = 0.044 \pm \frac{\pi n}{4}$
 $\therefore t = -1.46^\circ, -0.98^\circ, 0.11^\circ, 0.59^\circ$

(d) $5 \cos 4\phi + 1 = 0$
 $5 \cos 4\phi = -1$
 $\cos 4\phi = -\frac{1}{5}$
 $\arccos\left(-\frac{1}{5}\right) = 1.107^\circ$
 $4\phi = 1.107^\circ \pm 2\pi n$
 $\phi = 0.277 \pm \frac{\pi n}{4}$
 $\therefore \phi = 0.44^\circ, 1.13^\circ$

(e) $\tan\left(y + \frac{5\pi}{18}\right) = 3$
 $\tan\left(y + \frac{5\pi}{18}\right) = 3$
 $\arctan(3) = 1.107^\circ$
 $y + \frac{5\pi}{18} = 1.107^\circ \pm \pi n$
 $y = 0.38^\circ \pm \pi n$
 $\therefore y = 0.38^\circ, 3.52^\circ$

Question 12

Solve each of the following trigonometric equations in the range given.

a) $2 \sin 3x - 1 = 0.25$

$0 \leq \theta < \pi$

$x \approx 0.225^\circ, 0.822^\circ, 2.32^\circ, 2.92^\circ$

b) $5 \cos\left(\theta + \frac{\pi}{3}\right) - 1 = 2$

$0 \leq x < 2\pi$

$\theta = 4.31^\circ, 6.16^\circ$

c) $1 - \tan 4y = 3$

$0 \leq y < \pi$

$y \approx 0.509^\circ, 1.29^\circ, 2.08^\circ, 2.87^\circ$

d) $2 \sin\left(2\varphi - \frac{\pi}{3}\right) = 1$

$0 \leq \varphi < 2\pi$

$\varphi = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}$

e) $2 \cos\left(3t + \frac{\pi}{3}\right) = \sqrt{3}$

$-\frac{\pi}{2} \leq t < \frac{\pi}{2}$

$t = -\frac{\pi}{6}, -\frac{\pi}{18}$

Handwritten solutions for Question 12:

a) $2 \sin 3x - 1 = 0.25 \Rightarrow \sin 3x = 0.625$
 $3x = \sin^{-1}(0.625) \approx 0.675^\circ$
 $3x = 0.675^\circ + 2\pi n$
 $x = 0.225^\circ + \frac{2\pi n}{3}$
 $x = 0.225^\circ, 0.822^\circ, 2.32^\circ, 2.92^\circ$

b) $5 \cos\left(\theta + \frac{\pi}{3}\right) - 1 = 2 \Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \frac{3}{5}$
 $\theta + \frac{\pi}{3} = \cos^{-1}\left(\frac{3}{5}\right) \approx 0.927^\circ$
 $\theta = 0.927^\circ - \frac{\pi}{3} \approx 4.31^\circ$
 $\theta = 0.927^\circ - \frac{\pi}{3} + 2\pi \approx 6.16^\circ$

c) $1 - \tan 4y = 3 \Rightarrow \tan 4y = -2$
 $4y = \tan^{-1}(-2) \approx -1.107^\circ$
 $y = -0.277^\circ + \frac{\pi}{4}$
 $y = 0.509^\circ, 1.29^\circ, 2.08^\circ, 2.87^\circ$

d) $2 \sin\left(2\varphi - \frac{\pi}{3}\right) = 1 \Rightarrow \sin\left(2\varphi - \frac{\pi}{3}\right) = \frac{1}{2}$
 $2\varphi - \frac{\pi}{3} = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
 $2\varphi = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$
 $\varphi = \frac{\pi}{4}$
 $2\varphi - \frac{\pi}{3} = \frac{\pi}{6} + 2\pi$
 $2\varphi = \frac{\pi}{6} + \frac{\pi}{3} + 2\pi = \frac{5\pi}{6} + 2\pi$
 $\varphi = \frac{5\pi}{12}$
 $2\varphi - \frac{\pi}{3} = \frac{\pi}{6} + 4\pi$
 $2\varphi = \frac{\pi}{6} + \frac{\pi}{3} + 4\pi = \frac{5\pi}{6} + 4\pi$
 $\varphi = \frac{5\pi}{12} + 2\pi$

e) $2 \cos\left(3t + \frac{\pi}{3}\right) = \sqrt{3} \Rightarrow \cos\left(3t + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$
 $3t + \frac{\pi}{3} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$
 $3t = \frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{6}$
 $t = -\frac{\pi}{18}$
 $3t + \frac{\pi}{3} = \frac{\pi}{6} + 2\pi$
 $3t = \frac{\pi}{6} - \frac{\pi}{3} + 2\pi = \frac{11\pi}{6}$
 $t = \frac{11\pi}{18}$

Question 13

Solve each of the following trigonometric equations in the range given.

a) $\sin 2x = \sin\left(\frac{4\pi}{15}\right)$ $0 \leq \theta < 2\pi$

$$x = \frac{2\pi}{15}, \frac{11\pi}{30}, \frac{17\pi}{15}, \frac{41\pi}{30}$$

b) $\cos 3x = \cos\left(\frac{8\pi}{15}\right)$ $0 \leq x < \pi$

$$x = \frac{8\pi}{45}, \frac{22\pi}{45}, \frac{38\pi}{45}$$

c) $\tan\left(\theta + \frac{5\pi}{18}\right) = \tan\left(\frac{\pi}{12}\right)$ $0 \leq \theta < 2\pi$

$$\theta = \frac{29\pi}{36}, \frac{65\pi}{36}$$

d) $\sin 2x = \sin\left(\frac{29\pi}{15}\right)$ $0 \leq x < 2\pi$

$$x = \frac{8\pi}{15}, \frac{29\pi}{30}, \frac{23\pi}{15}, \frac{59\pi}{30}$$

e) $\cos 4x = \cos\left(\frac{4\pi}{3}\right)$ $0 \leq x < \pi$

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$$

Question 14

Solve each of the following trigonometric equations in the range given.

a) $\sin \theta = 2 \cos \theta$ $0 \leq \theta < 360^\circ$ $\theta \approx 63.4^\circ, 243.4^\circ$

b) $\sin \theta + 3 \cos \theta = 0$ $0 \leq \theta < 360^\circ$ $\theta \approx 108.4^\circ, 288.4^\circ$

c) $4 \cos x - 3 \sin x = 0$ $0 \leq x < 360^\circ$ $x \approx 53.1^\circ, 233.1^\circ$

d) $2 \sin x = \cos x$ $0 \leq x < 360^\circ$ $x \approx 26.6^\circ, 206.6^\circ$

e) $\sin 2\theta + \cos 2\theta = 0$ $0 \leq \theta < 360^\circ$ $\theta = 67.5^\circ, 157.5^\circ, 247.5^\circ, 337.5^\circ$

f) $\sqrt{3} \sin 3\theta = 3 \cos 3\theta$ $-90^\circ \leq \theta < 90^\circ$ $\theta = -40^\circ, 20^\circ, 80^\circ$

Handwritten solutions for Question 14:

- $\sin \theta = 2 \cos \theta$
 $\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{2 \cos \theta}{\cos \theta}$
 $\Rightarrow \tan \theta = 2$
 $\bullet \arctan(2) = 63.4^\circ$
 $(\theta = 63.4^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $\theta_1 = 63.4^\circ$
 $\theta_2 = 243.4^\circ$
- $\sin \theta + 3 \cos \theta = 0$
 $\Rightarrow \frac{\sin \theta}{\cos \theta} + \frac{3 \cos \theta}{\cos \theta} = \frac{0}{\cos \theta}$
 $\Rightarrow \tan \theta + 3 = 0$
 $\Rightarrow \tan \theta = -3$
 $\bullet \arctan(-3) = -71.6^\circ$
 $(\theta = -71.6^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $\theta_1 = 108.4^\circ$
 $\theta_2 = 288.4^\circ$
- $4 \cos x - 3 \sin x = 0$
 $\Rightarrow \frac{4 \cos x}{\cos x} - \frac{3 \sin x}{\cos x} = \frac{0}{\cos x}$
 $\Rightarrow 4 - 3 \tan x = 0$
 $\Rightarrow 4 = 3 \tan x$
 $\Rightarrow \tan x = \frac{4}{3}$
 $\bullet \arctan(\frac{4}{3}) = 53.1^\circ$
 $(x = 53.1^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $x_1 = 53.1^\circ$
 $x_2 = 233.1^\circ$
- $2 \sin x = \cos x$
 $\Rightarrow \frac{2 \sin x}{\cos x} = \frac{\cos x}{\cos x}$
 $\Rightarrow 2 \tan x = 1$
 $\Rightarrow \tan x = \frac{1}{2}$
 $\bullet \arctan(\frac{1}{2}) = 26.6^\circ$
 $(x = 26.6^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $x_1 = 26.6^\circ$
 $x_2 = 206.6^\circ$
- $\sin 2\theta + \cos 2\theta = 0$
 $\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos 2\theta}{\cos 2\theta} = \frac{0}{\cos 2\theta}$
 $\Rightarrow \tan 2\theta + 1 = 0$
 $\Rightarrow \tan 2\theta = -1$
 $\bullet \arctan(-1) = -45^\circ$
 $(2\theta = -45^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $(\theta = -22.5^\circ \pm 90^\circ)$
 $\theta = 67.5^\circ, 157.5^\circ, 247.5^\circ, 337.5^\circ$
- $\sqrt{3} \sin 3\theta = 3 \cos 3\theta$
 $\Rightarrow \frac{\sqrt{3} \sin 3\theta}{\cos 3\theta} = \frac{3 \cos 3\theta}{\cos 3\theta}$
 $\Rightarrow \sqrt{3} \tan 3\theta = 3$
 $\Rightarrow \tan 3\theta = \frac{3}{\sqrt{3}}$
 $\bullet \arctan(\frac{3}{\sqrt{3}}) = 60^\circ$
 $(3\theta = 60^\circ \pm 180^\circ \quad n=0,1,2,3, \dots)$
 $(\theta = 20^\circ \pm 60^\circ)$
 $\theta = -40^\circ, 20^\circ, 80^\circ$

Question 15

Solve each of the following trigonometric equations in the range given.

a) $\cos x = 3 \sin x$ $0 \leq x < 360^\circ$ $x \approx 18.4^\circ, 198.4^\circ$

b) $4 \cos x - 7 \sin x = 0$ $0 \leq x < 360^\circ$ $x \approx 29.7^\circ, 209.7^\circ$

c) $5 \sin x + 6 \cos x = 0$ $0 \leq x < 360^\circ$ $x \approx 129.8^\circ, 309.8^\circ$

d) $3 \sin 2\theta = \sqrt{3} \cos 2\theta$ $-180 \leq \theta < 180^\circ$ $\theta = -165^\circ, -75^\circ, 15^\circ, 105^\circ$

e) $\sin 3\theta = \cos 3\theta$ $0 \leq \theta < 180^\circ$ $\theta = 15^\circ, 75^\circ, 135^\circ$

f) $5 \cos 2x + 9 \sin 2x = 0$ $0 \leq x < 180^\circ$ $x \approx 75.5^\circ, 165.5^\circ$

Handwritten solutions for Question 15:

a) $\cos x = 3 \sin x$
 $\frac{\cos x}{\sin x} = \frac{3 \sin x}{\sin x}$
 $\frac{1}{\tan x} = 3 \tan x$
 $\tan x = \frac{1}{3}$
 $\arctan\left(\frac{1}{3}\right) = 18.43^\circ$
 $(x = 18.4 \pm 180^\circ) \text{ mod } 360^\circ$
 $\therefore x_1 = 18.4^\circ$
 $x_2 = 198.4^\circ$

b) $4 \cos x - 7 \sin x = 0$
 $\frac{4 \cos x}{\sin x} - \frac{7 \sin x}{\sin x} = \frac{0}{\sin x}$
 $\frac{4}{\tan x} - 7 \tan x = 0$
 $\tan x = \frac{4}{7}$
 $\arctan\left(\frac{4}{7}\right) = 29.7^\circ$
 $(x = 29.7 \pm 180^\circ) \text{ mod } 360^\circ$
 $\therefore x_1 = 29.7^\circ$
 $x_2 = 209.7^\circ$

c) $5 \sin x + 6 \cos x = 0$
 $\frac{5 \sin x}{\cos x} + \frac{6 \cos x}{\cos x} = \frac{0}{\cos x}$
 $\frac{5 \sin x}{\cos x} + 6 = 0$
 $\tan x = -\frac{6}{5}$
 $\arctan\left(-\frac{6}{5}\right) = -50.19^\circ$
 $(x = -50.2 \pm 180^\circ) \text{ mod } 360^\circ$
 $\therefore x_1 = 129.8^\circ$
 $x_2 = 309.8^\circ$

d) $3 \sin 2\theta = \sqrt{3} \cos 2\theta$
 $\frac{3 \sin 2\theta}{\cos 2\theta} = \frac{\sqrt{3} \cos 2\theta}{\cos 2\theta}$
 $3 \tan 2\theta = \sqrt{3}$
 $\tan 2\theta = \frac{\sqrt{3}}{3}$
 $\arctan\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$
 $(2\theta = 30 \pm 180^\circ) \text{ mod } 360^\circ$
 $(\theta = 15 \pm 90^\circ)$
 $\therefore \theta = 15^\circ, 105^\circ, -75^\circ, -165^\circ$

e) $\sin 3\theta = \cos 3\theta$
 $\frac{\sin 3\theta}{\cos 3\theta} = \frac{\cos 3\theta}{\cos 3\theta}$
 $\tan 3\theta = 1$
 $\arctan(1) = 45^\circ$
 $(3\theta = 45 \pm 180^\circ) \text{ mod } 360^\circ$
 $(\theta = 15 \pm 60^\circ)$
 $\therefore \theta = 15^\circ, 75^\circ, 135^\circ$

f) $5 \cos 2x + 9 \sin 2x = 0$
 $\frac{5 \cos 2x}{\sin 2x} + \frac{9 \sin 2x}{\sin 2x} = \frac{0}{\sin 2x}$
 $\frac{5}{\tan 2x} + 9 \tan 2x = 0$
 $\tan 2x = -\frac{5}{9}$
 $\arctan\left(-\frac{5}{9}\right) = -29.65^\circ$
 $(2x = -29.65 \pm 180^\circ) \text{ mod } 360^\circ$
 $(x = -14.82 \pm 90^\circ)$
 $\therefore x = 75.5^\circ, 165.5^\circ$

Question 16

Solve each of the following trigonometric equations in the range given.

a) $\sin \theta = 2 \cos \theta$

$0 \leq \theta < 2\pi$

$\theta \approx 1.11^\circ, 4.25^\circ$

b) $\sin \theta + 3 \cos \theta = 0$

$0 \leq \theta < 2\pi$

$\theta \approx 1.89^\circ, 5.03^\circ$

c) $4 \cos x - 3 \sin x = 0$

$0 \leq x < 2\pi$

$x \approx 0.927^\circ, 4.07^\circ$

d) $2 \sin x = \cos x$

$0 \leq x < 2\pi$

$x \approx 0.464^\circ, 3.61^\circ$

e) $\sin 2\theta + \cos 2\theta = 0$

$0 \leq \theta < 2\pi$

$\theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

f) $\sqrt{3} \sin 3\theta = 3 \cos 3\theta$

$-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

$\theta = -\frac{2\pi}{9}, \frac{\pi}{9}, \frac{4\pi}{9}$

Handwritten solutions for Question 16:

a) $\sin \theta = 2 \cos \theta$, $0 \leq \theta < 2\pi$
 $\Rightarrow \frac{\sin \theta}{\cos \theta} = 2$
 $\Rightarrow \tan \theta = 2$
 $\Rightarrow \theta = 1.107^\circ \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore \theta_1 = 1.107^\circ$
 $\theta_2 = 4.25^\circ$

b) $\sin \theta + 3 \cos \theta = 0$, $0 \leq \theta < 2\pi$
 $\Rightarrow \frac{\sin \theta}{\cos \theta} = -3$
 $\Rightarrow \tan \theta = -3$
 $\Rightarrow \theta = 1.107^\circ \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore \theta_1 = 1.107^\circ$
 $\theta_2 = 5.03^\circ$

c) $4 \cos x - 3 \sin x = 0$, $0 \leq x < 2\pi$
 $\Rightarrow \frac{4 \cos x}{3 \sin x} = 1$
 $\Rightarrow \frac{4}{3} \cot x = 1$
 $\Rightarrow \cot x = \frac{3}{4}$
 $\Rightarrow \tan x = \frac{4}{3}$
 $\Rightarrow x = 0.927^\circ \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore x_1 = 0.927^\circ$
 $x_2 = 4.07^\circ$

d) $2 \sin x = \cos x$, $0 \leq x < 2\pi$
 $\Rightarrow \frac{2 \sin x}{\cos x} = 1$
 $\Rightarrow 2 \tan x = 1$
 $\Rightarrow \tan x = \frac{1}{2}$
 $\Rightarrow x = 0.464^\circ \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore x_1 = 0.464^\circ$
 $x_2 = 3.61^\circ$

e) $\sin 2\theta + \cos 2\theta = 0$, $0 \leq \theta < 2\pi$
 $\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = -1$
 $\Rightarrow \tan 2\theta = -1$
 $\Rightarrow 2\theta = \frac{3\pi}{4} \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore \theta = \frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$

f) $\sqrt{3} \sin 3\theta = 3 \cos 3\theta$, $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$
 $\Rightarrow \frac{\sqrt{3} \sin 3\theta}{3 \cos 3\theta} = 1$
 $\Rightarrow \frac{\sqrt{3}}{3} \tan 3\theta = 1$
 $\Rightarrow \tan 3\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$
 $\Rightarrow 3\theta = \frac{\pi}{3} \pm n\pi$, $n = 0, 1, 2, \dots$
 $\therefore \theta = \frac{\pi}{9}, \frac{4\pi}{9}, -\frac{2\pi}{9}$

Question 17

Solve each of the following trigonometric equations in the range given.

a) $\cos x = 3 \sin x$

$0 \leq x < 2\pi$

$x \approx 0.32^\circ, 3.46^\circ$

b) $4 \cos x - 7 \sin x = 0$

$0 \leq x < 2\pi$

$x \approx 0.52^\circ, 3.66^\circ$

c) $5 \sin x + 6 \cos x = 0$

$0 \leq x < 2\pi$

$x \approx 2.27^\circ, 5.41^\circ$

d) $3 \sin 2\theta = \sqrt{3} \cos 2\theta$

$-\pi \leq \theta < \pi$

$\theta = -\frac{11\pi}{12}, -\frac{5\pi}{12}, \frac{\pi}{12}, \frac{7\pi}{12}$

e) $\sin 3\theta = \cos 3\theta$

$0 \leq \theta < \pi$

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

f) $5 \cos 2\theta + 9 \sin 2\theta = 0$

$0 \leq \theta < \pi$

$\theta \approx 1.32^\circ, 2.89^\circ$

Handwritten solutions for Question 17:

a) $\cos x = 3 \sin x$
 $\Rightarrow \frac{\cos x}{\sin x} = \frac{3 \sin x}{\sin x}$
 $\Rightarrow 1 = 3 \tan x$
 $\Rightarrow \tan x = \frac{1}{3}$
 $\Rightarrow \arctan\left(\frac{1}{3}\right) = 0.32^\circ$
 $(x = 0.32^\circ \pm n\pi, n \in \mathbb{Z})$
 $\therefore x = 0.32^\circ, 3.46^\circ$

b) $4 \cos x - 7 \sin x = 0$
 $\Rightarrow \frac{4 \cos x}{\cos x} - \frac{7 \sin x}{\cos x} = \frac{0}{\cos x}$
 $\Rightarrow 4 - 7 \tan x = 0$
 $\Rightarrow 4 = 7 \tan x$
 $\Rightarrow \tan x = \frac{4}{7}$
 $\Rightarrow \arctan\left(\frac{4}{7}\right) = 0.52^\circ$
 $(x = 0.52^\circ \pm n\pi, n \in \mathbb{Z})$
 $\therefore x = 0.52^\circ, 3.66^\circ$

c) $5 \sin x + 6 \cos x = 0$
 $\Rightarrow \frac{5 \sin x}{\sin x} + \frac{6 \cos x}{\sin x} = \frac{0}{\sin x}$
 $\Rightarrow 5 + 6 \cot x = 0$
 $\Rightarrow 6 \cot x = -5$
 $\Rightarrow \cot x = -\frac{5}{6}$
 $\Rightarrow \arctan\left(-\frac{5}{6}\right) = -0.57^\circ$
 $(x = -0.57^\circ \pm n\pi, n \in \mathbb{Z})$
 $\therefore x = 2.27^\circ, 5.41^\circ$

d) $3 \sin 2\theta = \sqrt{3} \cos 2\theta$
 $\Rightarrow \frac{3 \sin 2\theta}{\cos 2\theta} = \frac{\sqrt{3} \cos 2\theta}{\cos 2\theta}$
 $\Rightarrow 3 \tan 2\theta = \sqrt{3}$
 $\Rightarrow \tan 2\theta = \frac{\sqrt{3}}{3}$
 $\Rightarrow \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$
 $(2\theta = \frac{\pi}{6} \pm n\pi, n \in \mathbb{Z})$
 $\therefore \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

e) $\sin 3\theta = \cos 3\theta$
 $\Rightarrow \frac{\sin 3\theta}{\cos 3\theta} = \frac{\cos 3\theta}{\cos 3\theta}$
 $\Rightarrow \tan 3\theta = 1$
 $\Rightarrow \arctan(1) = \frac{\pi}{4}$
 $(3\theta = \frac{\pi}{4} \pm n\pi, n \in \mathbb{Z})$
 $\therefore \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$

f) $5 \cos 2\theta + 9 \sin 2\theta = 0$
 $\Rightarrow \frac{5 \cos 2\theta}{\cos 2\theta} + \frac{9 \sin 2\theta}{\cos 2\theta} = \frac{0}{\cos 2\theta}$
 $\Rightarrow 5 + 9 \tan 2\theta = 0$
 $\Rightarrow 9 \tan 2\theta = -5$
 $\Rightarrow \tan 2\theta = -\frac{5}{9}$
 $\Rightarrow \arctan\left(-\frac{5}{9}\right) = -0.51^\circ$
 $(2\theta = -0.51^\circ \pm n\pi, n \in \mathbb{Z})$
 $\therefore \theta = -0.26^\circ \pm \frac{n\pi}{2}$
 $\therefore \theta = 1.32^\circ, 2.89^\circ$

Question 18

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2\theta + 3\cos\theta - 2 = 0$ $0 \leq \theta < 360^\circ$

$\theta = 60^\circ, 300^\circ$

b) $6\sin^2x + 5\sin x + 1 = 0$ $0 \leq x < 360^\circ$

$x = 210^\circ, 330^\circ, x \approx 340.5^\circ, 199.5^\circ$

c) $2\tan^2\theta - \tan\theta - 6 = 0$ $0 \leq \theta < 360^\circ$

$\theta \approx 63.4^\circ, 123.7^\circ, 243.4^\circ, 303.7^\circ$

Handwritten solutions for Question 18:

(a) $2\cos^2\theta + 3\cos\theta - 2 = 0$
 $\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$
 $\therefore \cos\theta = \frac{1}{2}$
 $\arccos(\frac{1}{2}) = 60^\circ$
 $\theta = 60^\circ \pm 360^\circ n$
 $\theta = 300^\circ \pm 360^\circ n$
 $n = 0, 1, 2, \dots$
 $\theta = 60^\circ, 300^\circ$

(b) $6\sin^2x + 5\sin x + 1 = 0$
 $\Rightarrow (2\sin x + 1)(3\sin x + 1) = 0$
 $\therefore \sin x = -\frac{1}{2}$
 $\arcsin(-\frac{1}{2}) = -30^\circ$
 $x = -30^\circ \pm 360^\circ n$
 $x = 210^\circ \pm 360^\circ n$
 $\arcsin(-\frac{1}{2}) = -19.47^\circ$
 $x = -19.47^\circ \pm 360^\circ n$
 $x = 199.47^\circ \pm 360^\circ n$
 $n = 0, 1, 2, \dots$
 $x = 210^\circ, 330^\circ, 199.5^\circ, 340.5^\circ$

(c) $2\tan^2\theta - \tan\theta - 6 = 0$
 $\Rightarrow (2\tan\theta + 3)(\tan\theta - 2) = 0$
 $\therefore \tan\theta = -\frac{3}{2}$
 $\arctan(-\frac{3}{2}) = -63.4^\circ$
 $\theta = -63.4^\circ \pm 180^\circ n$
 $\arctan(-\frac{3}{2}) = -56.3^\circ$
 $\theta = -56.3^\circ \pm 180^\circ n$
 $n = 0, 1, 2, \dots$
 $\theta = 123.7^\circ, 243.4^\circ, 303.7^\circ, 63.4^\circ$

Question 19

Solve each of the following trigonometric equations in the range given.

a) $3\cos^2\theta - 13\cos\theta + 4 = 0$ $0 \leq \theta < 360^\circ$

$\theta \approx 70.5^\circ, 289.5^\circ$

b) $8\sin^2x + 6\sin x + 1 = 0$ $0 \leq x < 360^\circ$

$x = 210^\circ, 330^\circ, x \approx 194.5^\circ, 345.5^\circ$

c) $3\tan^2\theta + 8\tan\theta - 3 = 0$ $0 \leq \theta < 360^\circ$

$\theta \approx 18.4^\circ, 108.4^\circ, 198.4^\circ, 288.4^\circ$

Handwritten solutions for the trigonometric equations in Question 19:

a) $3\cos^2\theta - 13\cos\theta + 4 = 0$
 $(3\cos\theta - 1)(\cos\theta - 4) = 0$
 $\cos\theta = \frac{1}{3}$
 $\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.53^\circ$
 $\theta = 70.53^\circ \pm 360^\circ$
 $\theta = 289.47^\circ \pm 360^\circ$
 $\theta = 70.5^\circ$
 $\theta = 289.5^\circ$

b) $8\sin^2x + 6\sin x + 1 = 0$
 $(4\sin x + 1)(2\sin x + 1) = 0$
 $\sin x = -\frac{1}{4}$
 $\sin x = -\frac{1}{2}$
 $\bullet \sin x = -\frac{1}{4} \Rightarrow x = \sin^{-1}\left(-\frac{1}{4}\right) = -14.48^\circ$
 $x = -14.48^\circ \pm 360^\circ$
 $x = 345.52^\circ \pm 360^\circ$
 $\bullet \sin x = -\frac{1}{2} \Rightarrow x = \sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ$
 $x = -30^\circ \pm 360^\circ$
 $x = 330^\circ \pm 360^\circ$
 $\therefore x = 210^\circ, 330^\circ, 345.5^\circ, 194.5^\circ$

c) $3\tan^2\theta + 8\tan\theta - 3 = 0$
 $(3\tan\theta - 1)(\tan\theta + 3) = 0$
 $\tan\theta = \frac{1}{3}$
 $\bullet \tan\theta = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ$
 $\theta = 18.43^\circ \pm 180^\circ$
 $\bullet \tan\theta = -3 \Rightarrow \theta = \tan^{-1}(-3) = -71.57^\circ$
 $\theta = -71.57^\circ \pm 180^\circ$
 $\therefore \theta = 18.4^\circ, 198.4^\circ, 108.4^\circ, 288.4^\circ$

Question 20

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2 x - \sin x = 1$

$0 \leq x < 360^\circ$

$x = 30^\circ, 150^\circ, 270^\circ$

b) $2\cos^2 x + 1 = 5\sin x$

$0 \leq x < 360^\circ$

$x = 30^\circ, 150^\circ$

c) $2\sin^2 x - 5\cos x + 1 = 0$

$0 \leq x < 360^\circ$

$x = 60^\circ, 300^\circ$

Handwritten solutions for Question 20:

(a) $2\cos^2 x - \sin x = 1$
 $\Rightarrow 2(1 - \sin^2 x) - \sin x = 1$
 $\Rightarrow 2 - 2\sin^2 x - \sin x = 1$
 $\Rightarrow 0 = 2\sin^2 x + \sin x - 1$
 $\Rightarrow 0 = (2\sin x - 1)(\sin x + 1)$
 $\sin x = \frac{1}{2}$
 $\arcsin\left(\frac{1}{2}\right) = 30^\circ$
 $x = 30^\circ \pm 360^\circ$
 $x = 150^\circ \pm 360^\circ$
 $x = 30^\circ, 150^\circ, 270^\circ$

(b) $2\cos^2 x + 1 = 5\sin x$
 $\Rightarrow 2(1 - \sin^2 x) + 1 = 5\sin x$
 $\Rightarrow 2 - 2\sin^2 x + 1 = 5\sin x$
 $\Rightarrow 0 = 2\sin^2 x + 5\sin x - 3$
 $\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$
 $\sin x = \frac{1}{2}$
 $\arcsin\left(\frac{1}{2}\right) = 30^\circ$
 $x = 30^\circ \pm 360^\circ$
 $x = 150^\circ \pm 360^\circ$
 $x_1 = 30^\circ$
 $x_2 = 150^\circ$

(c) $2\sin^2 x - 5\cos x + 1 = 0$
 $\Rightarrow 2(1 - \cos^2 x) - 5\cos x + 1 = 0$
 $\Rightarrow 2 - 2\cos^2 x - 5\cos x + 1 = 0$
 $\Rightarrow 3 - 2\cos^2 x - 5\cos x = 0$
 $\Rightarrow (2\cos x - 1)(\cos x + 3) = 0$
 $\cos x = \frac{1}{2}$
 $\arccos\left(\frac{1}{2}\right) = 60^\circ$
 $x = 60^\circ \pm 360^\circ$
 $x = 300^\circ \pm 360^\circ$
 $x_1 = 60^\circ$
 $x_2 = 300^\circ$

Question 21

Solve each of the following trigonometric equations in the range given.

a) $2 \cos^2 x + 7 \sin x + 2 = 0$ $0 \leq x < 360^\circ$

$x = 210^\circ, 330^\circ$

b) $5 \cos^2 \theta - \sin \theta = 1$ $0 \leq \theta < 360^\circ$

$\theta \approx 53.1^\circ, 126.9^\circ, \theta = 270^\circ$

c) $5 \sin^2 y = 19 \cos y + 1$ $0 \leq y < 360^\circ$

$y \approx 78.5^\circ, 281.5^\circ$

Handwritten solutions for Question 21:

(a) $2 \cos^2 x + 7 \sin x + 2 = 0$
 $\Rightarrow 2(1 - \sin^2 x) + 7 \sin x + 2 = 0$
 $\Rightarrow 2 - 2 \sin^2 x + 7 \sin x + 2 = 0$
 $\Rightarrow 0 = 2 \sin^2 x - 7 \sin x - 4$
 $\Rightarrow 0 = (2 \sin x + 1)(\sin x - 4)$
 $\sin x = -\frac{1}{2}$
 $\arcsin(-\frac{1}{2}) = -30^\circ$
 $(x = -30 \pm 360)$
 $(x = 210 \pm 360)$
 $x = 330^\circ, 210^\circ$

(b) $5 \cos^2 \theta - \sin \theta = 1$
 $\Rightarrow 5(1 - \sin^2 \theta) - \sin \theta = 1$
 $\Rightarrow 5 - 5 \sin^2 \theta - \sin \theta = 1$
 $\Rightarrow 0 = 5 \sin^2 \theta + \sin \theta - 4$
 $\Rightarrow (5 \sin \theta - 4)(\sin \theta + 1)$
 $\sin \theta = -1$
 $\arcsin(-1) = -90^\circ$
 $(\theta = -90 \pm 360)$
 $(\theta = 270 \pm 360)$
 $\theta = 270^\circ$
 $\arcsin(\frac{4}{5}) = 53.13^\circ$
 $(\theta = 53.13^\circ \pm 360)$
 $(\theta = 186.87 \pm 360)$
 $\theta = 53.1^\circ, 126.9^\circ$

(c) $5 \sin^2 y = 19 \cos y + 1$
 $\Rightarrow 5(1 - \cos^2 y) = 19 \cos y + 1$
 $\Rightarrow 5 - 5 \cos^2 y = 19 \cos y + 1$
 $\Rightarrow 0 = 5 \cos^2 y + 19 \cos y - 4$
 $\Rightarrow 0 = (5 \cos y - 1)(\cos y + 4)$
 $\cos y = \frac{1}{5}$
 $\arccos(\frac{1}{5}) = 78.46^\circ$
 $(y = 78.46 \pm 360)$
 $(y = 281.54 \pm 360)$
 $y = 78.5^\circ, 281.5^\circ$

Question 22

Solve each of the following trigonometric equations in the range given.

a) $3\sin^2 \theta = 8 + 16\cos \theta$

$0 \leq \theta < 360^\circ$

$\theta \approx 109.5^\circ, 250.5^\circ$

b) $3\cos^2 y + 11\sin y = 9$

$0 \leq y < 360^\circ$

$y \approx 48.2^\circ, 138.2^\circ$

c) $5\sin^2 x + 8\cos x = 8$

$0 \leq x < 360^\circ$

$x = 0^\circ, x \approx 53.1^\circ, 306.9^\circ$

Handwritten solutions for Question 22:

a) $3\sin^2 \theta = 8 + 16\cos \theta$
 $\Rightarrow 3(1 - \cos^2 \theta) = 8 + 16\cos \theta$
 $\Rightarrow 3 - 3\cos^2 \theta = 8 + 16\cos \theta$
 $\Rightarrow 0 = 3\cos^2 \theta + 16\cos \theta + 5$
 $\Rightarrow 0 = (3\cos \theta + 1)(\cos \theta + 5)$
 $\Rightarrow \cos \theta = \begin{cases} -\frac{1}{3} \\ -5 \end{cases}$
 $\cos \theta = -\frac{1}{3}$
 $\theta = \cos^{-1}(-\frac{1}{3}) = 109.47^\circ$
 $\theta = 360^\circ - 109.47^\circ = 250.53^\circ$
 $\therefore \theta = 109.5^\circ, 250.5^\circ$

b) $3\cos^2 y + 11\sin y = 9$
 $\Rightarrow 3(1 - \sin^2 y) + 11\sin y = 9$
 $\Rightarrow 3 - 3\sin^2 y + 11\sin y = 9$
 $\Rightarrow 0 = 3\sin^2 y - 11\sin y + 6$
 $\Rightarrow (3\sin y - 2)(\sin y - 3) = 0$
 $\Rightarrow \sin y = \begin{cases} \frac{2}{3} \\ 3 \end{cases}$
 $\sin y = \frac{2}{3}$
 $y = \sin^{-1}(\frac{2}{3}) = 41.81^\circ$
 $y = 180^\circ - 41.81^\circ = 138.19^\circ$
 $\therefore y = 41.8^\circ, 138.2^\circ$

c) $5\sin^2 x + 8\cos x = 8$
 $\Rightarrow 5(1 - \cos^2 x) + 8\cos x = 8$
 $\Rightarrow 5 - 5\cos^2 x + 8\cos x = 8$
 $\Rightarrow 0 = 5\cos^2 x - 8\cos x + 3$
 $\Rightarrow (5\cos x - 3)(\cos x - 1) = 0$
 $\Rightarrow \cos x = \begin{cases} \frac{3}{5} \\ 1 \end{cases}$
 $\cos x = \frac{3}{5}$
 $x = \cos^{-1}(\frac{3}{5}) = 53.13^\circ$
 $x = 360^\circ - 53.13^\circ = 306.87^\circ$
 $\cos x = 1$
 $x = 0^\circ$
 $\therefore x = 0^\circ, 53.1^\circ, 306.9^\circ$

Question 23

Solve each of the following trigonometric equations in the range given.

a) $6\cos^2 x + \sin x = 4$

$0 \leq x < 360^\circ$

$x \approx 41.8^\circ, 138.2^\circ, x = 210^\circ, 330^\circ$

b) $\cos^2 \theta = 7\sin^2 \theta - 2\sin \theta$

$0 \leq \theta < 360^\circ$

$\theta \approx 194.5^\circ, 345.5^\circ, \theta = 30^\circ, 150^\circ$

c) $\sin y + 5 = 6\cos^2 y$

$0 \leq \theta < 360^\circ$

$y \approx 19.5^\circ, 160.5^\circ, y = 210^\circ, 330^\circ$

Handwritten solution for Question 23:

(a) $6\cos^2 x + \sin x = 4$
 $\Rightarrow 6(1 - \sin^2 x) + \sin x = 4$
 $\Rightarrow 6 - 6\sin^2 x + \sin x = 4$
 $\Rightarrow 0 = 6\sin^2 x - \sin x - 2$
 $\Rightarrow (2\sin x + 1)(3\sin x - 2) = 0$
 $\Rightarrow \sin x = \frac{2}{3}$
 $\Rightarrow x \approx 41.8^\circ, 138.2^\circ$
 $\bullet \arcsin\left(\frac{2}{3}\right) = 41.8^\circ$
 $\bullet \arcsin\left(\frac{2}{3}\right) = 138.2^\circ$
 $\bullet \arcsin\left(\frac{2}{3}\right) = 210^\circ$
 $\bullet \arcsin\left(\frac{2}{3}\right) = 330^\circ$

(b) $\cos^2 \theta = 7\sin^2 \theta - 2\sin \theta$
 $\Rightarrow (1 - \sin^2 \theta) = 7\sin^2 \theta - 2\sin \theta$
 $\Rightarrow 0 = 8\sin^2 \theta - 2\sin \theta - 1$
 $\Rightarrow (4\sin \theta + 1)(2\sin \theta - 1) = 0$
 $\Rightarrow \sin \theta = \frac{1}{2}$
 $\Rightarrow \theta = 30^\circ, 150^\circ$
 $\bullet \arcsin\left(\frac{1}{2}\right) = 30^\circ$
 $\bullet \arcsin\left(\frac{1}{2}\right) = 150^\circ$
 $\bullet \arcsin\left(\frac{1}{2}\right) = 194.5^\circ$
 $\bullet \arcsin\left(\frac{1}{2}\right) = 345.5^\circ$

(c) $\sin y + 5 = 6\cos^2 y$
 $\Rightarrow \sin y + 5 = 6(1 - \sin^2 y)$
 $\Rightarrow \sin y + 5 = 6 - 6\sin^2 y$
 $\Rightarrow 6\sin^2 y + \sin y - 1 = 0$
 $\Rightarrow (3\sin y - 1)(2\sin y + 1) = 0$
 $\Rightarrow \sin y = \frac{1}{3}$
 $\Rightarrow y \approx 19.5^\circ, 160.5^\circ$
 $\bullet \arcsin\left(\frac{1}{3}\right) = 19.5^\circ$
 $\bullet \arcsin\left(\frac{1}{3}\right) = 160.5^\circ$
 $\bullet \arcsin\left(\frac{1}{3}\right) = 210^\circ$
 $\bullet \arcsin\left(\frac{1}{3}\right) = 330^\circ$

Question 24

Solve each of the following trigonometric equations in the range given.

a) $4\cos^2\theta + 9\sin\theta - 6 = 0$ $0 \leq \theta < 360^\circ$

$\theta \approx 14.5^\circ, 165.5^\circ$

b) $8\cos^2x - 6\sin x - 9 = 0$ $0 \leq x < 360^\circ$

$x \approx 194.5^\circ, 345.5^\circ, x = 210^\circ, 330^\circ$

c) $6\sin^2y + \cos y - 5 = 0$ $0 \leq y < 360^\circ$

$y \approx 109.5^\circ, 250.5^\circ, y = 60^\circ, 300^\circ$

Handwritten solutions for Question 24:

a) $4\cos^2\theta + 9\sin\theta - 6 = 0$
 $\Rightarrow 4(1 - \sin^2\theta) + 9\sin\theta - 6 = 0$
 $\Rightarrow 4 - 4\sin^2\theta + 9\sin\theta - 6 = 0$
 $\Rightarrow 0 = 4\sin^2\theta - 9\sin\theta + 2 = 0$
 $\Rightarrow 0 = (4\sin\theta - 2)(\sin\theta - 1)$
 $\Rightarrow \sin\theta = \frac{1}{2}$
 $\theta = 14.5^\circ, 165.5^\circ$

b) $8\cos^2x - 6\sin x - 9 = 0$
 $\Rightarrow 8(1 - \sin^2x) - 6\sin x - 9 = 0$
 $\Rightarrow 8 - 8\sin^2x - 6\sin x - 9 = 0$
 $\Rightarrow 0 = 8\sin^2x + 6\sin x + 1 = 0$
 $\Rightarrow 0 = (4\sin x + 1)(2\sin x + 1) = 0$
 $\Rightarrow \sin x = -\frac{1}{4}$
 $x = 194.5^\circ, 345.5^\circ$
 $x = 210^\circ, 330^\circ$

c) $6\sin^2y + \cos y - 5 = 0$
 $\Rightarrow 6(1 - \cos^2y) + \cos y - 5 = 0$
 $\Rightarrow 6 - 6\cos^2y + \cos y - 5 = 0$
 $\Rightarrow 0 = 6\cos^2y - \cos y - 1$
 $\Rightarrow 0 = (3\cos y + 1)(2\cos y - 1)$
 $\Rightarrow \cos y = \frac{1}{2}$
 $y = 60^\circ, 300^\circ$
 $y \approx 109.5^\circ, 250.5^\circ$

Question 25

Solve each of the following trigonometric equations in the range given.

a) $4\cos^2\theta - \cos\theta = 2\sin^2\theta$

$0 \leq \theta < 360^\circ$

$\theta \approx 48.2^\circ, 311.8^\circ, \theta = 120^\circ, 240^\circ$

b) $3\cos^2 y - \sin^2 y - 6\cos y = 3$

$0 \leq y < 360^\circ$

$y = 120^\circ, 240^\circ$

c) $10\sin^2 x - 3\sin x = 4\cos^2 x + 1$

$0 \leq x < 360^\circ$

$x \approx 45.6^\circ, 134.4^\circ, x = 210^\circ, 330^\circ$

Handwritten solutions for Question 25:

a) $4\cos^2\theta - \cos\theta = 2\sin^2\theta$
 $\Rightarrow 4\cos^2\theta - \cos\theta = 2(1 - \cos^2\theta)$
 $\Rightarrow 4\cos^2\theta - \cos\theta = 2 - 2\cos^2\theta$
 $\Rightarrow 6\cos^2\theta - \cos\theta - 2 = 0$
 $\Rightarrow (2\cos\theta + 1)(3\cos\theta - 2) = 0$
 $\Rightarrow \cos\theta = -\frac{1}{2}$
 $\Rightarrow \theta = 120^\circ, 240^\circ$
 $\bullet \cos\theta = \frac{2}{3}$
 $\theta = 48.2^\circ, 311.8^\circ$
 $\therefore \theta = 120^\circ, 240^\circ, 48.2^\circ, 311.8^\circ$

b) $3\cos^2 y - \sin^2 y - 6\cos y = 3$
 $\Rightarrow 3\cos^2 y - (1 - \cos^2 y) - 6\cos y = 3$
 $\Rightarrow 3\cos^2 y - 1 + \cos^2 y - 6\cos y = 3$
 $\Rightarrow 4\cos^2 y - 6\cos y - 4 = 0$
 $\Rightarrow 2\cos^2 y - 3\cos y - 2 = 0$
 $\Rightarrow (2\cos y + 1)(\cos y - 2) = 0$
 $\Rightarrow \cos y = -\frac{1}{2}$
 $\Rightarrow y = 120^\circ, 240^\circ$
 $\bullet \cos y = \frac{2}{3}$
 $y = 48.2^\circ, 311.8^\circ$
 $\therefore y = 120^\circ, 240^\circ$

c) $10\sin^2 x - 3\sin x = 4\cos^2 x + 1$
 $\Rightarrow 10\sin^2 x - 3\sin x = 4(1 - \sin^2 x) + 1$
 $\Rightarrow 10\sin^2 x - 3\sin x = 4 - 4\sin^2 x + 1$
 $\Rightarrow 14\sin^2 x - 3\sin x - 5 = 0$
 $\Rightarrow (7\sin x - 5)(2\sin x + 1) = 0$
 $\Rightarrow \sin x = \frac{5}{7}$
 $\Rightarrow x = 45.6^\circ, 134.4^\circ$
 $\bullet \sin x = -\frac{1}{2}$
 $x = 210^\circ, 330^\circ$
 $\therefore x = 45.6^\circ, 134.4^\circ, 210^\circ, 330^\circ$

Question 26

Solve each of the following trigonometric equations in the range given.

a) $2\sin^2 x - 2\cos x - \cos^2 x = 1$ $0 \leq x < 360^\circ$ $x \approx 70.5^\circ, 289.5^\circ, x = 180^\circ$

b) $4\sin^2 \theta - \cos^2 \theta = 8\sin \theta + 3$ $0 \leq \theta < 360^\circ$ $\theta \approx 203.6^\circ, 336.4^\circ$

c) $3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6$ $0 \leq \phi < 360^\circ$ $\phi \approx 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$

a) $2\sin^2 x - 2\cos x - \cos^2 x = 1$
 $\Rightarrow 2(1 - \cos^2 x) - 2\cos x - \cos^2 x = 1$
 $\Rightarrow 2 - 2\cos^2 x - 2\cos x - \cos^2 x = 1$
 $\Rightarrow 0 = 3\cos^2 x + 2\cos x - 1$
 $\Rightarrow 0 = (3\cos x - 1)(\cos x + 1)$
 $\cos x = -\frac{1}{3}$
 $\cos x = -1$ $\cos x = \frac{1}{3}$
 $x = 180^\circ$ $x = 70.5^\circ$
 $x = 180^\circ \pm 360^\circ$ $x = 289.5^\circ \pm 360^\circ$
 $x = 180^\circ$ $x = 70.5^\circ$
 $x = 289.5^\circ$

b) $4\sin^2 \theta - \cos^2 \theta = 8\sin \theta + 3$
 $\Rightarrow 4\sin^2 \theta - (1 - \sin^2 \theta) = 8\sin \theta + 3$
 $\Rightarrow 4\sin^2 \theta - 1 + \sin^2 \theta = 8\sin \theta + 3$
 $\Rightarrow 5\sin^2 \theta - 8\sin \theta - 4 = 0$
 $\Rightarrow (5\sin \theta + 2)(\sin \theta - 2) = 0$
 $\sin \theta = -\frac{2}{5}$
 $\sin \theta = 2$
 $\theta = -23.58^\circ \pm 360^\circ$
 $\theta = 203.58^\circ \pm 360^\circ$
 $\theta = 336.4^\circ$
 $\theta = 203.6^\circ$

c) $3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6$ $0 \leq \phi < 360^\circ$
 $\Rightarrow 3\cos^2 2\phi - 4(1 - \cos^2 2\phi) = 15\cos 2\phi - 6$
 $\Rightarrow 3\cos^2 2\phi - 4 + 4\cos^2 2\phi = 15\cos 2\phi - 6$
 $\Rightarrow 7\cos^2 2\phi - 15\cos 2\phi + 2 = 0$
 $\Rightarrow (7\cos 2\phi - 1)(\cos 2\phi - 2) = 0$
 $\cos 2\phi = \frac{1}{7}$
 $\cos 2\phi = 2$
 $2\phi = 81.7867^\circ \pm 360^\circ$
 $2\phi = 278.2132^\circ \pm 360^\circ$
 $\phi = 40.89^\circ \pm 180^\circ$
 $\phi = 139.11^\circ \pm 180^\circ$
 $\therefore \phi = 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ$

Question 27

Solve each of the following trigonometric equations in the range given.

a) $12 \sin^2 \beta - 5 \cos \beta = 9$

$0 \leq \beta < 360^\circ$

$\beta \approx 70.5^\circ, 138.6^\circ, 221.4^\circ, 289.5^\circ$

b) $5 \cos^2 \psi + 2 = 3 \sin^2 \psi - 2 \cos \psi$

$0 \leq \psi < 360^\circ$

$\psi \approx 75.5^\circ, 284.5^\circ, \psi = 120^\circ, 240^\circ$

c) $3 - 7 \cos 3\theta = 6 \sin^2 3\theta$

$0 \leq \theta < 180^\circ$

$\theta \approx 36.5^\circ, 83.5^\circ, 156.5^\circ$

The image shows three handwritten solutions for the trigonometric equations.
Solution a) Starts with $12 \sin^2 \beta - 5 \cos \beta = 9$. It uses the identity $\sin^2 \beta = 1 - \cos^2 \beta$ to get $12(1 - \cos^2 \beta) - 5 \cos \beta = 9$, which simplifies to $12 - 12 \cos^2 \beta - 5 \cos \beta = 9$, then $3 = 12 \cos^2 \beta + 5 \cos \beta - 9$, and finally $0 = (4 \cos \beta + 3)(3 \cos \beta - 1)$. This gives $\cos \beta = -\frac{3}{4}$ or $\cos \beta = \frac{1}{3}$. For $\cos \beta = -\frac{3}{4}$, $\beta = \arccos(-\frac{3}{4}) = 138.6^\circ$ and $\beta = 180^\circ - 138.6^\circ = 221.4^\circ$. For $\cos \beta = \frac{1}{3}$, $\beta = \arccos(\frac{1}{3}) = 70.5^\circ$ and $\beta = 360^\circ - 70.5^\circ = 289.5^\circ$.
Solution b) Starts with $5 \cos^2 \psi + 2 = 3 \sin^2 \psi - 2 \cos \psi$. It uses the identity $\sin^2 \psi = 1 - \cos^2 \psi$ to get $5 \cos^2 \psi + 2 = 3(1 - \cos^2 \psi) - 2 \cos \psi$, which simplifies to $5 \cos^2 \psi + 2 = 3 - 3 \cos^2 \psi - 2 \cos \psi$, then $8 \cos^2 \psi + 2 \cos \psi - 1 = 0$. Factoring gives $(4 \cos \psi - 1)(2 \cos \psi + 1) = 0$. This gives $\cos \psi = \frac{1}{4}$ or $\cos \psi = -\frac{1}{2}$. For $\cos \psi = \frac{1}{4}$, $\psi = \arccos(\frac{1}{4}) = 75.5^\circ$ and $\psi = 360^\circ - 75.5^\circ = 284.5^\circ$. For $\cos \psi = -\frac{1}{2}$, $\psi = 120^\circ$ and $\psi = 240^\circ$.
Solution c) Starts with $3 - 7 \cos 3\theta = 6 \sin^2 3\theta$. It uses the identity $\sin^2 3\theta = 1 - \cos^2 3\theta$ to get $3 - 7 \cos 3\theta = 6(1 - \cos^2 3\theta)$, which simplifies to $3 - 7 \cos 3\theta = 6 - 6 \cos^2 3\theta$, then $6 \cos^2 3\theta - 7 \cos 3\theta - 3 = 0$. Factoring gives $(2 \cos 3\theta - 3)(3 \cos 3\theta + 1) = 0$. This gives $\cos 3\theta = \frac{3}{2}$ (which is impossible) or $\cos 3\theta = -\frac{1}{3}$. For $\cos 3\theta = -\frac{1}{3}$, $3\theta = \arccos(-\frac{1}{3}) = 109.47^\circ$. This gives $3\theta = 109.47^\circ \pm 360^\circ$ or $3\theta = 250.53^\circ \pm 360^\circ$. Dividing by 3 gives $\theta = 36.49^\circ \pm 120^\circ$ or $\theta = 83.51^\circ \pm 120^\circ$. The solutions are $\theta_1 = 36.5^\circ$, $\theta_2 = 156.5^\circ$, and $\theta_3 = 83.5^\circ$.

Question 28

Solve each of the following trigonometric equations in the range given.

a) $12 \cos^2 \theta - 5 \sin \theta = 10$

$0 \leq \theta < 360^\circ$

$\theta \approx 14.5^\circ, 165.5^\circ, 221.8^\circ, 318.2^\circ$

b) $6 \sin^2 \theta + 19 \cos \theta = 16$

$0 \leq \theta < 360^\circ$

$\theta \approx 48.2^\circ, 311.8^\circ$

c) $\cos^2 x - 9 \sin^2 x = \sin x - 1$

$0 \leq x < 360^\circ$

$x \approx 23.6^\circ, 156.4^\circ, x = 210^\circ, 330^\circ$

$$\begin{aligned} \text{a) } 12 \cos^2 \theta - 5 \sin \theta &= 10 \\ \Rightarrow 12(1 - \sin^2 \theta) - 5 \sin \theta &= 10 \\ \Rightarrow 12 - 12 \sin^2 \theta - 5 \sin \theta &= 10 \\ \Rightarrow 0 &= 12 \sin^2 \theta + 5 \sin \theta - 2 \\ \Rightarrow 0 &= (3 \sin \theta + 2)(4 \sin \theta - 1) \\ \therefore \sin \theta &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \bullet \arcsin\left(\frac{1}{4}\right) &= 14.5^\circ \\ \left\{ \begin{aligned} \theta &= 14.5^\circ \pm 360^\circ \\ \theta &= 165.5^\circ \pm 360^\circ \end{aligned} \right. & \quad \bullet \arcsin\left(-\frac{1}{4}\right) = -14.5^\circ \\ & \quad \left\{ \begin{aligned} \theta &= -14.5^\circ \pm 360^\circ \\ \theta &= 221.8^\circ \pm 360^\circ \end{aligned} \right. \end{aligned}$$

$$\therefore \theta = 14.5^\circ, 165.5^\circ, 221.8^\circ, 318.2^\circ$$

$$\begin{aligned} \text{b) } 6 \sin^2 \theta + 19 \cos \theta &= 16 \\ \Rightarrow 6(1 - \cos^2 \theta) + 19 \cos \theta &= 16 \\ \Rightarrow 6 - 6 \cos^2 \theta + 19 \cos \theta &= 16 \\ \Rightarrow 0 &= 6 \cos^2 \theta - 19 \cos \theta + 10 \\ \Rightarrow 0 &= (3 \cos \theta - 2)(2 \cos \theta - 5) \\ \therefore \cos \theta &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \bullet \arccos\left(\frac{2}{3}\right) &= 48.2^\circ \\ \left\{ \begin{aligned} \theta &= 48.2^\circ \pm 360^\circ \\ \theta &= 311.8^\circ \pm 360^\circ \end{aligned} \right. & \quad \bullet \arccos\left(-\frac{2}{3}\right) = 131.8^\circ \\ & \quad \left\{ \begin{aligned} \theta &= 131.8^\circ \pm 360^\circ \\ \theta &= 228.2^\circ \pm 360^\circ \end{aligned} \right. \end{aligned}$$

$$\therefore \theta = 48.2^\circ, 311.8^\circ$$

$$\begin{aligned} \cos^2 x - 9 \sin^2 x &= \sin x - 1 \\ \Rightarrow (1 - \sin^2 x) - 9 \sin^2 x &= \sin x - 1 \\ \Rightarrow 1 - 10 \sin^2 x &= \sin x - 1 \\ \Rightarrow 0 &= 10 \sin^2 x + \sin x - 2 \\ \Rightarrow 0 &= (5 \sin x - 2)(2 \sin x + 1) \\ \therefore \sin x &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} \bullet \arcsin\left(\frac{2}{5}\right) &= 23.6^\circ \\ \left\{ \begin{aligned} x &= 23.6^\circ \pm 360^\circ \\ x &= 156.4^\circ \pm 360^\circ \end{aligned} \right. & \quad \bullet \arcsin\left(-\frac{2}{5}\right) = -23.6^\circ \\ & \quad \left\{ \begin{aligned} x &= -23.6^\circ \pm 360^\circ \\ x &= 336.4^\circ \pm 360^\circ \end{aligned} \right. \end{aligned}$$

$$x = 23.6^\circ, 156.4^\circ, 210^\circ, 330^\circ$$

Question 29

Solve each of the following trigonometric equations in the range given.

a) $4\sin^2\psi = 15\cos\psi$

$0 \leq \psi < 2\pi$

$\psi \approx 1.32^\circ, 4.97^\circ$

b) $6\cos^2x + \sin x = 4$

$0 \leq x < 2\pi$

$x \approx 0.73^\circ, 2.41^\circ, 3.67^\circ, 5.76^\circ$

c) $4\sin^2 2\phi - \cos^2 2\phi = 3 + 8\sin 2\phi$

$0 \leq \phi < 2\pi$

$\phi \approx 1.78^\circ, 2.94^\circ, 4.92^\circ, 6.08^\circ$

a) $4\sin^2\psi = 15\cos\psi, \quad 0 \leq \psi < 2\pi$
 $4\sin^2\psi = 15\cos\psi$
 $\Rightarrow 4(1 - \cos^2\psi) = 15\cos\psi$
 $\Rightarrow 4 - 4\cos^2\psi = 15\cos\psi$
 $\Rightarrow 0 = 4\cos^2\psi + 15\cos\psi - 4$
 $\Rightarrow 0 = (4\cos\psi - 1)(\cos\psi + 4)$
 $\Rightarrow \cos\psi = \frac{1}{4}$
 $\arccos\left(\frac{1}{4}\right) = 1.32^\circ$
 $\psi = 1.32^\circ \pm 2\pi n$
 $\psi = 4.97^\circ \pm 2\pi n$
 $n = 0, 1, 2, \dots$
 $\psi_1 = 1.32^\circ$
 $\psi_2 = 4.97^\circ$

b) $6\cos^2x + \sin x = 4, \quad 0 \leq x < 2\pi$
 $6\cos^2x + \sin x = 4$
 $\Rightarrow 6(1 - \sin^2x) + \sin x = 4$
 $\Rightarrow 6 - 6\sin^2x + \sin x = 4$
 $\Rightarrow 0 = 6\sin^2x - \sin x - 2$
 $\Rightarrow (2\sin x + 1)(3\sin x - 2) = 0$
 $\sin x = -\frac{1}{3}$
 $\arcsin\left(-\frac{1}{3}\right) = -0.33^\circ$
 $x = -0.33^\circ \pm 2\pi n$
 $x = \frac{19}{18}^\circ \pm 2\pi n$
 $n = 0, 1, 2, \dots$
 $\arcsin\left(\frac{2}{3}\right) = 0.73^\circ$
 $x = 0.73^\circ \pm 2\pi n$
 $x = 2.41^\circ \pm 2\pi n$
 $n = 0, 1, 2, \dots$
 $x_1 = \frac{19}{18}^\circ \approx 5.76^\circ$
 $x_2 = \frac{19}{18}^\circ \approx 3.67^\circ$
 $x_3 = 0.73^\circ$
 $x_4 = 2.41^\circ$

c) $4\sin^2 2\phi - \cos^2 2\phi = 3 + 8\sin 2\phi, \quad 0 \leq \phi < 2\pi$
 $4\sin^2 2\phi - \cos^2 2\phi = 3 + 8\sin 2\phi$
 $\Rightarrow 4\sin^2 2\phi - (1 - \sin^2 2\phi) = 3 + 8\sin 2\phi$
 $\Rightarrow 4\sin^2 2\phi - 1 + \sin^2 2\phi = 3 + 8\sin 2\phi$
 $\Rightarrow 5\sin^2 2\phi - 8\sin 2\phi - 4 = 0$
 $\Rightarrow (5\sin 2\phi + 2)(\sin 2\phi - 2) = 0$
 $\Rightarrow \sin 2\phi = -\frac{2}{5}$
 $\arcsin\left(-\frac{2}{5}\right) = -0.4115^\circ$
 $2\phi = -0.4115^\circ \pm 2\pi n$
 $2\phi = 3.553^\circ \pm 2\pi n$
 $n = 0, 1, 2, \dots$
 $\phi = -0.205^\circ \pm \pi n$
 $\phi = 1.776^\circ \pm \pi n$
 $\phi_1 = 2.94^\circ$
 $\phi_2 = 6.08^\circ$
 $\phi_3 = 1.78^\circ$
 $\phi_4 = 4.92^\circ$

Question 30

Solve each of the following trigonometric equations in the range given.

a) $3 - 7\cos\theta = 6\sin^2\theta$

$0 \leq \theta < 2\pi$

$\theta \approx 1.91^\circ, 4.37^\circ$

b) $4\cos^2x - \cos x = 2\sin^2x$

$0 \leq x < 2\pi$

$x \approx 0.84^\circ, 2.09^\circ, 4.19^\circ, 5.44^\circ$

c) $5\cos^2y + 2 = 3\sin^2y - 2\cos y$

$0 \leq y < 2\pi$

$y \approx 1.32^\circ, 2.09^\circ, 4.19^\circ, 4.19^\circ$

a) $3 - 7\cos\theta = 6\sin^2\theta$
 $\Rightarrow 3 - 7\cos\theta = 6(1 - \cos^2\theta)$
 $\Rightarrow 3 - 7\cos\theta = 6 - 6\cos^2\theta$
 $\Rightarrow 6\cos^2\theta - 7\cos\theta - 3 = 0$
 $\Rightarrow (2\cos\theta - 3)(3\cos\theta + 1) = 0$
 $\Rightarrow \cos\theta = \frac{3}{2}$ or $-\frac{1}{3}$
 $\bullet \arccos(-\frac{1}{3}) = 1.91^\circ$
 $(\theta = 1.91^\circ \pm 2\pi n)$
 $(\theta = 4.37^\circ \pm 2\pi n)$ where $n = 0, 1, 2, 3, \dots$
 $\theta_1 = 1.91^\circ$
 $\theta_2 = 4.37^\circ$

b) $4\cos^2x - \cos x = 2\sin^2x$
 $\Rightarrow 4\cos^2x - \cos x = 2(1 - \cos^2x)$
 $\Rightarrow 4\cos^2x - \cos x = 2 - 2\cos^2x$
 $\Rightarrow 6\cos^2x - \cos x - 2 = 0$
 $\Rightarrow (3\cos x - 2)(2\cos x + 1) = 0$
 $\Rightarrow \cos x = \frac{2}{3}$ or $-\frac{1}{2}$
 $\bullet \arccos(\frac{2}{3}) = 0.84^\circ$
 $x = \frac{2\pi}{3} \pm 2\pi n$
 $x = \frac{4\pi}{3} \pm 2\pi n$ where $n = 0, 1, 2, 3, \dots$
 $x_1 = 0.84^\circ$
 $x_2 = 2.09^\circ$
 $x_3 = 4.19^\circ$
 $x_4 = 5.44^\circ$

c) $5\cos^2y + 2 = 3\sin^2y - 2\cos y$
 $\Rightarrow 5\cos^2y + 2 = 3(1 - \cos^2y) - 2\cos y$
 $\Rightarrow 5\cos^2y + 2 = 3 - 3\cos^2y - 2\cos y$
 $\Rightarrow 8\cos^2y + 2\cos y - 1 = 0$
 $\Rightarrow (4\cos y - 1)(2\cos y + 1) = 0$
 $\Rightarrow \cos y = \frac{1}{4}$ or $-\frac{1}{2}$
 $\bullet \arccos(\frac{1}{4}) = 1.32^\circ$
 $y = 1.32^\circ \pm 2\pi n$
 $y = 4.96^\circ \pm 2\pi n$ where $n = 0, 1, 2, 3, \dots$
 $y_1 = 1.32^\circ$
 $y_2 = 2.09^\circ$
 $y_3 = 4.19^\circ$
 $y_4 = 4.19^\circ$

Question 31

Solve each of the following trigonometric equations in the range given.

a) $2\cos^2 x - \sin x = 1$

$0 \leq x < 2\pi$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) $2\sin^2 \theta + 1 = 5\cos \theta$

$0 \leq \theta < 2\pi$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $3\cos^2 2y - \sin^2 2y - 6\cos 2y = 3$

$0 \leq y < 2\pi$

$y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

The image shows three panels of handwritten solutions for the trigonometric equations.
 Panel (a) solves $2\cos^2 x - \sin x = 1$ by substituting $\sin x = t$, leading to $2(1-t^2) - t = 1$, which simplifies to $2t^2 + t - 1 = 0$. The roots are $t = \frac{1}{2}$ and $t = -1$. For $t = \frac{1}{2}$, $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$. For $t = -1$, $x = \frac{3\pi}{2}$.
 Panel (b) solves $2\sin^2 \theta + 1 = 5\cos \theta$ by substituting $\cos \theta = t$, leading to $2(1-t^2) + 1 = 5t$, which simplifies to $2t^2 + 5t - 3 = 0$. The roots are $t = \frac{1}{2}$ and $t = -3$. For $t = \frac{1}{2}$, $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$.
 Panel (c) solves $3\cos^2 2y - \sin^2 2y - 6\cos 2y = 3$ by substituting $\cos 2y = t$, leading to $3t^2 - (1-t^2) - 6t = 3$, which simplifies to $4t^2 - 6t - 4 = 0$. The roots are $t = \frac{3}{2}$ and $t = -\frac{1}{2}$. For $t = -\frac{1}{2}$, $2y = \frac{2\pi}{3}$ and $2y = \frac{4\pi}{3}$, giving $y = \frac{\pi}{3}$ and $y = \frac{2\pi}{3}$. For $t = \frac{3}{2}$, $2y = \frac{4\pi}{3}$ and $2y = \frac{2\pi}{3}$, giving $y = \frac{2\pi}{3}$ and $y = \frac{\pi}{3}$. The final solutions are $y = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

Question 32

Solve each of the following trigonometric equations in the range given.

a) $12 \sin^2 x - 5 \cos x = 9$

$0 \leq x < 2\pi$

$x \approx 1.23^\circ, 2.42^\circ, 3.86^\circ, 5.05^\circ$

b) $10 \sin^2 \theta - 3 \sin \theta = 4 \cos^2 \theta + 1$

$0 \leq \theta < 2\pi$

$\theta \approx 0.52^\circ, 0.80^\circ, 2.62^\circ, 2.36^\circ$

c) $\cos^2 \phi = 7 \sin^2 \phi - 2 \sin \phi$

$0 \leq \phi < 2\pi$

$\phi \approx 0.52^\circ, 2.62^\circ, 3.39^\circ, 6.03^\circ$

The image shows three handwritten solutions for the trigonometric equations. Each solution is written in a clear, step-by-step manner, showing the algebraic manipulation and the final solutions in degrees.

a) $12 \sin^2 x - 5 \cos x = 9$, $0 \leq x < 2\pi$

$\Rightarrow 12(1 - \cos^2 x) - 5 \cos x = 9$
 $\Rightarrow 12 - 12 \cos^2 x - 5 \cos x = 9$
 $\Rightarrow 0 = 12 \cos^2 x + 5 \cos x - 3$
 $\Rightarrow 0 = (3 \cos x - 1)(4 \cos x + 3)$
 $\cos x = \frac{1}{3}$ or $\cos x = -\frac{3}{4}$
 $\arccos(\frac{1}{3}) = 1.23^\circ$ or $\arccos(-\frac{3}{4}) = 2.42^\circ$
 $x = 1.23^\circ, 5.05^\circ, 2.42^\circ, 3.86^\circ$

b) $10 \sin^2 \theta - 3 \sin \theta = 4 \cos^2 \theta + 1$, $0 \leq \theta < 2\pi$

$\Rightarrow 10 \sin^2 \theta - 3 \sin \theta = 4(1 - \sin^2 \theta) + 1$
 $\Rightarrow 10 \sin^2 \theta - 3 \sin \theta = 4 - 4 \sin^2 \theta + 1$
 $\Rightarrow 14 \sin^2 \theta - 3 \sin \theta - 5 = 0$
 $\Rightarrow (7 \sin \theta - 5)(2 \sin \theta + 1) = 0$
 $\sin \theta = \frac{5}{7}$ or $\sin \theta = -\frac{1}{2}$
 $\arcsin(\frac{5}{7}) = 0.714^\circ$ or $\arcsin(-\frac{1}{2}) = 0.785^\circ$
 $\theta = 0.714^\circ, 2.36^\circ, 0.785^\circ, 2.62^\circ$

c) $\cos^2 \phi = 7 \sin^2 \phi - 2 \sin \phi$, $0 \leq \phi < 2\pi$

$\Rightarrow \cos^2 \phi = 7(1 - \cos^2 \phi) - 2 \sin \phi$
 $\Rightarrow \cos^2 \phi = 7 - 7 \cos^2 \phi - 2 \sin \phi$
 $\Rightarrow 8 \cos^2 \phi + 2 \sin \phi - 7 = 0$
 $\Rightarrow 8(1 - \sin^2 \phi) + 2 \sin \phi - 7 = 0$
 $\Rightarrow -8 \sin^2 \phi + 2 \sin \phi - 1 = 0$
 $\Rightarrow 8 \sin^2 \phi - 2 \sin \phi + 1 = 0$
 $\Rightarrow (4 \sin \phi - 1)(2 \sin \phi + 1) = 0$
 $\sin \phi = \frac{1}{4}$ or $\sin \phi = -\frac{1}{2}$
 $\arcsin(\frac{1}{4}) = 0.25^\circ$ or $\arcsin(-\frac{1}{2}) = 0.785^\circ$
 $\phi = 0.25^\circ, 3.39^\circ, 0.785^\circ, 6.03^\circ$

Question 33

Solve each of the following trigonometric equations in the range given.

a) $4 \tan^2 \theta \cos \theta = 15$

$0 \leq \theta < 360^\circ$

$\theta \approx 75.5^\circ, 284.5^\circ$

b) $\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$

$0 \leq \theta < 360^\circ$

$\theta = 120^\circ, 240^\circ$

c) $2 \sin x = \tan x$

$0 \leq x < 360^\circ$

$x = 0^\circ, 60^\circ, 180^\circ, 300^\circ$

Handwritten solutions for Question 33:

(a) $4 \tan^2 \theta \cos \theta = 15$
 $\Rightarrow 4 \frac{\sin^2 \theta}{\cos^2 \theta} \cos \theta = 15$
 $\Rightarrow \frac{4 \sin^2 \theta}{\cos \theta} = 15$
 $\Rightarrow 4 \sin^2 \theta = 15 \cos \theta$
 $\Rightarrow 4(1 - \cos^2 \theta) = 15 \cos \theta$
 $\Rightarrow 4 - 4 \cos^2 \theta = 15 \cos \theta$
 $\Rightarrow 0 = 4 \cos^2 \theta + 15 \cos \theta - 4$
 $\Rightarrow (4 \cos \theta - 1)(\cos \theta + 4) = 0$
 $\Rightarrow \cos \theta = \frac{1}{4}$
 $\Rightarrow \cos^{-1}(\frac{1}{4}) = 75.5^\circ$
 $\theta = 75.5^\circ \pm 360^\circ$
 $\theta = 284.5^\circ \pm 360^\circ$
 $\theta_1 = 75.5^\circ$
 $\theta_2 = 284.5^\circ$

(b) $\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$
 $\Rightarrow 3 + \sin^2 \theta = 3 \cos \theta (\cos \theta - 2)$
 $\Rightarrow 3 + \sin^2 \theta = 3 \cos^2 \theta - 6 \cos \theta$
 $\Rightarrow 3 + (1 - \cos^2 \theta) = 3 \cos^2 \theta - 6 \cos \theta$
 $\Rightarrow 4 - \cos^2 \theta = 3 \cos^2 \theta - 6 \cos \theta$
 $\Rightarrow 0 = 4 \cos^2 \theta - 6 \cos \theta - 4$
 $\Rightarrow 0 = 2 \cos^2 \theta - 3 \cos \theta - 2$
 $\Rightarrow 0 = (2 \cos \theta + 1)(\cos \theta - 2)$
 $\Rightarrow \cos \theta = -\frac{1}{2}$
 $\Rightarrow \cos^{-1}(-\frac{1}{2}) = 120^\circ$
 $\theta = 120^\circ \pm 360^\circ$
 $\theta = 240^\circ \pm 360^\circ$
 $\theta_1 = 120^\circ$
 $\theta_2 = 240^\circ$

(c) $2 \sin x = \tan x$
 $\Rightarrow 2 \sin x = \frac{\sin x}{\cos x}$
 $\Rightarrow 2 \sin x \cos x = \sin x$
 $\Rightarrow 2 \sin x \cos x - \sin x = 0$
 $\Rightarrow \sin x (2 \cos x - 1) = 0$
 $\Rightarrow \sin x = 0 \quad \cos x = \frac{1}{2}$
 $\Rightarrow x = 0^\circ \pm 360^\circ$
 $\Rightarrow x = 180^\circ \pm 360^\circ$
 $\Rightarrow x = 60^\circ \pm 360^\circ$
 $\Rightarrow x = 300^\circ \pm 360^\circ$
 $\Rightarrow x = 0^\circ, 60^\circ, 180^\circ, 300^\circ$

Question 34

Solve each of the following trigonometric equations in the range given.

a) $3 \cos x = 8 \tan x$

$0 \leq x < 360^\circ$

$x \approx 19.5^\circ, 160.5^\circ$

b) $2 \tan x \sin x = 3$

$0 \leq x < 360^\circ$

$x = 60^\circ, 300^\circ$

c) $2 \cos x = 3 \tan x$

$0 \leq x < 360^\circ$

$x = 30^\circ, 150^\circ$

Handwritten solutions for the three trigonometric equations:

(a) $3 \cos x = 8 \tan x$
 $\Rightarrow 3 \cos x = 8 \left(\frac{\sin x}{\cos x} \right)$
 $\Rightarrow 3 \cos^2 x = 8 \sin x$
 $\Rightarrow 3(1 - \sin^2 x) = 8 \sin x$
 $\Rightarrow 3 - 3 \sin^2 x = 8 \sin x$
 $\Rightarrow 0 = 3 \sin^2 x + 8 \sin x - 3$
 $\Rightarrow 0 = (3 \sin x - 1)(\sin x + 3)$
 $\Rightarrow \sin x = \frac{1}{3}$
 $\Rightarrow x = 19.47^\circ \approx 19.5^\circ$
 $x = 160.53^\circ \approx 160.5^\circ$

(b) $2 \tan x \sin x = 3$
 $\Rightarrow 2 \left(\frac{\sin x}{\cos x} \right) \sin x = 3$
 $\Rightarrow \frac{2 \sin^2 x}{\cos x} = 3$
 $\Rightarrow 2 \sin^2 x = 3 \cos x$
 $\Rightarrow 2(1 - \cos^2 x) = 3 \cos x$
 $\Rightarrow 2 - 2 \cos^2 x = 3 \cos x$
 $\Rightarrow 0 = 2 \cos^2 x + 3 \cos x - 2$
 $\Rightarrow 0 = (2 \cos x - 1)(\cos x + 2)$
 $\Rightarrow \cos x = \frac{1}{2}$
 $\Rightarrow x = 60^\circ$
 $x = 300^\circ$

(c) $2 \cos x = 3 \tan x$
 $\Rightarrow 2 \cos x = 3 \left(\frac{\sin x}{\cos x} \right)$
 $\Rightarrow 2 \cos^2 x = 3 \sin x$
 $\Rightarrow 2(1 - \sin^2 x) = 3 \sin x$
 $\Rightarrow 2 - 2 \sin^2 x = 3 \sin x$
 $\Rightarrow 0 = 2 \sin^2 x + 3 \sin x - 2$
 $\Rightarrow 0 = (2 \sin x - 1)(\sin x + 2)$
 $\Rightarrow \sin x = \frac{1}{2}$
 $\Rightarrow x = 30^\circ$
 $x = 150^\circ$

Question 35

Solve each of the following trigonometric equations in the range given.

a) $4 \tan x \sin x = 15$

$0 \leq x < 2\pi$

$x \approx 1.32^\circ, 4.97^\circ$

b) $6 \cos x = 5 \tan x$

$0 \leq x < 2\pi$

$x \approx 0.73^\circ, 2.41^\circ$

c) $\tan x \sin x = 2$

$0 \leq x < 2\pi$

$x \approx 2.00^\circ, 4.29^\circ$

Handwritten solutions for Question 35:

a) $4 \tan x \sin x = 15$
 $\Rightarrow 4 \left(\frac{\sin x}{\cos x} \right) \sin x = 15$
 $\Rightarrow \frac{4 \sin^2 x}{\cos x} = 15$
 $\Rightarrow 4 \sin^2 x = 15 \cos x$
 $\Rightarrow 4(1 - \cos^2 x) = 15 \cos x$
 $\Rightarrow 4 - 4 \cos^2 x = 15 \cos x$
 $\Rightarrow 0 = 4 \cos^2 x + 15 \cos x - 4$
 $\Rightarrow (4 \cos x - 1)(\cos x + 4) = 0$
 $\Rightarrow \cos x = \frac{1}{4}$
 $\Rightarrow \arccos\left(\frac{1}{4}\right) = 1.32^\circ$
 $\alpha = 1.32^\circ \pm 2\pi n$
 $\alpha = 4.97^\circ \pm 2\pi n$
 $n = 0, 1, 2, 3$
 $\alpha_1 = 1.32^\circ$
 $\alpha_2 = 4.97^\circ$

b) $6 \cos x = 5 \tan x$
 $\Rightarrow 6 \cos x = 5 \left(\frac{\sin x}{\cos x} \right)$
 $\Rightarrow 6 \cos^2 x = 5 \sin x$
 $\Rightarrow 6(1 - \sin^2 x) = 5 \sin x$
 $\Rightarrow 6 - 6 \sin^2 x = 5 \sin x$
 $\Rightarrow 0 = 6 \sin^2 x + 5 \sin x - 6$
 $\Rightarrow (3 \sin x - 2)(2 \sin x + 3) = 0$
 $\Rightarrow \sin x = \frac{2}{3}$
 $\Rightarrow \arcsin\left(\frac{2}{3}\right) = 0.73^\circ$
 $\alpha = 0.73^\circ \pm 2\pi n$
 $\alpha = 2.41^\circ \pm 2\pi n$
 $n = 0, 1, 2, 3$
 $\alpha_1 = 0.73^\circ$
 $\alpha_2 = 2.41^\circ$

c) $\tan x \sin x = 2$
 $\Rightarrow \frac{\sin x}{\cos x} \sin x = 2$
 $\Rightarrow \frac{\sin^2 x}{\cos x} = 2$
 $\Rightarrow \sin^2 x = 2 \cos x$
 $\Rightarrow 1 - \cos^2 x = 2 \cos x$
 $\Rightarrow 0 = \cos^2 x + 2 \cos x - 1$
 $\Rightarrow 0 = (\cos x + 1)^2 - 1$
 $\Rightarrow (\cos x - 1)^2 = 2$
 $\Rightarrow \cos x - 1 = \pm \sqrt{2}$
 $\Rightarrow \cos x = 1 \pm \sqrt{2}$
 $\Rightarrow \arccos(1 - \sqrt{2}) = 1.107^\circ$
 $\alpha = 1.107^\circ \pm 2\pi n$
 $\alpha = 4.29^\circ \pm 2\pi n$
 $n = 0, 1, 2, 3$
 $\alpha_1 = 2.00^\circ$
 $\alpha_2 = 4.29^\circ$

Question 36

Solve each of the following trigonometric equations in the range given.

a) $2 \cos x - 3 \tan x = 0$

$0 \leq x < 2\pi$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

b) $3 \tan x \sin x = 8$

$0 \leq x < 2\pi$

$$x \approx 1.23^\circ, 5.05^\circ$$

c) $3 \tan x + 2 \cos x = 0$

$0 \leq x < 2\pi$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Handwritten solutions for Question 36:

(a) $2 \cos x - 3 \tan x = 0$
 $\Rightarrow 2 \cos x - \frac{3 \sin x}{\cos x} = 0$
 $\Rightarrow 2 \cos^2 x - 3 \sin x = 0$
 $\Rightarrow 2(1 - \sin^2 x) - 3 \sin x = 0$
 $\Rightarrow 2 - 2 \sin^2 x - 3 \sin x = 0$
 $\Rightarrow 0 = 2 \sin^2 x + 3 \sin x - 2$
 $\Rightarrow (2 \sin x - 1)(\sin x + 2) = 0$
 $\Rightarrow \sin x = \frac{1}{2}$
 $\Rightarrow x = \frac{\pi}{6} \pm 2n\pi$
 $\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

(b) $3 \tan x \sin x = 8$
 $\Rightarrow 3 \frac{\sin x}{\cos x} \sin x = 8$
 $\Rightarrow \frac{3 \sin^2 x}{\cos x} = 8$
 $\Rightarrow 3 \sin^2 x = 8 \cos x$
 $\Rightarrow 3(1 - \cos^2 x) = 8 \cos x$
 $\Rightarrow 3 - 3 \cos^2 x = 8 \cos x$
 $\Rightarrow 0 = 3 \cos^2 x + 8 \cos x - 3$
 $\Rightarrow (3 \cos x - 1)(\cos x + 3) = 0$
 $\Rightarrow \cos x = \frac{1}{3}$
 $\Rightarrow x = \arccos\left(\frac{1}{3}\right) \approx 1.23^\circ$
 $\therefore x = 1.23^\circ, 5.05^\circ$

(c) $3 \tan x + 2 \cos x = 0$
 $\Rightarrow 3 \frac{\sin x}{\cos x} + 2 \cos x = 0$
 $\Rightarrow \frac{3 \sin x}{\cos x} + 2 \cos x = 0$
 $\Rightarrow 3 \sin x + 2 \cos^2 x = 0$
 $\Rightarrow 3 \sin x + 2(1 - \sin^2 x) = 0$
 $\Rightarrow 3 \sin x + 2 - 2 \sin^2 x = 0$
 $\Rightarrow 0 = 2 \sin^2 x - 3 \sin x - 2$
 $\Rightarrow (2 \sin x + 1)(\sin x - 2) = 0$
 $\Rightarrow \sin x = -\frac{1}{2}$
 $\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 37

Solve each of the following trigonometric equations in the range given.

a) $\frac{1}{2} \tan x - \sin x = 0$

$0 \leq x < 360^\circ$

$x = 0^\circ, 60^\circ, 180^\circ, 300^\circ$

b) $\frac{\sin x - \cos x}{\cos x} = 2$

$0 \leq x < 360^\circ$

$x \approx 71.6^\circ, 251.6^\circ$

c) $4 \tan y \sin y \cos y + 4 \tan y \cos y + 1 = 0$

$0 \leq y < 360^\circ$

$y = 210^\circ, 330^\circ$

(a) $\frac{1}{2} \tan x - \sin x = 0$
 $\Rightarrow \tan x = 2 \sin x$
 $\Rightarrow \frac{\sin x}{\cos x} = 2 \sin x$
 $\Rightarrow \sin x = 2 \sin x \cos x$
 $\Rightarrow \sin x - 2 \sin x \cos x = 0$
 $\Rightarrow \sin x (1 - 2 \cos x) = 0$
 $\bullet \sin x = 0$
 $\Rightarrow \sin x(0) = 0$
 $\bullet \cos x = \frac{1}{2}$
 $\Rightarrow \cos x(3) = 60^\circ$
 $\bullet \cos x = \frac{1}{2}$
 $\Rightarrow \cos x = \frac{1}{2} \Rightarrow x = 60^\circ \pm 360^\circ$
 $\Rightarrow x = 60^\circ, 300^\circ$
 $\bullet \sin x = 0$
 $\Rightarrow x = 0^\circ, 180^\circ$
 $\therefore x = 0^\circ, 60^\circ, 180^\circ, 300^\circ$
 (b) $\frac{\sin x - \cos x}{\cos x} = 2$
 $\Rightarrow \sin x - \cos x = 2 \cos x$
 $\Rightarrow \sin x = 3 \cos x$
 $\Rightarrow \frac{\sin x}{\cos x} = \frac{3 \cos x}{\cos x}$
 $\Rightarrow \tan x = 3$
 $\Rightarrow \tan^{-1} 3 = 71.6^\circ$
 $\bullet \tan^{-1} 3 = 71.6^\circ$
 $\bullet \tan^{-1} 3 = 251.6^\circ$
 $\therefore x = 71.6^\circ, 251.6^\circ$
 (c) $4 \tan y \sin y \cos y + 4 \tan y \cos y + 1 = 0$
 $\Rightarrow 4 \tan y \cos y (\sin y + 1) = -1$
 $\Rightarrow \frac{4 \sin y \cos y (\sin y + 1)}{\cos y} = -1$
 $\Rightarrow 4 \sin y (\sin y + 1) = -1$
 $\Rightarrow 4 \sin^2 y + 4 \sin y + 1 = 0$
 $\Rightarrow (2 \sin y + 1)^2 = 0$
 $\Rightarrow 2 \sin y + 1 = 0$
 $\Rightarrow \sin y = -\frac{1}{2}$
 $\bullet \sin y = -\frac{1}{2}$
 $\Rightarrow \sin^{-1}(-\frac{1}{2}) = -30^\circ$
 $\Rightarrow y = -30^\circ \pm 360^\circ$
 $\Rightarrow y = 330^\circ, 210^\circ$
 $\therefore y = 210^\circ, 330^\circ$

Question 38

Prove the validity of each of the following trigonometric identities.

a) $(2 \cos x + \sin x)^2 + (\cos x - 2 \sin x)^2 \equiv 5.$

b) $\cos x + \sin x \tan x \equiv \frac{1}{\cos x}.$

c) $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} \equiv \frac{2}{\sin \theta}.$

(a) LHS = $(2 \cos x + \sin x)^2 + (\cos x - 2 \sin x)^2$
 $= 4 \cos^2 x + 4 \cos x \sin x + \sin^2 x + \cos^2 x - 4 \cos x \sin x + 4 \sin^2 x$
 $= 5 \cos^2 x + 5 \sin^2 x = 5 (\cos^2 x + \sin^2 x) = 5 \times 1 = 5 = \text{RHS}$

(b) LHS = $\cos x + \sin x \tan x = \cos x + \sin x \left(\frac{\sin x}{\cos x} \right)$
 $= \cos x + \frac{\sin^2 x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}$

(c) LHS = $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)(1 + \cos \theta)}{(1 + \cos \theta) \sin \theta}$
 $= \frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{(1 + \cos \theta) \sin \theta} = \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$
 $= \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta} = \text{RHS}$

ALTERNATIVE
 LHS = $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta (1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} + \frac{1 + \cos \theta}{\sin \theta}$
 $= \frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta} + \frac{1 + \cos \theta}{\sin \theta}$
 $= \frac{1 - \cos \theta}{\sin \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{2}{\sin \theta} = \text{RHS}$

Question 39

Prove the validity of each of the following trigonometric identities.

a) $\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} \equiv 2 \tan^2 x.$

b) $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} \equiv \frac{2}{\cos x}.$

c) $\frac{1 + \sin x}{\cos x} \equiv \frac{\cos x}{1 - \sin x}.$

(a) $LHS = \frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} = \frac{\sin x(1 + \sin x) - \sin x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$
 $= \frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{1 - \sin^2 x} = \frac{2\sin^2 x}{\cos^2 x} = 2\tan^2 x \equiv RHS$
 (b) $LHS = \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = \frac{\cos^2 x + (1 - \sin x)(1 - \sin x)}{(1 - \sin x)\cos x}$
 $= \frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{(1 - \sin x)\cos x} = \frac{2 - 2\sin x}{(1 - \sin x)\cos x} = \frac{2(1 - \sin x)}{(1 - \sin x)\cos x}$
 $= \frac{2}{\cos x} \equiv RHS$
 (c) $LHS = \frac{1 + \sin x}{\cos x} = \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)} = \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$
 $= \frac{\cos^2 x}{\cos x(1 - \sin x)} = \frac{\cos x}{1 - \sin x} \equiv RHS$
 Alternative for (b) using idea from part (c):
 $LHS = \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} + \frac{1 - \sin x}{\cos x}$
 $= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} + \frac{1 - \sin x}{\cos x} = \frac{\cos x(1 + \sin x)}{\cos^2 x} + \frac{1 - \sin x}{\cos x}$
 $= \frac{1 + \sin x}{\cos x} + \frac{1 - \sin x}{\cos x} = \frac{2}{\cos x} \equiv RHS$