# Createu w TRIGONOSETRY MAR COMPOUND ANGLE SINALISCOM L.Y.C.B. MARIASINALISCOM I.Y.C.B. MARIASIN,

#### Question 1

Prove the validity of each of the following trigonometric identities. 

- **a**)  $\sin\left(x+\frac{\pi}{4}\right) \equiv \cos\left(x-\frac{\pi}{4}\right)$
- **b**)  $\cos\left(x+\frac{\pi}{3}\right)+\sqrt{3}\sin\left(x+\frac{\pi}{3}\right) = 2\cos x$

c) 
$$\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$$

 $\frac{\sin(x+y)}{\cos x \cos y} \equiv \tan x + \tan y$ 

e)  $\tan\left(x+\frac{\pi}{4}\right)\tan\left(x-\frac{\pi}{4}\right) \equiv -1$ I.Y.C.B. Madasman

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(a) Ufs=sh(x+要)=smales要+losesm要= 握sma+ 怪lose
$H = \frac{\pi}{2} \cos $
(b) $U_{1}^{t} = \omega_{S}(x + \frac{\pi}{2}) + M^{2} sm(x + \frac{\pi}{2})$
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= 1/00 - 13 pm2 + 3 + 1 un2 + 13 + 15 + 15 cora
$=\frac{1}{2}\log_2 + \frac{3}{2}\log_2 = 2\log_2 = 245$
$(\overline{y} - c_{2})aa + (\overline{y} + c_{2})aa = 2\mu$ (3)
= COS2LOST - SURDERT + COS2LSINT + SIN 2+SINT
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#### **Question 2**

I.V.G.B.

Prove the validity of each of the following trigonometric identities. 

a) 
$$\sin\left(x + \frac{\pi}{3}\right) - \sqrt{3}\cos\left(x + \frac{\pi}{3}\right) \equiv 2\sin x$$
  
b)  $\frac{\cos x}{\sin y} - \frac{\sin x}{\cos y} \equiv \frac{\cos(x + y)}{\sin y \cos y}$ 

**b)** 
$$\frac{\cos x}{\sin y} - \frac{\sin x}{\cos y} \equiv \frac{\cos(x+y)}{\sin y \cos y}$$

c) 
$$\tan(x+60^\circ)\tan(x-60^\circ) \equiv \frac{\tan^2 x - 3}{1 - 3\tan^2 x}$$

d) 
$$\sin(x+y)\sin(x-y) \equiv \cos^2 y - \cos^2 x$$
  
e)  $\cot(x+y) \equiv \frac{\cot x \cot y - 1}{\cot x + \cot y}$ 

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e)  $\cot(x+y) \equiv \frac{\cot x \cot y - 1}{\cot x + \cot y}$ I.V.C.B. Madasmaths.Com

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	= 2 312 + 12 602 - 12 602	+ 콜= 3142	
	$= 2s_{1}m_{A} = R4+S$		
k.	(b) UAS = Cosx - Sma = Cosacosu - Smassing = Cos	<u>a(a+y)</u> = RHS Mycoey	
	(c) LHS = tw(2+60) tw(2-60) = tw2+tw60 × ton2 1-tonto60 × 1++	- toylo tans toxo	
	$= \frac{\tan_{12} + v_{1}^{2}}{1 - v_{1}^{2} \tan^{2}} \times \frac{\tan_{12} - v_{1}^{2}}{1 + v_{1}^{2} \tan^{2}} = \frac{\tan_{12} - 3}{1 - 3 \tan_{12}^{2}}$	= R45	
Ø.	(d) 443 = Sm(2+4) sm(2-4) = (smx (agg + werseny) (smx we	- ouszeny)	
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#### **Question 3**

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I.F.C.B

Prove the validity of each of the following trigonometric identities.

a)  $\cos(x+y)\cos(x-y) \equiv \cos^2 x - \sin^2 y$ 

**b**) 
$$\sin P - \sin Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

c) 
$$\sin^2\left(\theta + \frac{\pi}{4}\right) + \sin^2\left(\theta - \frac{\pi}{4}\right) \equiv 1$$

 $\cos x$ **d**)  $\cos x + \sin x \tan 2x \equiv$  $\cos 2x$ 

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e) 
$$\cos P + \cos Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

- [Less cosy sinalsing][c ny] = dilformit = 60ईदावहेंy - รพธิธรพรุ้y  $= (a \hat{s}_{\alpha}(i - s \hat{n}_{y}) - (i - c \hat{s}_{\alpha}) s \hat{n}_{y}$ = cost - costering - sing + costa sing
- = cosa snzy
- = RHS
- sm(4+B) = sm4cas8+coa4sm8 sm(4-B) = sm4cas8-coa4sm8 (6) sobbrad - equations sm(A+B) - sm(A-B) = 2005AsmB
  - P = A + B Q = A B Q = A B Add Guudions: P+Q = 2A P = Q P = Q
  - Subtract Equations: P-ap = 2B
  - TANS 🛞 BEROMAS  $\sin \frac{p}{2} - \sin \varphi = 2 \cos \left(\frac{p+\varphi}{2}\right) \sin \left(\frac{p-\varphi}{2}\right)$
- $( \mathbf{j}_{1}^{\mathrm{max}} + \mathbf{j}_{2}^{\mathrm{max}} + \mathbf{j}_{2}^{\mathrm{max}} \mathbf{j}_{2}^{\mathrm{max}} + \mathbf{j}_{2}^{\mathrm{max}} \mathbf$  $= \frac{\left(\frac{1}{2} \cos \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4}\right)^{2}}{\left(\frac{1}{2} \cos \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4}\right)^{2}} + \frac{\left(\frac{1}{2} \cos \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4}\right)^{2}}{\left(\frac{\pi}{2} \cos \theta \sin \frac{\pi}{4} + \sin \frac{\pi}{4} \cos \frac{\pi}{4} + \sin \frac{\pi}{4} \sin \frac{\pi}{4}\right)^{2}}$ 

  - =  $s_{M}^{2}\Theta + c_{0}s_{0}^{2}\Theta = 1$

(b) 145 = 000 + SM2 tong 2 = 000 + SM2 (b)  $= \frac{(c_{-3/2})_{2/0}}{(c_{-3/2})_{2/0}} = \frac{(c_{1/2})_{2/1/2} + c_{2/2/0}}{(c_{2/2})_{2/1/2}} =$ 2#15 = x200 = Bm2Am2 - Beccher = (8+4) 200 Bm2Am2 + Becchero = (8-4) 200 ) Add Equations

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 $\Im \int Bau Aau \nabla = (E - Bau + (B + A)au)$ Let P=4+B) Q=A-B) add fguudi. P+Q= 24  $\frac{P-Q=23}{\left\lceil \frac{P-Q}{2}-B\right\rceil}$ 

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 $\frac{1}{2}$  becomes  $\frac{1}{2}$  b

#### Question 4

If  $\sin(\theta + \alpha) = 2\sin\theta$ , show clearly that

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 $\tan\theta = \frac{\sin\alpha}{2 - \cos\alpha}$ 



$\begin{split} & Sm\left(\Theta t, \alpha\right) = 2 \text{SmB} \\ & SmB(cSH + CosB(SmH) = 2 \text{SmB} \\ & CosB(SmH) = 2 \text{SmB} - SmB(csH) \\ & CosB(SmH) = -SmB(csH) \\ & CosB(SmH) = -SmB(csH) \\ & SmB(sH) = -SmB(sH) \\ & SmB(sH) \\ & SmB(sH) = -SmB(sH) \\ & SmB(sH) \\$	- tanga - smar 2- tang 44 Espurito

### **Question 5**

By expanding  $tan(\theta + 45^\circ)$  with a suitable value for  $\theta$ , show clearly that

 $\tan 75^\circ = 2 + \sqrt{3}$ .

proof

$$\begin{split} & \mathsf{sur}(75) = \mathsf{bur}_1(4(33)) = \frac{\mathsf{bur}_1(5+\mathsf{dow}_35)}{(-\mathsf{bur}_1(5+\mathsf{dow}_35))} = \frac{\mathsf{l} + \frac{\mathsf{st}_1^2}{2}}{\mathsf{l} - \mathsf{l} \times \mathsf{st}_2^2} \\ & = \frac{\mathsf{3}_+ \mathsf{st}_2^2}{\mathsf{3}_- \mathsf{st}_1^2} = \frac{(\mathsf{3}_+ \mathsf{st}_1^2)(\mathsf{3}_+ \mathsf{st}_2^2)}{(\mathsf{3}_- \mathsf{st}_1^2)(\mathsf{3}_+ \mathsf{st}_2^2)} = \frac{\mathsf{q}_+ \mathsf{f} \mathsf{st}_2^2 + \mathsf{st}_2}{\mathsf{q} - \mathsf{3}_-} = \frac{\mathsf{1}_2 + \mathsf{st}_2^2}{\mathsf{q}} \\ & = 2 + \mathsf{st}_2^2 \end{split}$$

**Question 6** 

E.

By expanding  $sin(45^\circ - x)$  with a suitable value for x, show clearly that

 $\csc 15^\circ = \sqrt{2} + \sqrt{6} \, .$ 

proof

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$$\begin{split} f_{-1}(\tau, \tau, s) &= g_{0}(d_{1}, -s_{0}) = g_{0}(d_{2}, -s_{0}) =$$

#### Question 7

By expanding  $tan(\theta + 45^\circ)$  with a suitable value for  $\theta$ , show clearly that

 $\tan 105^\circ = -2 - \sqrt{3}$ .



$$\begin{split} &\mathcal{H} \stackrel{\text{des}}{=} & \mathcal{G}_{\text{ch}} \left( \mathcal{G}_{\text{ch}} + \mathcal{G}_{\text{ch}} \right) = \frac{\mathcal{G}_{\text{ch}} \mathcal{G}_{\text{ch}} + \frac{\mathcal{G}_{\text{ch}} \mathcal{G}_{\text{ch}}}{\left[ - \mathcal{G}_{\text{ch}} \right]} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} + 1}{1 - \mathcal{G}} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} = \frac{\mathcal{G}_{\text{ch}} (1) \left( + \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( - \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( - \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( - \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( - \mathcal{G}_{\text{ch}} \right)}{\left( - \mathcal{G}_{\text{ch}} \right)} \\ &= \frac{\mathcal{G}_{\text{ch}} (1) \left( - \mathcal{G}_{\text{$$

#### Question 8

By expanding  $\cos(y+45^\circ)$  with a suitable value for y, show clearly that

 $\sec 75^\circ = \sqrt{2} + \sqrt{6} \ .$ 

proof

ic75 =	$\frac{1}{6573} = \frac{1}{65430} = \frac{1}{654300}$
Ξ.	$\frac{1}{\frac{q_{1}^{2}}{2}\frac{q_{1}^{2}}{2}-\frac{q_{2}^{2}}{2}x_{2}^{\frac{1}{2}}} = \frac{1}{\frac{q_{1}^{2}}{4}-\frac{q_{2}^{2}}{4}} = \frac{4}{\kappa^{2}-q_{2}^{2}} = \frac{4(j\zeta_{+}^{2}(\zeta_{+}^{2}))}{(q_{+}^{2}-q_{2}^{2})(q_{+}^{2}+q_{2}^{2})}$
=	$\frac{4(\sqrt{6}+\sqrt{2})}{6} = \sqrt{4(\sqrt{6}+\sqrt{2})} = \sqrt{6} + \sqrt{2}$

#### **Question 9**

By considering the expansion of tan(A+B) with suitable values for A and B, show clearly that

 $\cot 75^\circ = 2 - \sqrt{3}.$ 

proof

$$\begin{split} & = \frac{1}{1 - \frac{\sqrt{3}}{2}} = \frac{1}{\frac{3 - \sqrt{3}}{1 - \sqrt{3}}} = \frac{1}{\frac{\sqrt{3} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{1 - \sqrt{3}}} = \frac{1}{\frac{\sqrt{3} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{1 - \sqrt{3}}} \\ & = \frac{1 - \frac{\sqrt{3}}{2}}{1 - \sqrt{3}} = \frac{3 - \sqrt{3}}{2 - \sqrt{3}} = \frac{\sqrt{3} - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}} \\ & = \frac{1 - \sqrt{3}}{2 - \sqrt{3}} = \frac{1 - \sqrt{3}$$

#### Question 10

Show clearly, by using the compound angle identities, that

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Show clearly, by using the compound angle identities, that

 $\cos 105^\circ = \frac{\sqrt{2} - \sqrt{6}}{4}.$ 

proof

proof

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 $\begin{array}{l} (\alpha_{1},\beta_{2$ 

Question 12

Show clearly, by using the compound angle identities, that

 $\tan 15^\circ = 2 - \sqrt{3} \ .$ 



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 $m_1 U'' = t_{m_1} (60'-45'') = \frac{t_{m_1} b_0 - t_{m_1} 45}{1 + t_{m_1} b_0 t_{m_1} 45} = \frac{15' - 1}{1 + \sqrt{3} \times 1}$ 

 $= \frac{\sqrt{3^{2}-1}}{\sqrt{3^{2}+1}} = \frac{(\sqrt{3}-1)(\sqrt{3^{2}+1})}{(\sqrt{3^{2}+1})(\sqrt{3^{2}-1})} = \frac{3-2\sqrt{3^{2}+1}}{3-1} = \frac{4-2\sqrt{3^{2}}}{2}$   $= 2-\sqrt{3^{2}}$ 

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#### Question 13

 $\sin(A+B) \equiv \sin A \cos B + \cos A \sin B.$ 

a) Use the above trigonometric identity with suitable values for A and B, to show that

 $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}.$ 

**b**) Hence by using the trigonometric expansion of  $cos(75^\circ + \alpha)$  with a suitable value for  $\alpha$ , show clearly that

 $\cos 165^\circ = -\sin 75^\circ.$ 

Question 14

 $\sin A = \frac{12}{13}$  and  $\cos B =$ 5

If A is obtuse and B is acute, show clearly that

 $\sin\left(A+B\right)=\frac{33}{65}.$ 

proof

proof

# Question 15

$$n\theta = \frac{8}{17}$$
 and  $\cos \varphi = \frac{5}{13}$ .

If  $\theta$  is obtuse and  $\varphi$  is acute, show clearly that

$$\cos\left(\theta+\varphi\right)=-\frac{171}{221}$$



proof

#### Question 16

The constants a and b are such so that

$$\tan a = \frac{1}{3}$$
 and  $\tan b = \frac{1}{7}$ 

Determine the exact value of  $\cot(a-b)$ , showing all the steps in the workings.

$$\cot(a-b) = \frac{11}{2}$$

# Question 17

$$nx = \frac{12}{13}$$
 and  $\cos y = \frac{15}{17}$ .

If x is obtuse and y is acute, show clearly that

 $\sin(x-y) = \frac{220}{221}.$ 

Question 18

.K.C.

 $\sin P = \frac{8}{17}$ and  $\tan Q =$ 

If P is obtuse and Q is reflex, show clearly that

 $\cos\left(P-Q\right)=\frac{13}{85}.$ 



proof

2×15 - (-5)×1

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8 9 20	T C	$Gas P = -\frac{15}{17}$	
S IQ J	S A A	$Contp = -\frac{3}{2}$ $Contp = -\frac{4}{2}$	
as(P−9) = =	$m_Z + Q_Z \omega q^2 \cos \frac{1}{2}$ $m_Z + \left(\frac{1}{2}\right) \times \frac{2i}{7i} - \frac{3}{2}$	$\varphi_{ii}Q_{ij} = \left(\frac{\varphi}{2}\right) \cdot \left(\frac{\varphi}{2}\right) \cdot$	$\frac{51}{28} = \frac{55}{28}$



 $\sin\theta = \frac{5}{13}$  and  $\sin\varphi = -\frac{7}{25}$ 

If  $\theta$  is obtuse and  $\varphi$  is such so that  $180^{\circ} < \varphi < 270^{\circ}$ , show clearly that

 $\sin\left(\theta+\varphi\right)=-\frac{36}{325}$ 

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Question 20

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N.C.

$$\cos\theta = -\frac{3}{5}$$
 and  $\tan\varphi = \frac{24}{7}$ 

If  $\theta$  is reflex, and  $\varphi$  is also reflex, show clearly that

$$\sin\left(\theta-\varphi\right)=-\frac{44}{125}$$

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$\cos \theta = -\frac{3}{5}$		$SWD = -\frac{4}{5}$ $Cost \phi = -\frac{2}{5}$ $Sh \phi = -\frac{24}{25}$
sn(b-i	$ \sum_{k=1\\k=1\\k=1\\k=1\\k=1\\k=1\\k=1\\k=1\\k=1\\k=1\\$	
	= 44	* p





#### Question 23

$$nA = \frac{1}{3}$$
 and  $\cos B = \frac{1}{2}$ .

If A is obtuse and B is reflex, show clearly that

$$\sin\left(A+B\right)=\frac{1-2\sqrt{6}}{6}.$$



The point A lies on the y axis above the origin O and the point B lies on the y axis below the origin O.

The point C(12,0) is at a distance of 20 units from A and at a distance of 13 units from B.

By considering the tangent ratios of  $\angle OCA$  and  $\angle OCB$ , show that the tangent of the angle ACB is exactly  $\frac{63}{16}$ .



proof

proof

#### **Question 25**

R

Solve each of the following trigonometric equations.

- **a**)  $\cos(\theta + 30^\circ) = \sin \theta$ ,  $0 \le \theta < 360^\circ$
- **b**)  $3\cos(x+30^\circ) = \sin(x-60^\circ), \quad 0 \le x < 360^\circ$
- c)  $\sin(y-30^\circ) = \sin(y+45^\circ)$ ,  $0 \le y < 360^\circ$
- **d**)  $\sin(\varphi + 30^\circ) = \cos(\varphi 45^\circ), \quad 0 \le \varphi < 360^\circ$
- e)  $\cos(\alpha 60^\circ) = \cos(\alpha 45^\circ), \quad 0 \le \alpha < 360^\circ$

$$\theta = 30^{\circ}, 210^{\circ}, x = 60^{\circ}, 240^{\circ}, y = 82.5^{\circ}, 262.5^{\circ}, \varphi = 52.5^{\circ}, 232.5^{\circ}, z = 52.5^{\circ}, 232.5^{\circ}, z = 52.5^{\circ}, z = 52.5^$$

(m) (celan-)- and	(C. D) D.
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- Allan Love - 0	7 tony 2 42+1
	and (Sett) - that
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	= 4= 80.0° ± 180, Mars122
620-	in sis mars //
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#### **Question 26**

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Solve each of the following trigonometric equations.

- **a**)  $\sin(\theta 45^\circ) = \sin \theta$ ,  $0 \le \theta < 360^\circ$
- **b**)  $\cos(x-30^\circ) = \sin(x+30^\circ), \quad 0 \le x < 360^\circ$
- c)  $\cos(y-30^\circ) = \sin(y+45^\circ), \quad 0 \le y < 360^\circ$
- **d**)  $\sin(\varphi 30^\circ) = \cos(\varphi 45^\circ), \quad 0 \le \varphi < 360^\circ$
- e)  $\cos(\alpha 60^\circ) = \cos(\alpha + 60^\circ), \quad 0 \le \alpha < 360^\circ$

 $\theta = 112.5^{\circ}, 292.5^{\circ}, x = 45^{\circ}, 225^{\circ}, y = 37.5^{\circ}, 217.5^{\circ}, \varphi = 82.5^{\circ}, 262.5^{\circ}, z_{0} = 82.5^{\circ}, 262.5^{\circ}, z_{0} = 82.5^{\circ}, z_{0} = 82$ 



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 $\alpha = 0^{\circ}, 180^{\circ}$ 

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#### **Question 27**

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Solve each of the following trigonometric equations.

- **a**)  $\sin\left(\theta + \frac{\pi}{4}\right) = \sin\theta$ ,  $0 \le \theta < 2\pi$
- b)  $\cos\left(x+\frac{\pi}{6}\right) = \cos\left(x-\frac{3}{3}\right)$ c)  $\sin\left(\frac{\pi}{3}-y\right) = \cos\left(y+\frac{5\pi}{6}\right), \quad 0 \le y < 2\pi \text{ (very hard)}$   $\left(\pi\right) = 0 \le \varphi < 2\pi$ 

  - e)  $\sqrt{2}\cos\left(\alpha + \frac{\pi}{4}\right) = \sin\left(\alpha + \frac{\pi}{6}\right), \quad 0 \le \alpha < 2\pi$

$$s\left(\varphi + \frac{\pi}{2}\right) + \sin\left(\varphi + \frac{\pi}{3}\right) = 0, \quad 0 \le \varphi < 2\pi$$

$$\cos\left(\alpha + \frac{\pi}{4}\right) = \sin\left(\alpha + \frac{\pi}{6}\right), \quad 0 \le \alpha < 2\pi$$

$$\theta = \frac{3\pi}{8}, \frac{11\pi}{8}, \quad \left[x = \frac{7\pi}{12}, \frac{19\pi}{12}\right], \quad y = \frac{\pi}{2}, \frac{3\pi}{2}, \quad \varphi = \frac{\pi}{6}, \frac{7\pi}{6}, \quad \alpha = \frac{\pi}{12}, \frac{13\pi}{12}$$



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#### **Question 28**

Solve each of the following trigonometric equations.

- **a**)  $\sin(\theta 20^\circ) = \sin(\theta + 60^\circ), \quad 0 \le \theta < 360^\circ$
- **b**)  $\cos(x-35^\circ) = \cos(x-55^\circ)$ ,  $0 \le x < 360^\circ$
- c)  $\sin(y-48^\circ) = \cos(y+12^\circ), \quad 0 \le y < 360^\circ$
- **d**)  $\sin(\varphi + 72^\circ) = \cos(\varphi 38^\circ)$ ,  $0 \le \varphi < 360^\circ$
- e)  $\cos(\alpha 36^\circ) = \cos(\alpha 72^\circ)$ ,  $0 \le \alpha < 360^\circ$



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$ \left\{ \begin{array}{c} (a) + b \\ (a) + b \\ (b) +$	19447) = 70 Bon 100023. 70 45 45 Bau 100023	$\begin{cases} (37.4)^{2} = (27.4)^{2} (37.4$	ocp.(2.217)= 50 4.22.*. cocp.(2.217)= 50 0.20.*.
$ \left. \begin{array}{c} (y_1+y_2) = (y_2+y_3) = (y_2+y_3) e_2 \\ (y_1+z_2e_1e_2-(y_2-(y_2e_1e_2)) = (y_2e_1e_2(y_2-(y_2e_1e_2)) e_2e_1e_2) \\ (y_1+z_2e_1e_2e_2) = (y_1e_1e_1e_2e_1e_2) = (y_1e_1e_1e_2e_1e_2) e_2e_2 \\ (y_1+z_2e_1e_2) = (y_1e_1e_2+(y_1e_2)) e_2e_2 \\ (y_1+z_2e_1e_2) = (y_1e_2e_1e_2e_2) e_2e_2 \\ (y_1+z_2e_1e_2e_2) = (y_1e_2e_1e_2e_2) e_2e_2 \\ (y_1+z_2e_1e_2e_2) = (y_1e_2e_1e_2e_2) e_2e_2 \\ (y_1+z_2e_1e_2e_2) = (y_1e_2e_1e_2e_2) e_2e_2 \\ (y_1+z_2e_1e_2e_2e_2) = (y_1e_2e_1e_2e_2e_2) e_2e_2 \\ (y_1+z_2e_2e_2e_2e_2e_2e_2e_2) = (y_1e_2e_2e_2e_2e_2e_2e_2e_2e_2e_2e_2e_2e_2e$	626)	$\begin{split} & \left( \mathrm{ccs}\left( \mathbf{x}' \rightarrow \mathrm{cds}\right) = \cos\left( \mathbf{x}' - 7\mathbf{x} \right) \\ & \mathrm{ccs}_{\mathcal{H}}(\mathrm{ccd}_{\mathcal{H}}^{2} + \mathrm{Sup}(\mathrm{Sup}(\mathcal{L}) = \mathrm{Sup}(\mathrm{ccd}_{\mathcal{H}}^{2} + \mathrm{Sup}(\mathrm{Sup}(\mathcal{L}) \in \mathrm{Sup}(\mathrm{Sup}(\mathcal{L}) = \mathrm{Sup}($	$\begin{array}{l} t_{20,4,4}\\ \\ \text{orthan}(1/3164) = 94\\ \\ \\ x'_{1} \leq 5\frac{4}{3} \pm 1/804,  \forall x = q_{1}r_{1}, \\ \\ x'_{1} \leq 5\frac{4}{3} \pm \frac{1}{3} \\ \\ \\ x'_{2} + 2\frac{4}{3} \\ \end{array}$