BINOMIAL SERIES EXPANSIONS

Question 1

a) Expand $(1+2x)^{-1}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-2x+4x^2-8x^3+16x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

(a) $(1+2x)^{-1} = 1 + \frac{(-1)}{1}(2x) + \frac{(-1)(-2)}{1\times 2}(2x)^2 + \frac{(-1)(-2)(-3)}{1\times 2\times 3}(2x)^3 + \frac{(-1)(-2)(-3)(-4)}{1\times 2\times 3\times 4}(2x)^4 + O(x^5)$

$= 1 - 2x + 4x^2 - 8x^3 + 16x^4 + O(x^5)$

(b) VALID FOR $\left.\begin{array}{l}|2x|<1\\ |x|<\frac{1}{2}\end{array}\right\}$ I.E. $-\frac{1}{2}<x<\frac{1}{2}$

Question 2

a) Expand $(1-4x)^{-\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+2x+6x^2+20x^3+70x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

(a) $(1-4x)^{-\frac{1}{2}} = 1 + \frac{-\frac{1}{2}}{1}(-4x)^1 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2}(-4x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}(-4x)^3 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{1\times 2\times 3\times 4}(-4x)^4$

$= 1 + 2x + \frac{3}{8}(16x^2) - \frac{5}{16}(-64x^3) + \frac{35}{128}(256x^4) + O(x^5)$

$= 1 + 2x + 6x^2 + 20x^3 + 70x^4 + O(x^5)$

(b) VALID FOR $|4x|<1$

$|x|<\frac{1}{4}$ $\quad\therefore -\frac{1}{4}<x<\frac{1}{4}$

Question 3

a) Expand $(1+2x)^{-2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-4x+12x^2-32x^3+80x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

(a) $(1+2x)^{-2} = 1 + \frac{-2}{1}(2x) + \frac{(-2)(-3)}{1\times 2}(2x)^2 + \frac{(-2)(-3)(-4)}{1\times 2\times 3}(2x)^3 + \frac{(-2)(-3)(-4)(-5)}{1\times 2\times 3\times 4}(2x)^4$

$= 1 - 4x + 12x^2 - 32x^3 + 80x^4 + O(x^5)$

(b) VALID FOR $|2x|<1$
$|x|<\frac{1}{2}$ $\quad -\frac{1}{2}<x<\frac{1}{2}$

Question 4

a) Expand $(1+3x)^{-\frac{1}{3}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-x+2x^2-\frac{14}{3}x^3+\frac{35}{3}x^4+O\left(x^5\right)}, \quad \boxed{-\frac{1}{3}<x<\frac{1}{3}}$$

(a) $(1+3x)^{-\frac{1}{3}} = 1 + \frac{\left(-\frac{1}{3}\right)}{1}(3x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{1\times 2}(3x)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{1\times 2\times 3}(3x)^3 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(-\frac{10}{3}\right)}{1\times 2\times 3\times 4}(3x)^4 + O(x^5)$

$= 1 - x + \frac{2}{9}(9x^2) - \frac{14}{81}(27x^3) + \frac{35}{243}(81x^4) + O(x^5)$

$= 1 - x + 2x^2 - \frac{14}{3}x^3 + \frac{35}{3}x^4 + O(x^5)$

(b) VALID FOR $\left.\begin{array}{l}|3x|<1\\ |x|<\frac{1}{3}\end{array}\right\}$ i.e. $-\frac{1}{3}<x<\frac{1}{3}$

Question 5

a) Expand $\dfrac{1}{(1-2x)^2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+4x+12x^2+32x^3+80x^4+O\left(x^5\right)}, \quad \boxed{-\tfrac{1}{2}<x<\tfrac{1}{2}}$$

(a) $\frac{1}{(1-2x)^2}=(1-2x)^{-2}=1+\frac{-2}{1}(-2x)^1+\frac{(-2)(-3)}{1\times2}(-2x)^2+\frac{(-2)(-3)(-4)}{1\times2\times3}(-2x)^3+\frac{(-2)(-3)(-4)(-5)}{1\times2\times3\times4}(-2x)^4+O(x^5)$

$=1+4x+3(4x^2)-4(-8x^3)+5(16x^4)+O(x^5)$

$=1+4x+12x^2+32x^3+80x^4+O(x^5)$

(b) VALID FOR $|2x|<1$

$|x|<\frac{1}{2}$ $\quad\therefore -\frac{1}{2}<x<\frac{1}{2}$

Question 6

a) Expand $\sqrt[4]{1+2x}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+\tfrac{1}{2}x-\tfrac{3}{8}x^2+\tfrac{7}{16}x^3-\tfrac{77}{128}x^4+O\left(x^5\right)}, \quad \boxed{-\tfrac{1}{2}<x<\tfrac{1}{2}}$$

(a) $\sqrt[4]{1+2x}=(1+2x)^{\frac{1}{4}}$

$=1+\frac{\frac{1}{4}}{1}(2x)^1+\frac{\frac{1}{4}\left(-\frac{3}{4}\right)}{1\times2}(2x)^2+\frac{\frac{1}{4}\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)}{1\times2\times3}(2x)^3+\frac{\frac{1}{4}\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)\left(-\frac{11}{4}\right)}{1\times2\times3\times4}(2x)^4+O(x^5)$

$=1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{7}{128}(8x^3)-\frac{77}{2048}\times16x^4+O(x^5)$

$=1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{7}{16}x^3-\frac{77}{128}x^4+O(x^5)$

(b) $|2x|<1$

$|x|<\frac{1}{2}$ $\quad -\frac{1}{2}<x<\frac{1}{2}$

Question 7

a) Expand $\dfrac{1}{(1+2x)^3}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-6x+24x^2-80x^3+240x^4+O\left(x^5\right)}, \quad \boxed{-\tfrac{1}{2}<x<\tfrac{1}{2}}$$

(a) $\frac{1}{(1+2x)^3}=(1+2x)^{-3}=1+\frac{-3}{1}(2x)^1+\frac{(-3)(-4)}{1\times2}(2x)^2+\frac{(-3)(-4)(-5)}{1\times2\times3}(2x)^3+\frac{(-3)(-4)(-5)(-6)}{1\times2\times3\times4}(2x)^4+O(x^5)$

$=1-6x+6(4x^2)-10(8x^3)+15(16x^4)+O(x^5)$

$=1-6x+24x^2-80x^3+240x^4+O(x^5)$

(b) VALID FOR $|2x|<1$

$|x|<\frac{1}{2}$ $\quad\therefore -\frac{1}{2}<x<\frac{1}{2}$

Question 8

a) Expand $\dfrac{1}{(1-3x)^2}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+6x+27x^2+108x^3+405x^4+O\left(x^5\right)}, \quad \boxed{-\tfrac{1}{3}<x<\tfrac{1}{3}}$$

(a) $\frac{1}{(1-3x)^2}=(1-3x)^{-2}=1+\frac{(-2)}{1}(-3x)+\frac{(-2)(-3)}{1\times2}(-3x)^2+\frac{(-2)(-3)(-4)}{1\times2\times3}(-3x)^3+\frac{(-2)(-3)(-4)(-5)}{1\times2\times3\times4}(-3x)^4+O(x^5)$

$=1+6x+27x^2+108x^3+405x^4+O(x^5)$

(b) VALID FOR $\left.\begin{matrix}|3x|<1\\|x|<\frac{1}{3}\end{matrix}\right\}$ ie $-\frac{1}{3}<x<\frac{1}{3}$

Question 9

a) Expand $(1+3x)^{-\frac{5}{3}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-5x+20x^2-\frac{220}{3}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{3}<x<\frac{1}{3}}$$

(a) $(1+3x)^{-\frac{5}{3}} = 1+\frac{-\frac{5}{3}}{1}(3x)^1+\frac{\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)}{1\times 2}(3x)^2+\frac{\left(-\frac{5}{3}\right)\left(-\frac{8}{3}\right)\left(-\frac{11}{3}\right)}{1\times 2\times 3}(3x)^3+O(x^4)$

$= 1-5x+20x^2-\frac{220}{3}x^3+O(x^4)$ //

(b) VALID FOR $|3x|<1$

$|x|<\frac{1}{3}$ $\quad \therefore -\frac{1}{3}<x<\frac{1}{3}$ //

Question 10

a) Expand $(1+5x)^{-\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-\frac{5}{2}x+\frac{75}{8}x^2-\frac{625}{16}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{5}<x<\frac{1}{5}}$$

(a) $(1+5x)^{-\frac{1}{2}} = 1+\frac{-\frac{1}{2}}{1}(5x)^1+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{1\times 2}(5x)^2+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}(5x)^3+O(x^4)$

$= 1-\frac{5}{2}x+\frac{75}{8}x^2-\frac{625}{16}x^3+O(x^4)$ //

(b) VALID FOR $|5x|<1$

$|x|<\frac{1}{5}$ $\quad -\frac{1}{5}<x<\frac{1}{5}$ //

Question 11

a) Expand $(1-4x)^{\frac{1}{2}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-2x-2x^2-4x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

(a) $(1-4x)^{\frac{1}{2}} = 1 + \frac{\frac{1}{2}}{1}(-4x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}(-4x)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}(-4x)^3 + \dots$

$= 1 - 2x - 2x^2 - 4x^3 + O(x^4)$

(b) VALID FOR $|4x| < 1$

$|x| < \frac{1}{4}$ $\quad -\frac{1}{4} < x < \frac{1}{4}$

Question 12

a) Expand $\dfrac{1}{(1+4x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1-12x+96x^2-640x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

(a) $\frac{1}{(1+4x)^3} = (1+4x)^{-3} = 1 + \frac{-3}{1}(4x)^1 + \frac{(-3)(-4)}{1\times 2}(4x)^2 + \frac{(-3)(-4)(-5)}{1\times 2\times 3}(4x)^3 + O(x^4)$

$= 1 - 12x + 96x^2 - 640x^3 + O(x^4)$

(b) VALID FOR $|4x| < 1$

$|x| < \frac{1}{4}$ $\quad -\frac{1}{4} < x < \frac{1}{4}$

Question 13

a) Expand $\dfrac{1}{\sqrt{1-2x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}$$

(a) $\dfrac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1+\dfrac{\left(-\frac{1}{2}\right)}{1}(-2x)^1+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2}(-2x)^2+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}(-2x)^3+O(x^4)$

$= 1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+O(x^4)$

(b) VALID FOR $|2x|<1$

$|x|<\frac{1}{2}$ $\quad\therefore\; -\frac{1}{2}<x<\frac{1}{2}$

Question 14

a) Expand $\sqrt{1+2x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) By using $x=0.01$ in the above expansion find an approximation to $\sqrt{1.02}$, giving the answer correct to 5 decimal places.

$$\boxed{1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{2}<x<\frac{1}{2}}, \quad \boxed{1.00995}$$

(a) $\sqrt{1+2x} = (1+2x)^{\frac{1}{2}} = 1+\dfrac{\left(\frac{1}{2}\right)}{1}(2x)+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1\times 2}(2x)^2+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}(2x)^3+O(x^4)$

$= 1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+O(x^4)$

(b) VALID FOR $|2x|<1$, IE $|x|<\frac{1}{2}$, IE $-\frac{1}{2}<x<\frac{1}{2}$

(c) IF $x=0.01$ $\quad\sqrt{1+2(0.01)} = 1+(0.01)-\frac{1}{2}(0.01)^2+\frac{1}{2}(0.01)^3+O((0.01)^4)$

$\sqrt{1.02} \approx 1+0.01-0.00005+0.0000005$

$\sqrt{1.02} \simeq 1.00995$

(5 d.p)

MORE BINOMIAL EXPANSIONS

Question 1

a) Expand $\sqrt{4-9x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{9}{4}x-\frac{81}{64}x^2-\frac{729}{512}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{9}<x<\frac{4}{9}}$$

(a) $\sqrt{4-9x} = (4-9x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{9}{4}x\right)^{\frac{1}{2}} = 2\left(1-\frac{9}{4}x\right)^{\frac{1}{2}}$

$= 2\left[1+\frac{\left(\frac{1}{2}\right)}{1}\left(-\frac{9}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1\times 2}\left(-\frac{9}{4}x\right)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}\left(-\frac{9}{4}x\right)^3+O(x^4)\right]$

$= 2\left[1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+O(x^4)\right]$

$= 2-\frac{9}{4}x-\frac{81}{64}x^2-\frac{729}{512}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{9}{4}x\right|<1$

$|x|<\frac{4}{9}$ $\quad\therefore\quad -\frac{4}{9}<x<\frac{4}{9}$

Question 2

a) Expand $\dfrac{1}{(2-5x)^2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+\frac{125}{8}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{5}<x<\frac{2}{5}}$$

(a) $\frac{1}{(2-8x)^3} = (2-8x)^{-3} = 2^{-3}\left[1-4x\right]^{-3} = \frac{1}{8}(1-4x)^{-3}$

$= \frac{1}{8}\left[1+\frac{(-3)}{1}(-4x)+\frac{(-3)(-4)}{1\times 2}(-4x)^2+\frac{(-3)(-4)(-5)}{1\times 2\times 3}(-4x)^3+O(x^4)\right]$

$= \frac{1}{8}\left[1+12x+96x^2+640x^3+O(x^4)\right]$

$= \frac{1}{8}+\frac{3}{2}x+12x^2+80x^3+O(x^4)$

(b) VALID FOR $|4x|<1$

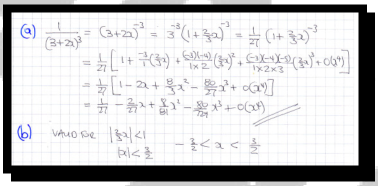
$|x|<\frac{1}{4}$ $\quad\therefore\quad -\frac{1}{4}<x<\frac{1}{4}$

Question 3

a) Expand $\dfrac{1}{(3+2x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^2-\frac{80}{729}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{3}{2}<x<\frac{3}{2}}$$



(a) $\frac{1}{(3+2x)^3} = (3+2x)^{-3} = 3^{-3}\left(1+\frac{2}{3}x\right)^{-3} = \frac{1}{27}\left(1+\frac{2}{3}x\right)^{-3}$

$= \frac{1}{27}\left[1+\frac{-3}{1}\left(\frac{2}{3}x\right)+\frac{(-3)(-4)}{1\times 2}\left(\frac{2}{3}x\right)^2+\frac{(-3)(-4)(-5)}{1\times 2\times 3}\left(\frac{2}{3}x\right)^3+O(x^4)\right]$

$= \frac{1}{27}\left[1-2x+\frac{8}{3}x^2-\frac{80}{27}x^3+O(x^4)\right]$

$= \frac{1}{27}-\frac{2}{27}x+\frac{8}{81}x^2-\frac{80}{729}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{2}{3}x\right|<1$

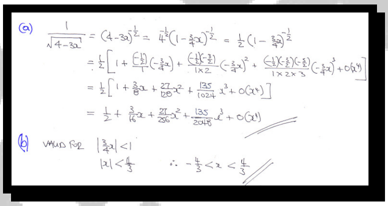
$|x|<\frac{3}{2}$ $\quad -\frac{3}{2}<x<\frac{3}{2}$

Question 4

a) Expand $\dfrac{1}{\sqrt{4-3x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\frac{135}{2048}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{3}<x<\frac{4}{3}}$$



(a) $\frac{1}{\sqrt{4-3x}} = (4-3x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left[1+\frac{\left(-\frac{1}{2}\right)}{1}\left(-\frac{3}{4}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2}\left(-\frac{3}{4}x\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}\left(-\frac{3}{4}x\right)^3+O(x^4)\right]$

$= \frac{1}{2}\left[1+\frac{3}{8}x+\frac{27}{128}x^2+\frac{135}{1024}x^3+O(x^4)\right]$

$= \frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+\frac{135}{2048}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{3}{4}x\right|<1$

$|x|<\frac{4}{3}$ $\quad \therefore -\frac{4}{3}<x<\frac{4}{3}$

Question 5

a) Expand $(2+3x)^{-3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2-\frac{135}{32}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

(a) $(2+3x)^{-3} = 2^{-3}\left(1+\frac{3}{2}x\right)^{-3} = \frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$

$= \frac{1}{8}\left[1+\frac{-3}{1}\left(\frac{3}{2}x\right)^1+\frac{-3(-4)}{1\times 2}\left(\frac{3}{2}x\right)^2+\frac{-3(-4)(-5)}{1\times 2\times 3}\left(\frac{3}{2}x\right)^3+O(x^4)\right]$

$= \frac{1}{8}\left[1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3+O(x^4)\right]$

$= \frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2-\frac{135}{32}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{3}{2}x\right|<1$

$|x|<\frac{2}{3}$ $\quad -\frac{2}{3}<x<\frac{2}{3}$

Question 6

a) Expand $\sqrt{4-2x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{1}{2}x-\frac{1}{16}x^2-\frac{1}{64}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$

(a) $\sqrt{4-2x} = (4-2x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{1}{2}x\right)^{\frac{1}{2}} = 2\left(1-\frac{1}{2}x\right)^{\frac{1}{2}}$

$= 2\left[1+\frac{\frac{1}{2}}{1}\left(-\frac{1}{2}x\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}\left(-\frac{1}{2}x\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}\left(-\frac{1}{2}x\right)^3+O(x^4)\right]$

$= 2\left[1-\frac{1}{4}x-\frac{1}{32}x^2-\frac{1}{128}x^3+O(x^4)\right]$

$= 2-\frac{1}{2}x-\frac{1}{16}x^2-\frac{1}{64}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{1}{2}x\right|<1$

$|x|<2$ $\quad \therefore -2<x<2$

Question 7

a) Expand $\dfrac{1}{(2-8x)^3}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{8}+\frac{3}{2}x+12x^2+80x^3+O\left(x^4\right)}, \quad \boxed{-\frac{1}{4}<x<\frac{1}{4}}$$

(a) $\frac{1}{(2-8x)^3}=(2-8x)^{-3}=2^{-3}(1-4x)^{-3}=\frac{1}{8}(1-4x)^{-3}$

$=\frac{1}{8}\left[1+\frac{-3}{1}(-4x)^1+\frac{(-3)(-4)}{1\times 2}(-4x)^2+\frac{(-3)(-4)(-5)}{1\times 2\times 3}(-4x)^3+O(x^4)\right]$

$=\frac{1}{8}\left[1+12x+96x^2+640x^3+O(x^4)\right)$

$=\frac{1}{8}+\frac{3}{2}x+12x^2+80x^3+O(x^4)$

(b) VALID $|4x|<1$

$|x|<\frac{1}{4}$ $\quad\therefore\ -\frac{1}{4}<x<\frac{1}{4}$

Question 8

a) Expand $\sqrt{4-6x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{3}{2}x-\frac{9}{16}x^2-\frac{27}{64}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

(a) $\sqrt{4-6x}=(4-6x)^{\frac{1}{2}}=4^{\frac{1}{2}}(1-\frac{3}{2}x)^{\frac{1}{2}}=2(1-\frac{3}{2}x)^{\frac{1}{2}}$

$=2\left[1+\frac{\frac{1}{2}}{1}(-\frac{3}{2}x)+\frac{\frac{1}{2}(-\frac{1}{2})}{1\times 2}(-\frac{3}{2}x)^2+\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1\times 2\times 3}(-\frac{3}{2}x)^3+O(x^4)\right]$

$=2\left[1-\frac{3}{4}x-\frac{9}{32}x^2-\frac{27}{128}x^3+O(x^4)\right]$

$=2-\frac{3}{2}x-\frac{9}{16}x^2-\frac{27}{64}x^3+O(x^4)$

(b) VALID FOR $|\frac{3}{2}x|<1$

$|x|<\frac{2}{3}$ $\quad\therefore\ -\frac{2}{3}<x<\frac{2}{3}$

Question 9

a) Expand $\dfrac{1}{\sqrt{4-5x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\frac{625}{2048}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{4}{5}<x<\frac{4}{5}}$$

(a) $\frac{1}{\sqrt{4-5x}} = (4-5x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{5}{4}x\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{5}{4}x\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left[1+\frac{-\frac{1}{2}}{1}\left(-\frac{5}{4}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2}\left(-\frac{5}{4}x\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3}\left(-\frac{5}{4}x\right)^3+O(x^4)\right]$

$= \frac{1}{2}\left[1+\frac{5}{8}x+\frac{75}{128}x^2+\frac{625}{1024}x^3+O(x^4)\right]$

$= \frac{1}{2}+\frac{5}{16}x+\frac{75}{256}x^2+\frac{625}{2048}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{5}{4}x\right|<1$

$|x|<\frac{4}{5}$ $\quad\therefore\ -\frac{4}{5}<x<\frac{4}{5}$

Question 10

a) Expand $\dfrac{1}{(2-x)^2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{4}+\frac{1}{4}x+\frac{3}{16}x^2+\frac{1}{8}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$

(a) $\frac{1}{(2-x)^2} = (2-x)^{-2} = 2^{-2}\left(1-\frac{1}{2}x\right)^{-2} = \frac{1}{4}\left(1-\frac{1}{2}x\right)^{-2}$

$= \frac{1}{4}\left[1+\frac{-2}{1}\left(-\frac{1}{2}x\right)^1+\frac{(-2)(-3)}{1\times 2}\left(-\frac{1}{2}x\right)^2+\frac{(-2)(-3)(-4)}{1\times 2\times 3}\left(-\frac{1}{2}x\right)^3+O(x^4)\right]$

$= \frac{1}{4}\left[1+x+\frac{3}{4}x^2+\frac{1}{2}x^3+O(x^4)\right]$

$= \frac{1}{4}+\frac{1}{4}x+\frac{3}{16}x^2+\frac{1}{8}x^3+O(x^4)$

(b) $\left|\frac{1}{2}x\right|<1$

$|x|<2$ $\quad -2<x<2$

Question 11

a) Expand $\sqrt{4-x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2-\frac{1}{4}x-\frac{1}{64}x^2-\frac{1}{512}x^3+O\left(x^4\right)}, \quad \boxed{-4<x<4}$$

(a) $\sqrt{4-x} = (4-x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$

$= 2\left[1+\frac{\frac{1}{2}}{1}\left(-\frac{1}{4}x\right)^1+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}\left(-\frac{1}{4}x\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}\left(-\frac{1}{4}x\right)^3+O(x^4)\right]$

$= 2\left[1-\frac{1}{8}x-\frac{1}{128}x^2-\frac{1}{1024}x^3+O(x^4)\right]$

$= 2-\frac{1}{4}x-\frac{1}{64}x^2-\frac{1}{512}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{1}{4}x\right|<1$

$|x|<4$ $\quad\therefore -4<x<4$

Question 12

a) Expand $\dfrac{1}{2-3x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{2}+\frac{3}{4}x+\frac{9}{8}x^2+\frac{27}{16}x^3+O\left(x^4\right)}, \quad \boxed{-\frac{2}{3}<x<\frac{2}{3}}$$

(a) $\frac{1}{2-3x} = (2-3x)^{-1} = 2^{-1}\left(1-\frac{3}{2}x\right)^{-1} = \frac{1}{2}\left(1-\frac{3}{2}x\right)^{-1}$

$= \frac{1}{2}\left[1+\frac{-1}{1}\left(-\frac{3}{2}x\right)+\frac{(-1)(-2)}{1\times 2}\left(-\frac{3}{2}x\right)^2+\frac{(-1)(-2)(-3)}{1\times 2\times 3}\left(-\frac{3}{2}x\right)^3+O(x^4)\right]$

$= \frac{1}{2}\left[1+\frac{3}{2}x+\frac{9}{4}x^2+\frac{27}{8}x^3+O(x^4)\right]$

$= \frac{1}{2}+\frac{3}{4}x+\frac{9}{8}x^2+\frac{27}{16}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{3}{2}x\right|<1$

$|x|<\frac{2}{3}$ $\quad\therefore -\frac{2}{3}<x<\frac{2}{3}$

Question 13

a) Expand $\dfrac{1}{\sqrt{25-x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{5}+\frac{1}{250}x+\frac{3}{25000}x^2+\frac{1}{250000}x^3+O\left(x^4\right)}, \quad \boxed{-25<x<25}$$

(a) $\dfrac{1}{\sqrt{25-x}}=(25-x)^{-\frac{1}{2}}=25^{-\frac{1}{2}}\left(1-\frac{1}{25}x\right)=\frac{1}{5}\left(1-\frac{1}{25}x\right)^{-\frac{1}{2}}$

$=\frac{1}{5}\left[1+\frac{-\frac{1}{2}}{1}\left(-\frac{1}{25}x\right)^1+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{1\times2}\left(-\frac{1}{25}x\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times2\times3}\left(-\frac{1}{25}x\right)^3+O(x^4)\right]$

$=\frac{1}{5}\left[1+\frac{1}{50}x+\frac{3}{5000}x^2+\frac{1}{50000}x^3+O(x^4)\right]$

$=\frac{1}{5}+\frac{1}{250}x+\frac{3}{25000}x^2+\frac{1}{250000}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{1}{25}x\right|<1$

$|x|<25$ $\quad$ ∴ $-25<x<25$

Question 14

a) Expand $\sqrt[3]{8+24x}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

$$\boxed{2+2x-2x^2+\frac{10}{3}x^3+O\left(x^4\right)}, \quad \boxed{-2<x<2}$$

(a) $\sqrt[3]{8+24x}=(8+24x)^{\frac{1}{3}}=8^{\frac{1}{3}}(1+3x)^{\frac{1}{3}}=2(1+3x)^{\frac{1}{3}}$

$=2\left[1+\frac{\frac{1}{3}}{1}(3x)^1+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{1\times2}(3x)^2+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{1\times2\times3}(3x)^3+O(x^4)\right]$

$=2\left[1+x-\frac{1}{9}(9x^2)+\frac{5}{81}(27x^3)+O(x^4)\right]$

$=2\left[1+x-x^2+\frac{5}{3}x^3+O(x^4)\right]$

$=2+2x-2x^2+\frac{10}{3}x^3+O(x^4)$

(b) VALID FOR $|3x|<1$

$|x|<\frac{1}{3}$ $\quad$ ∴ $-\frac{1}{3}<x<\frac{1}{3}$

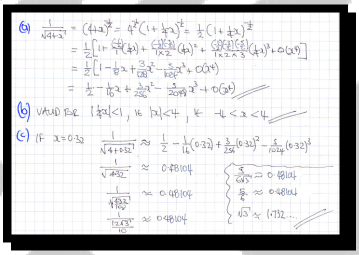
Question 15

a) Expand $\dfrac{1}{\sqrt{4+x}}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) By substituting $x = 0.32$ into the expansion show that $\sqrt{3} \approx 1.732$.

$$\boxed{\frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + O\left(x^4\right)}, \quad \boxed{-4 < x < 4}$$

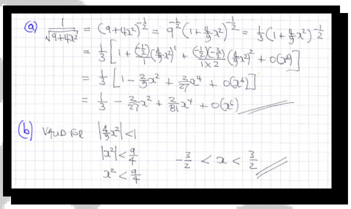


Question 16

a) Expand $\dfrac{1}{\sqrt{9+4x^2}}$ as an infinite convergent binomial series, up and including the term in x^4.

b) State the range of values of x for which the expansion is valid.

$$\boxed{\frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4 + O\left(x^6\right)}, \quad \boxed{-\frac{3}{2} < x < \frac{3}{2}}$$



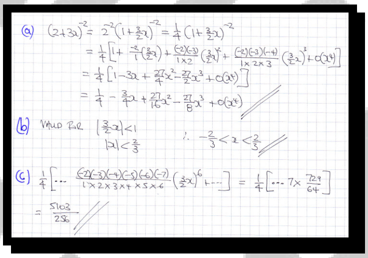
Question 17

a) Expand $(2+3x)^{-2}$ as an infinite convergent binomial series, up and including the term in x^3.

b) State the range of values of x for which the expansion is valid.

c) Find the coefficient of x^6 in the above expansion.

$$\boxed{\frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2-\frac{27}{8}x^3+O\left(x^4\right)},\ \boxed{-\frac{2}{3}<x<\frac{2}{3}},\ \boxed{\frac{5103}{256}}$$



(a) $(2+3x)^{-2} = 2^{-2}\left(1+\frac{3}{2}x\right)^{-2} = \frac{1}{4}\left(1+\frac{3}{2}x\right)^{-2}$

$= \frac{1}{4}\left[1+\frac{-2}{1}\left(\frac{3}{2}x\right)+\frac{(-2)(-3)}{1\times2}\left(\frac{3}{2}x\right)^2+\frac{(-2)(-3)(-4)}{1\times2\times3}\left(\frac{3}{2}x\right)^3+O(x^4)\right]$

$= \frac{1}{4}\left[1-3x+\frac{27}{4}x^2-\frac{27}{2}x^3+O(x^4)\right]$

$= \frac{1}{4}-\frac{3}{4}x+\frac{27}{16}x^2-\frac{27}{8}x^3+O(x^4)$

(b) VALID FOR $\left|\frac{3}{2}x\right|<1$

$|x|<\frac{2}{3}$ $\quad\therefore\ -\frac{2}{3}<x<\frac{2}{3}$

(c) $\frac{1}{4}\left[\cdots\frac{(-2)(-3)(-4)(-5)(-6)(-7)}{1\times2\times3\times4\times5\times6}\left(\frac{3}{2}x\right)^6+\cdots\right] = \frac{1}{4}\left[\cdots 7\times\frac{729}{64}\right]$

$= \frac{5103}{256}$