DIFFERENTIATION PRACTICE

THE CHAIN RULE WITH ALGEBRAIC FUNCTIONS

Question 1

1. $y=(2x+1)^4$
2. $y=(3x-2)^6$
3. $y=(8x-1)^{-1}$
4. $y=(6x+1)^{\frac{1}{2}}$
5. $y=(4x+3)^{\frac{3}{2}}$
6. $y=(1-x)^4$
7. $y=2(5x-2)^3$
8. $y=2(3x+1)^{\frac{1}{2}}$
9. $y=4(1-5x)^{-\frac{1}{2}}$
10. $y=6(1-2x)^{\frac{1}{3}}$
11. $y=(3x^2+1)^5$
12. $y=(2x^2-3x)^4$
13. $y=(5-2x+x^2)^{-2}$
14. $y=\dfrac{4}{(5x+9)^3}$

15. $y = \dfrac{3}{\sqrt{3-2x}}$

16. $y = \dfrac{1}{4x+1}$

17. $y = \dfrac{2}{3(2x+7)^2}$

18. $y = \dfrac{3}{x^2+1}$

19. $y = \dfrac{4}{\sqrt{4-3x^2}}$

20. $y = \dfrac{1}{1-x^3}$

Question 2

1. $y=(4x+1)^3$
2. $y=(2x-1)^7$
3. $y=(6x-5)^{-1}$
4. $y=(4x+1)^{\frac{1}{2}}$
5. $y=(6x-3)^{\frac{5}{2}}$
6. $y=(1-2x)^8$
7. $y=4(2x-3)^5$
8. $y=3(6x-1)^{\frac{1}{2}}$
9. $y=6(1-3x)^{-\frac{1}{2}}$
10. $y=9(1-5x)^{\frac{1}{3}}$
11. $y=(2x^2-1)^5$
12. $y=(3x^2-4x)^3$
13. $y=(1-4x+x^2)^{-2}$
14. $y=\dfrac{2}{(4x-1)^3}$
15. $y=\dfrac{2}{\sqrt{5-2x}}$

16. $y = \dfrac{3}{2x+1}$

17. $y = \dfrac{3}{2(4x+1)^2}$

18. $y = \dfrac{4}{x^2-3}$

19. $y = \dfrac{5}{6\sqrt{1-3x^2}}$

20. $y = \dfrac{4}{4-x^3}$

Question 3

1. $y=(6x-5)^3$
2. $y=(1-3x)^6$
3. $y=(2x-5)^{-2}$
4. $y=(8x+3)^{\frac{1}{2}}$
5. $y=3(4x-1)^{\frac{5}{2}}$
6. $y=4\left(1-\frac{1}{2}x\right)^6$
7. $y=8\left(\frac{15}{16}x-3\right)^{1.2}$
8. $y=5\left(\frac{3}{5}x-1\right)^{\frac{1}{3}}$
9. $y=2\left(1-\frac{5}{2}x\right)^{1.4}$
10. $y=\dfrac{2}{4x-1}$
11. $y=\left(4x^2-3\right)^3$
12. $y=\left(4x^2+2x\right)^5$
13. $y=\left(5-4x-2x^2\right)^{-3}$
14. $y=\dfrac{5}{\left(4x^2-3\right)^2}$
15. $y=\dfrac{2}{x^4+1}$

16. $y = \dfrac{5}{2\sqrt{2x+1}}$

17. $y = \dfrac{5}{6\sqrt{1-x^3}}$

18. $y = \dfrac{4}{1+2\sqrt{x}}$

19. $y = \sqrt{2+\sqrt{x}}$

20. $y = \sqrt{3x+(2x+1)^4}$

THE CHAIN RULE WITH EXPONENTIALS & LOGS

Question 4

1. $y = \mathrm{e}^{2x}$
2. $y = \ln 2x$
3. $y = \mathrm{e}^{4x-1}$
4. $y = \ln(3x-4)$
5. $y = \mathrm{e}^{-3x}$
6. $y = \ln(x^2-4)$
7. $y = 3\mathrm{e}^{x^2}$
8. $y = (\ln x)^4$
9. $y = (1+\mathrm{e}^x)^4$
10. $y = \ln x^4$
11. $y = 3(1+\mathrm{e}^{2x})^3$
12. $y = (2x+\ln x)^4$
13. $y = (4+\mathrm{e}^{-5x})^4$
14. $y = (x^3-3\ln x)^5$
15. $y = (x^2+\mathrm{e}^{2x})^4$

16. $y = \sqrt{5x - 3\ln x}$

17. $y = \sqrt{1 + \mathrm{e}^{2x}}$

18. $y = \ln\left(\mathrm{e}^{2x} + 3\right)$

19. $y = \mathrm{e}^{(2x+1)^5}$

20. $y = \ln\left(\mathrm{e}^{4} + 1\right)$

Question 5

1. $y = \mathrm{e}^{4x}$
2. $y = \ln 4x$
3. $y = \mathrm{e}^{2x-5}$
4. $y = \ln(5x-4)$
5. $y = 3\mathrm{e}^{-2x}$
6. $y = 3\ln\left(x^2+1\right)$
7. $y = 2\mathrm{e}^{-x^2}$
8. $y = 2(\ln x)^3$
9. $y = 2\left(1+\mathrm{e}^{x}\right)^3$
10. $y = 2\ln x^{\frac{1}{2}}$
11. $y = 4\sqrt{2+\mathrm{e}^{-2x}}$
12. $y = \dfrac{3}{2x-\ln x}$
13. $y = 10\left(x^2+\mathrm{e}^{-\frac{1}{2}x}\right)^3$
14. $y = \dfrac{1}{2}\left(\mathrm{e}^{3x}-3\ln x\right)^{\frac{4}{3}}$
15. $y = \left(\mathrm{e}^{2x}+\mathrm{e}^{-2x}\right)^4$
16. $y = \dfrac{2}{\sqrt{4x-\ln x}}$

17. $y=\sqrt{1+2\mathrm{e}^{2x^2}}$

18. $y=\ln\left(\ln x\right)$

19. $y=\mathrm{e}^{\mathrm{e}^{x}}$

20. $y=\ln\left(\mathrm{e}^{4+\ln 2}+1\right)$

Question 6

1. $y = \mathrm{e}^{3x}$
2. $y = \ln 5x$
3. $y = \mathrm{e}^{3x+2}$
4. $y = \ln\left(2x+7\right)$
5. $y = 4\mathrm{e}^{-5x}$
6. $y = 2\ln\left(4x^2-3\right)$
7. $y = \mathrm{e}^{3x} - 3\mathrm{e}^{-x^2}$
8. $y = \ln x^5 + \left(\ln x\right)^5$
9. $y = 3\left(2-\mathrm{e}^{x}\right)^4$
10. $y = \ln\sqrt{x} + \sqrt{\ln x}$
11. $y = 3\sqrt{1+\mathrm{e}^{-3x}}$
12. $y = \dfrac{1}{x^3 + \ln x}$
13. $y = 10\left(x^4 - 2\mathrm{e}^{-\frac{1}{2}x}\right)^4$
14. $y = \dfrac{1}{3}\left(\mathrm{e}^{6x} + 3\ln x\right)^{\frac{1}{3}}$
15. $y = \left(2\mathrm{e}^{3x} - 3\mathrm{e}^{-2x}\right)^3$
16. $y = \dfrac{4}{\sqrt{2x - 3\ln x}}$

17. $y=\sqrt{1-2\mathrm{e}^{-3x^2}}$

18. $y=\ln\left(\ln\left(\ln x\right)\right)$

19. $y=2\mathrm{e}^{2\mathrm{e}^{2x}}$

20. $y=3\ln\left(\mathrm{e}^{\sqrt{5}+\ln 2}-\ln 3\right)$

THE CHAIN RULE WITH SINES & COSINES

Question 7

1. $y = \sin 4x$
2. $y = 2\sin 3x$
3. $y = \cos 3x$
4. $y = 6\cos\left(\frac{2}{3}x\right)$
5. $y = 4\sin\left(\frac{x}{2}\right)$
6. $y = 3\sin(5x-1)$
7. $y = 3\cos\left(2x-\frac{\pi}{3}\right)$
8. $y = 2\cos\left(\frac{\pi}{4}-2x\right)$
9. $y = 6\cos\left(1-\frac{3x}{2}\right)$
10. $y = 2\sin x^4$
11. $y = 2\sin^4 x$
12. $y = 4\cos x^3$
13. $y = 4\cos^3 x$
14. $y = 3\sin^5 x$
15. $y = 2\cos\sqrt{x}$
16. $y = 2\sin^3 2x$

17. $y = 2(3\cos 2x + 1)^4$

18. $y = \sqrt{1 - 2\cos x}$

19. $y = (2\sin 3x - 3\cos 2x)^3$

20. $y = \sin^3\left(\frac{\pi}{2}\right)$

Question 8

1. $y = \sin 5x$
2. $y = 4\sin 2x$
3. $y = \cos 4x$
4. $y = 8\cos\left(\frac{1}{2}x\right)$
5. $y = 4\sin\left(\frac{x}{4}\right)$
6. $y = 2\sin(3x-2)$
7. $y = 5\cos\left(3x-\frac{\pi}{4}\right)$
8. $y = 4\cos\left(\frac{\pi}{2}-3x\right)$
9. $y = 8\cos\left(3-\frac{5x}{2}\right)$
10. $y = \frac{1}{2}\sin x^6$
11. $y = \frac{1}{2}\sin^6 x$
12. $y = 10\cos x^5$
13. $y = 10\cos^5 x$
14. $y = 3\sin^7 x$
15. $y = 2\cos\left(x^{\frac{3}{2}}\right)$
16. $y = \frac{1}{6}\sin^4 3x$
17. $y = \frac{1}{3}(2\sin 3x+3)^5$

18. $y = \sqrt{1-\cos 6x}$

19. $y = \left(4\sin 3x - 3\cos 4x\right)^3$

20. $y = \sin\left(\frac{\pi}{4}\right)$

THE CHAIN RULE WITH TAN, COT, SEC, COSEC

Question 9

1. $y = 4\tan 3x$
2. $y = 2\tan\left(2x + \frac{\pi}{4}\right)$
3. $y = 3\tan^4 x$
4. $y = 3\tan 2x$
5. $y = 12\tan\left(\frac{\pi x}{4}\right)$
6. $y = \cot 2x$
7. $y = 3\tan 2x - \cot 3x$
8. $y = 4\sec 2x$
9. $y = 2\operatorname{cosec} 3x$
10. $y = 4\sec\frac{x}{2} - 6\operatorname{cosec}\frac{2x}{3}$
11. $y = 2\cot 4x - 2\sec 3x$
12. $y = 3\tan 5x - 6\operatorname{cosec} 2x$
13. $y = \tan^6 x$
14. $y = 3\cot^4 x$
15. $y = 3\sec^2 x$
16. $y = 4\operatorname{cosec}^4 x$

17. $y = 2\tan^4 3x$

18. $y = 2\cot^2 4x$

19. $y = 4\sec^4 2x$

20. $y = 6\operatorname{cosec}^3\left(\frac{x}{2}\right)$

Question 10

1. $y = 3\tan 5x$
2. $y = 4\tan\left(3x + \frac{\pi}{3}\right)$
3. $y = \frac{1}{2}\tan^6 x$
4. $y = 10\tan\frac{1}{2}x$
5. $y = 12\tan\left(\frac{5\pi x}{6}\right)$
6. $y = \cot 7x$
7. $y = 4\tan 3x - 2\cot 2x$
8. $y = 3\sec 4x$
9. $y = 6\operatorname{cosec} 2x$
10. $y = 6\sec\frac{x}{3} - 4\operatorname{cosec}\frac{3x}{4}$
11. $y = 7\cot 2x - 3\sec 3x$
12. $y = 2\tan 7x - 7\operatorname{cosec} 2x$
13. $y = \tan^3 x$
14. $y = 8\cot^5 x$
15. $y = \frac{1}{2}\sec^4 x$
16. $y = \frac{3}{4}\operatorname{cosec}^6 x$
17. $y = 2\tan^6 2x$
18. $y = 2\cot^3 3x$

19. $y = 3\sec^3 3x$

20. $y = 12\operatorname{cosec}^3\left(\frac{x}{4}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS

Question 11

1. $y = \sin 2x$
2. $y = 3\cos 2x$
3. $y = 4\tan 3x$
4. $y = 6\sin\left(\frac{1}{2}x\right)$
5. $y = 3\cos\left(\frac{x}{3}\right)$
6. $y = 2\sin(3x-1)$
7. $y = 2\cos\left(4x-\frac{\pi}{3}\right)$
8. $y = 2\tan\left(2x+\frac{\pi}{4}\right)$
9. $y = 9\cos\left(\frac{\pi}{6}-3x\right)$
10. $y = 2\sin x^3$
11. $y = 2\sin^3 x$
12. $y = 4\cos x^2$
13. $y = 5\cos^2 x$
14. $y = 3\tan^4 x$

15. $y = 4\sin\sqrt{x}$

16. $y = \sin^5 2x$

17. $y = (3\sin x + 2)^4$

18. $y = \sqrt{1+4\sin x}$

19. $y = (\sin x - \cos x)^3$

20. $y = \sin^3\left(\frac{\pi}{6}\right)$

Question 12

1. $y = 3\sin 3x$
2. $y = 2\cos 4x$
3. $y = 3\tan 2x$
4. $y = 4\sin\left(\frac{3}{2}x\right)$
5. $y = 2\cos\left(\frac{x}{4}\right)$
6. $y = \frac{2}{3}\sin(6x-5)$
7. $y = 3\cos\left(\frac{\pi}{4}-4x\right)$
8. $y = 12\tan\left(\frac{\pi x}{4}\right)$
9. $y = \frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}-\sqrt{3}x\right)$
10. $y = \sin x^4$
11. $y = \sin^4 x$
12. $y = \cos^3 x$
13. $y = 2\sin^5 x$
14. $y = \tan^6 x$
15. $y = 4\cos^2 x$
16. $y = 3\cot^4 x$

17. $y = 3\sec^2 x$

18. $y = 4\operatorname{cosec}^4 x$

19. $y = 2\sin^5 2x$

20. $y = 4\cos^3 2x$

21. $y = 2\tan^4 3x$

22. $y = 2\cot^2 4x$

23. $y = 4\sec^4 2x$

24. $y = 6\operatorname{cosec}^3\left(\frac{x}{2}\right)$

THE CHAIN RULE WITH TRIGONOMETRIC FUNCTIONS, EXPONENTIALS AND LOGARITHMS

Question 13

1. $y = 4\mathrm{e}^{\sin x}$
2. $y = \sin\left(\mathrm{e}^{2x}\right)$
3. $y = \ln\left(\sin x\right)$
4. $y = \sin\left(\ln x\right)$
5. $y = \mathrm{e}^{2\tan x}$
6. $y = \tan\left(\mathrm{e}^{-x}\right)$
7. $y = \cos\left(3\ln x\right)$
8. $y = 2\mathrm{e}^{\cos 2x}$
9. $y = \sin^4\left(\mathrm{e}^{x}\right)$
10. $y = \mathrm{e}^{\sin^2 x}$

MIXED CHAIN RULE

Question 14

1. $y=(3x+1)^8$
2. $y=\ln 2x$
3. $y=3\sin 2x+2\cos 3x$
4. $y=\mathrm{e}^{3-2x}$
5. $y=\ln\left(x^2+1\right)$
6. $y=\dfrac{4}{(2x-1)^2}$
7. $y=4\cot 3x$
8. $y=\ln\left(4-x^3\right)$
9. $y=\tan\left(2x^2+3\right)$
10. $y=2\sin 4x-3\cos 2x$
11. $y=2\mathrm{e}^{3x}$
12. $y=\ln(\sin x)$
13. $y=\cos(\ln x)$
14. $y=\mathrm{e}^{\sin x}$
15. $y=4\cos 3x-2\sin 4x$

16. $y=\dfrac{3}{(4x-2)^3}$

17. $y=(3-6x)^{\frac{5}{2}}$

18. $y=\ln\left(2x^2+3x-1\right)$

19. $y=\mathrm{e}^{x^2}$

20. $y=\sin\left(x^2\right)$

21. $y=\sin^3 x$

22. $y=\cos\left(x^2-1\right)$

23. $y=3\cot 4x$

24. $y=\mathrm{e}^{\tan x}$

25. $y=\left(\mathrm{e}^{2x}+2\right)^3$

26. $y=3(2x+1)^6$

27. $y=3\ln 4x$

28. $y=4\sin 3x-3\cos 2x$

29. $y=4\mathrm{e}^{1-4x}$

30. $y=3\ln\left(2x^2+1\right)$

31. $y=\dfrac{4}{\sqrt{2x-1}}$

32. $y=4\tan 2x$

33. $y=\ln(\sin 2x)$

34. $y = \tan^4 x$

35. $y = 2\sin\left(\frac{1}{2}x\right) - 3\cos\left(\frac{2}{3}x\right)$

36. $y = 2\mathrm{e}^{x^2}$

37. $y = \ln(3\sin 2x)$

38. $y = 2\cos(2\ln x)$

39. $y = 2\mathrm{e}^{\sin 3x}$

40. $y = 2\sin^4 3x$

41. $y = \frac{2}{(2x-1)^4}$

42. $y = \left(3 - 6\mathrm{e}^x\right)^{\frac{3}{2}}$

43. $y = \ln(\sec x + \tan x)$

44. $y = 2\mathrm{e}^{-x^4}$

45. $y = 4\sin\left(\sqrt{x}\right)$

46. $y = 4\sin^3 2x$

47. $y = \cos\left(\mathrm{e}^{2x} - 1\right)$

48. $y = 6\tan 2x - 2\cot 3x$

49. $y = \mathrm{e}^{4\tan^2 x}$

50. $y = \ln(\cos 3x)$

THE PRODUCT RULE

Question 15

1. $y = x \sin x$
2. $y = 4x^2 \cos x$
3. $y = x^4 \, \mathrm{e}^x$
4. $y = x^3 \, \mathrm{e}^{-2x}$
5. $y = x^2 (2x-1)^5$
6. $y = 2\mathrm{e}^{3x} (3x-1)^4$
7. $y = 3\mathrm{e}^{-4x} \sin 2x$
8. $y = \cos 2x \tan 2x$
9. $y = (2x-1)^4 \sqrt{x}$
10. $y = (2x+1)^{\frac{3}{2}} (6x-1)^{\frac{1}{2}}$

Question 16

1. $y = x\cos x$
2. $y = 2x^4 \sin x$
3. $y = x^3 \mathrm{e}^{-x}$
4. $y = 4x\mathrm{e}^{2x}$
5. $y = x^2 (3x-1)^4$
6. $y = 3\mathrm{e}^{-2x} (2x-1)^3$
7. $y = \mathrm{e}^{-2x} \tan 2x$
8. $y = \sin 2x \cot 2x$
9. $y = 2(1-4x)^3 \sqrt{x}$
10. $y = (6x+1)^{\frac{1}{2}} (2x-1)^{-\frac{1}{2}}$

MIXED PRODUCT RULE

Question 17

1. $y = x^4 (4x-1)^3$
2. $y = 2x^3 (2x+3)^5$
3. $y = 6x^{\frac{1}{2}} (2x-1)^4$
4. $y = \mathrm{e}^{3x} \cos x$
5. $y = x^2 \mathrm{e}^{4x}$
6. $y = (4x+1)\mathrm{e}^{2x}$
7. $y = x^2 \tan x$
8. $y = 3x^2 \sin 2x$
9. $y = x^3 \tan 2x$
10. $y = x^4 \ln x$
11. $y = \mathrm{e}^{4x} \cos x$
12. $y = 4x^{\frac{1}{2}} \ln x$
13. $y = 4\mathrm{e}^{-x} \tan 2x$
14. $y = \mathrm{e}^{2x} (4\sin 2x + 3\cos 2x)$
15. $y = (2x+1)\cot 4x$
16. $y = (3x^2 - 4x)\tan 2x$

17. $y = x^4 \sin^2 x$

18. $y = 3x^2 \sec 2x$

19. $y = (4x+5)^{\frac{3}{2}} \mathrm{e}^{-2x}$

20. $y = x^3 \left(\sin 2x - 3\cos 2x\right)$

21. $y = \mathrm{e}^{6x} \cos^3 x$

22. $y = \sin x \tan^2 x$

23. $y = 4x^3 \operatorname{cosec} 3x$

24. $y = 3\mathrm{e}^{-4x} \cot 6x$

25. $y = 4x^{\frac{5}{2}} \sin^5 x$

26. $y = 5x \ln\left(x^2 - 2\right)$

27. $y = \sin x \tan x$

28. $y = x^2 \sin 4x$

29. $y = \mathrm{e}^{2x} \cos 3x$

30. $y = \left(4x-1\right)\mathrm{e}^{-x}$

31. $y = x^3 \tan 2x$

32. $y = x^4 \left(4x-1\right)^3$

33. $y = \left(3-2x^2\right)\cos 2x$

34. $y = \left(3x-1\right)^{\frac{1}{2}} \mathrm{e}^{x}$

35. $y = 2x^4 \ln x$

36. $y = \mathrm{e}^{x}\left(\sin x - \cos x\right)$

37. $y = \mathrm{e}^{-2x}\left(4x-1\right)^{3}$

38. $y = \left(x^{2}-2x+1\right)\mathrm{e}^{2x}$

39. $y = \left(4x+1\right)^{3}\left(1-3x\right)^{2}$

40. $y = x^{4}\sqrt{4x-1}$

41. $y = x\sin^{2} x$

42. $y = x^{-2}\ln x$

43. $y = \operatorname{cosec} x \cot x$

44. $y = \sqrt{x}\sin 2x$

45. $y = \left(4x+3\right)\tan 2x$

46. $y = \left(3x-1\right)^{5}\left(2x+1\right)^{\frac{3}{2}}$

47. $y = x^{2}\sin^{4} x$

48. $y = 6\mathrm{e}^{2x}\cos^{3} x$

49. $y = x^{2}\left(\mathrm{e}^{x}+\mathrm{e}^{-x}\right)$

50. $y = \left(1-2x\right)^{\frac{3}{2}}\left(3x+1\right)^{-\frac{1}{2}}$

51. $y = x^{5}\sqrt{x^{2}-1}$

THE QUOTIENT RULE

Question 18

1. $y = \dfrac{2x-5}{3x-1}$

2. $y = \dfrac{4x-1}{1-5x}$

3. $y = \dfrac{2x^2+1}{3x^2-1}$

4. $y = \dfrac{\ln x}{x}$

5. $y = \dfrac{\sin x}{x}$

6. $y = \dfrac{\mathrm{e}^{2x}}{x}$

7. $y = \dfrac{x^2}{(3x-1)^2}$

8. $y = \dfrac{\sin x}{\cos x}$

9. $y = \dfrac{x^2-1}{2x+3}$

10. $y = \dfrac{8x^2+8x+3}{(2x+1)^2}$

Question 19

1. $y=\dfrac{4x-3}{2x-5}$

2. $y=\dfrac{3x-1}{1-2x}$

3. $y=\dfrac{5x^2+1}{2x^2-3}$

4. $y=\dfrac{\ln x}{x^3}$

5. $y=\dfrac{\sin 2x}{x^2}$

6. $y=\dfrac{\mathrm{e}^{3x}}{2x}$

7. $y=\dfrac{4x^2}{(2x+3)^2}$

8. $y=\dfrac{\cos x}{\sin x}$

9. $y=\dfrac{3x^2+2}{x-1}$

10. $y=\dfrac{3x^2-6x+4}{(x-1)^2}$

MIXED QUOTIENT RULE

Question 20

1. $y = \dfrac{4x+3}{2x-3}$

2. $y = \dfrac{3-4x}{2x+1}$

3. $y = \dfrac{2x^2+1}{3x^2+1}$

4. $y = \dfrac{1+\cos x}{1+\sin x}$

5. $y = \dfrac{\ln x}{x^2}$

6. $y = \dfrac{x^2+1}{x^2-2}$

7. $y = \dfrac{3x^2+2}{x^2+5}$

8. $y = \dfrac{1-\cos x}{1+\cos x}$

9. $y = \dfrac{\sec x}{\tan x}$

10. $y = \dfrac{\mathrm{e}^x+2}{\mathrm{e}^x-2}$

11. $y = \dfrac{2x-1}{\sqrt{x+1}}$

12. $y = \dfrac{\sin^2 x}{\tan x}$

13. $y = \dfrac{\ln x}{x^4}$

14. $y = \dfrac{\sin 2x}{x}$

15. $y = \dfrac{x^2 - 1}{\sqrt{x+1}}$

16. $y = \dfrac{3x}{(4x-2)^3}$

17. $y = \dfrac{3\mathrm{e}^x}{2\mathrm{e}^x - 1}$

18. $y = \dfrac{4x-1}{2x+1}$

19. $y = \dfrac{1-2x}{3x+2}$

20. $y = \dfrac{4x^3+1}{2x^3+1}$

21. $y = \dfrac{1-\sin x}{1+\sin x}$

22. $y = \dfrac{\ln 2x}{x^3}$

23. $y = \dfrac{2x^2+3}{x^2-1}$

24. $y = \dfrac{2x^2+3}{x+1}$

25. $y = \dfrac{1+\cos x}{1-\cos x}$

26. $y = \dfrac{\sec x}{\sin x}$

27. $y=\dfrac{\mathrm{e}^{2x}+2}{\mathrm{e}^{2x}-1}$

28. $y=\dfrac{2x-5}{\sqrt{4x+1}}$

29. $y=\dfrac{\cos^2 x}{\tan x}$

30. $y=\dfrac{\ln x}{\sqrt{x}}$

31. $y=\dfrac{\sin 2x}{x^2}$

32. $y=\dfrac{4x}{\sqrt{2x+1}}$

33. $y=\dfrac{x^3}{(2x-1)^4}$

34. $y=\dfrac{4\mathrm{e}^x}{\mathrm{e}^x+2}$

Question 21

Prove that:

1. $\frac{d}{dx}\left(\mathrm{e}^{2x}\left(4\sin 2x+3\cos 2x\right)\right)=2\mathrm{e}^{2x}\left(\sin 2x+7\cos 2x\right)$ **(**)**

2. $\frac{d}{dx}\left(\mathrm{e}^{2x}\left(x^2-4x-2\right)\right)=2\mathrm{e}^{2x}\left(x^2-3x-4\right)$ **(**)**

3. $\frac{d}{dx}\left(\frac{4x}{4x-3}\right)=-\frac{12}{\left(4x-3\right)^2}$ **(**)**

4. $\frac{d}{dx}\left(\frac{4x+3}{2x-3}\right)=-\frac{18}{\left(2x-3\right)^2}$ **(**)**

5. $\frac{d}{dx}\left(\frac{3-4x}{2x+1}\right)=-\frac{10}{\left(2x+1\right)^2}$ **(**)**

6. $\frac{d}{dx}\left(\frac{2x^2+1}{3x^2+1}\right)=-\frac{2x}{\left(3x^2+1\right)^2}$ **(**)**

7. $\frac{d}{dx}\left(\ln\left(\sec x+\tan x\right)\right)=\sec x$ **(***)**

8. $\frac{d}{dx}\left(x^4\left(4x-1\right)^3\right)=4x^3\left(7x-1\right)\left(4x-1\right)^2$ **(***)**

9. $\frac{d}{dx}\left(2x^3\left(2x+3\right)^5\right)=2x^2\left(16x+9\right)\left(2x+3\right)^4$ **(***)**

10. $\frac{d}{dx}\left(x^4\left(4x-1\right)^3\right)=4x^3\left(7x-1\right)\left(4x-1\right)^2$ **(***)**

11. $\frac{d}{dx}\left(2\mathrm{e}^{-3x}\left(2x+1\right)^{\frac{3}{2}}\right)=-12x\mathrm{e}^{-3x}\left(2x+1\right)^{\frac{1}{2}}$ **(****)**

12. $\frac{d}{dx}\left(\frac{x-4}{\sqrt{x}+2}\right)=\frac{1}{2\sqrt{x}}$ **(****+)**

13. $\dfrac{d}{dx}\left(\mathrm{e}^{-2x}\left(4x-1\right)^{3}\right)=2\left(7-4x\right)\left(4x-1\right)^{2}\mathrm{e}^{-2x}$ **(***)**

14. $\dfrac{d}{dx}\left(\left(x^{2}-2x+1\right)\mathrm{e}^{2x}\right)=2x\left(x-1\right)\mathrm{e}^{2x}$ **(***)**

15. $\dfrac{d}{dx}\left(x^{4}\sqrt{4x-1}\right)=\dfrac{2x^{3}\left(9x-2\right)}{\sqrt{4x-1}}$ **(****)**

16. $\dfrac{d}{dx}\left(4\sqrt{x}\ln x\right)=\dfrac{2\left(2+\ln x\right)}{\sqrt{x}}$ **(****)**

17. $\dfrac{d}{dx}\left(\left(4x+5\right)^{\frac{3}{2}}\mathrm{e}^{-2x}\right)=-4\left(2x+1\right)\sqrt{4x+5}\,\mathrm{e}^{-2x}$ **(****)**

18. $\dfrac{d}{dx}\left(6\sqrt{x}\left(2x-1\right)^{4}\right)=\dfrac{3\left(18x-1\right)\left(2x-1\right)^{3}}{\sqrt{x}}$ **(****)**

19. $\dfrac{d}{dx}\left(\dfrac{1-\cos x}{1+\cos x}\right)=\dfrac{2\sin x}{\left(1+\cos x\right)^{2}}$ **(***)**

20. $\dfrac{d}{dx}\left(\dfrac{3+\sin 2x}{2+\cos 2x}\right)=\dfrac{6\sin 2x+4\cos 2x+2}{\left(2+\cos 2x\right)^{2}}$ **(***)**

21. $\dfrac{d}{dx}\left(\dfrac{5x^{2}-10x+8}{\left(x-1\right)^{2}}\right)=-\dfrac{6}{\left(x-1\right)^{3}}$ **(***)**

22. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\tan x}\right)=-\operatorname{cosec}x\cot x$ **(****)**

23. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{x}+2}{\mathrm{e}^{x}-2}\right)=-\dfrac{4}{\left(\mathrm{e}^{x}-2\right)^{2}}$ **(***)**

24. $\dfrac{d}{dx}\left(\dfrac{2x+1}{\sqrt{x+1}}\right)=\dfrac{2x+3}{2\left(x+1\right)^{\frac{3}{2}}}$ **(****)**

25. $\dfrac{d}{dx}\left(\dfrac{x-1}{\sqrt{x}+1}\right)=\dfrac{1}{2\sqrt{x}}$ **(****)**

26. $\dfrac{d}{dx}\left(\dfrac{\sin^2 x}{\tan x}\right)=\cos 2x$ **(****)**

27. $\dfrac{d}{dx}\left(\dfrac{3\mathrm{e}^{x}}{2\mathrm{e}^{x}-1}\right)=-\dfrac{3\mathrm{e}^{x}}{\left(2\mathrm{e}^{x}-1\right)^2}$ **(***)**

28. $\dfrac{d}{dx}\left(\dfrac{4x-1}{2x+1}\right)=\dfrac{6}{\left(2x+1\right)^2}$ **(**)**

29. $\dfrac{d}{dx}\left(\dfrac{1-2x}{3x+2}\right)=-\dfrac{7}{\left(3x+2\right)^2}$ **(**)**

30. $\dfrac{d}{dx}\left(\dfrac{4x^3+1}{2x^3+1}\right)=\dfrac{6x^2}{\left(2x^3+1\right)^2}$ **(**)**

31. $\dfrac{d}{dx}\left(\dfrac{1-\sin x}{1+\sin x}\right)=-\dfrac{2\cos x}{\left(1+\sin x\right)^2}$ **(**)**

32. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1-\sin x}{1+\sin x}\right)\right)=-\sec x$ **(****)**

33. $\dfrac{d}{dx}\left(\ln\left(\dfrac{x+1}{x-1}\right)\right)=-\dfrac{2}{x^2-1}$ **(***)**

34. $\dfrac{d}{dx}\left(\dfrac{2\sin 3x}{1+\cos 3x}\right)=\dfrac{6}{1+\cos 3x}$ **(***)**

35. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x^2-1}\right)=-\dfrac{10x}{\left(x^2-1\right)^2}$ **(**)**

36. $\dfrac{d}{dx}\left(\dfrac{2x^2+3}{x+1}\right)=\dfrac{2x^2+4x-3}{\left(x+1\right)^2}$ **(***)**

37. $\dfrac{d}{dx}\left(\dfrac{3x^2+6x-5}{\left(x+1\right)^2}\right)=\dfrac{16}{\left(x+1\right)^3}$ **(***)**

38. $\dfrac{d}{dx}\left(\dfrac{1+\cos x}{1-\cos x}\right)=-\cot\dfrac{x}{2}\operatorname{cosec}^2\dfrac{x}{2}$ **(*****)**

39. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\sin x}\right)=\sec^2 x-\operatorname{cosec}^2 x$ **(****)**

40. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1}{1-\sin x}\right)\right)=\dfrac{1+\sin x}{\cos x}$ **(****)**

41. $\dfrac{d}{dx}\left(\dfrac{\mathrm{e}^{2x}+2}{\mathrm{e}^{2x}-1}\right)=-\dfrac{6\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^2}$ **(***)**

42. $\dfrac{d}{dx}\left(\dfrac{2x-5}{\sqrt{4x+1}}\right)=\dfrac{4(x+3)}{(4x+1)^{\frac{3}{2}}}$ **(****)**

43. $\dfrac{d}{dx}\left(\dfrac{\ln x}{\sqrt{x}}\right)=\dfrac{2-\ln x}{2x^{\frac{3}{2}}}$ **(****)**

44. $\dfrac{d}{dx}\left(\dfrac{4x}{\sqrt{2x+1}}\right)=\dfrac{4(x+1)}{(2x+1)^{\frac{3}{2}}}$ **(****)**

45. $\dfrac{d}{dx}\left(\dfrac{x^3}{(2x-1)^4}\right)=-\dfrac{x^2(2x+3)}{(2x-1)^5}$ **(****)**

46. $\dfrac{d}{dx}\left(\ln\sqrt{4x+1}\right)=\dfrac{2}{4x+1}$ **(**)**

47. $\dfrac{d}{dx}\left(\dfrac{4\mathrm{e}^x}{\mathrm{e}^x+2}\right)=\dfrac{8\mathrm{e}^x}{\left(\mathrm{e}^x+2\right)^2}$ **(***)**

48. $\dfrac{d}{dx}\left(\dfrac{\sec x}{\sin x}\right)=-4\operatorname{cosec}2x\cot 2x$ **(****)**

49. $\dfrac{d}{dx}\left(\ln\left(\tan x\right)\right)=2\operatorname{cosec}2x$ **(***)**

50. $\dfrac{d}{dx}\left(\dfrac{2x\left(x^2+6x+12\right)}{\left(x+2\right)^3}\right)=\dfrac{48}{\left(x+2\right)^4}$ (****)

51. $\dfrac{d}{dx}\left(\sqrt{\dfrac{x+1}{x-1}}\right)=-\dfrac{1}{\left(x-1\right)\sqrt{x^2-1}}$ (****)

52. $\dfrac{d}{dx}\left(\ln\left(\dfrac{1+\cos x}{1-\cos x}\right)\right)=-2\operatorname{cosec} x$ (****)

53. $\dfrac{d}{dx}\left(\cos 2x+\tan x\sin 2x\right)=0$ (*****)

54. $\dfrac{d}{dx}\left(\ln\left(\dfrac{\sqrt{\mathrm{e}^x+1}-1}{\sqrt{\mathrm{e}^x+1}+1}\right)\right)=\dfrac{1}{\sqrt{\mathrm{e}^x+1}}$ (*****)

55. $\dfrac{d}{dx}\left(\ln\left(\dfrac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\right)=\dfrac{2}{x\sqrt{1-x^2}}$ (*****)

56. $\dfrac{d}{dx}\left(\dfrac{\cos 2x}{\sqrt{1+\sin 2x}}\right)=-\sin x-\cos x$ (*****)

57. $\dfrac{d}{dx}\left(\ln\left(x+\sqrt{x^2+8}\right)-\dfrac{x}{x^2+8}\right)=\dfrac{x^2}{\left(x^2+8\right)^{\frac{3}{2}}}$ (*****)

58. $\dfrac{d}{dx}\left(\dfrac{\sqrt{\mathrm{e}^{2x}-9}}{\mathrm{e}^x}\right)=\dfrac{9}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-9}}$ (****)

59. $\dfrac{d}{dx}\left(\ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-9}\right)\right)=\dfrac{\mathrm{e}^x}{\sqrt{\mathrm{e}^{2x}-9}}$ (*****)

60. $\dfrac{d}{dx}\left(\dfrac{\sqrt{\mathrm{e}^{2x}+4}}{\mathrm{e}^x}\right)=-\dfrac{4}{\mathrm{e}^x\sqrt{\mathrm{e}^{2x}+4}}$ (****)

61. $\dfrac{d}{dx}\left(\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-1}-\ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-1}\right)\right)=2\mathrm{e}^x\sqrt{\mathrm{e}^{2x}-1}$ (*****)